Resonant Systems for Measurement of Electromagnetic Properties of Substances at V-Band Frequencies

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Additional information is available at the end of the chapter

Abstract

Hemispherical open resonator (OR) with the segment of the oversized circular waveguide is considered. The cavity is formed by cylindrical, conical and spherical surfaces, and only axial-symmetric modes are excited. The power and spectral characteristics of such cavity with a dielectric bead and a rod have been studied. The quasi-periodic behavior of those dependencies was found out. Their qualitative agreement with similar dependencies for a cylindrical cavity is shown. It was found that physical processes in the cavity and in the hemispherical OR with the segment of the oversized circular waveguide are identical. Dielectric permittivity and loss-angle tangent measurements have been carried out in millimeter wavelength range for the Teflon and Plexiglas samples having the shape of the bead as well as for the fused quartz and glass samples of the rod shape. It was found out that such a resonant system allows measuring samples with high losses, that is, especially important for quality control of food stuffs and analysis of biological liquids. Energy analysis of the OR with the segment of the oversized rectangular waveguide has been performed. Basic possibility to apply such a resonant system for measurement of the dielectric permittivity of cylindrical samples with high losses has been shown as well.

Keywords: permittivity, cavity, open resonator, circular waveguide, rectangular waveguide, oversized waveguide

1. Introduction

Measurement of electromagnetic properties of existing and novel materials in new frequency ranges is an important issue of the day. Recently emerged new class of artificial materials—composite materials (composites) are characterized by the negative refraction coefficient. Investigations showed that devices based on such materials can possess entirely unique

properties and characteristics [1]. Caused by technology progress, advance of composite materials into millimeter and especially sub-millimeter ranges requires knowing of information about their electromagnetic characteristics. On the other hand, the real part of dielectric permittivity ε' and dielectric loss-angle tangent tan δ of water, which is the main component of the whole series of food stuff and biological liquids, decrease with the wavelength shortening. Therefore, for effective control of their quality, we also should move to the specified ranges of the wavelengths. For measurement of electromagnetic characteristics of such substances, the application of the resonant techniques is needed due to their higher accuracy. The main point of such methods consists of observation of resonant curves of the oscillatory circuit, in which the sample of the studied substance is placed.

Comparing the resonance curves corresponding to the cases of resonator with and without sample allows determining both ε' and tg δ using the Q-factor and frequency shift. Open resonators (OR) are used as circuits in millimeter and sub-millimeter ranges for such measurements. The peculiarity of such resonant systems consists of the fact that, apart from high Q-factor, their geometrical dimensions account a few tens of wavelengths, and coupling with free space provides an additional mode selection and free access to the resonant volume. However, such resonant systems are applicable to use just for investigations of substances with low losses. In the case of high losses, sample thickness should not exceed the size of the skin layer since it can result in oscillation suppression in such resonant systems. This circumstance imposes limitations to the application of the OR in the research of electromagnetic characteristics of composite materials and biological liquids, quality control of food stuffs since they are characterized by high losses. Therefore, the most promising resonant system to use for investigations of such substances is the OR proposed in [2, 3]. It represents symbiosis of the OR and the segment of the oversized waveguide, which could be both circular and rectangular. A distinctive feature of such resonant systems is that they are characterized by the single frequency response in the wide range of frequencies [3]. It is an advantage at investigations of electromagnetic characteristics of substances. At placement of the sample in the waveguide part of the OR, the measurement accuracy increases due to keeping high Q-factor, and therefore, the range of the analyzed values ε' and tanδ extends.

The studied sample having the shape of a bead is located in the bottom of the circular waveguide segment, in which there is a plane wave front of the propagating TE_{01} mode. It allows measuring samples, the thickness of which exceeds the wavelength of the excited oscillation. At the research of substances with the application of the OR having a cylindrical shape, difficulties related to their positioning in resonant volume may arise. At each measurement, the samples should be placed in the area with the same electric field intensity. The proposed resonator allows solving of this problem. The sample should be located along the OR axis, where the electric field intensity of the excited oscillation is minimal. It provides analyzing of substances with high losses. In the case of the OR having the segment of the oversized rectangular waveguide with the TE_{10} mode, it is expedient to use the samples of a cylindrical shape. They should be located in the waveguide part parallel to the vector of the electric field intensity of the mode.

On the basis of the all above-stated, we can summarize that the goal of investigations, performed in this chapter, is theoretical and experimental research of the considered OR, which will allow to measure, in millimeter and in sub-millimeter ranges, the electromagnetic characteristics of composite materials and biological liquids, as well as to control the quality of food stuff.

2. Open resonator with a dielectric bead

2.1. Resonator model

In the OR, axial-symmetric modes are confined by caustics and hence they are with low diffraction losses. Placing of perfectly conducting boundary (**Figure 1**, dotted lines) in the area of exponentially vanishing intensity, almost does not affect the field pattern in OR. Our method is based on such physical principles.

Therefore, the task transforms to the study of the cavity resonator and approximate solution for the OR is achieved by selecting only modes with near axis distributed intensity (exponentially vanishing near conical boundary) from the cavity spectrum. We noticed that such approach for the electrodynamic model of the OR was proposed in [4].

Let us consider the cavity as a body of revolution with perfectly conducting boundary and dielectric bead located in the bottom of the cylindrical part (**Figure 1**). We assume that the resonator is filled with a homogeneous isotropic medium having specific dielectric and magnetic conductivities ε_1, μ_1 in the area (1) and ε_2, μ_2 in the area (2). We consider only axial-symmetric TE modes with E_φ, H_r and E_z are components of the electromagnetic field in the cylindrical coordinate system with the axis z, coinciding with the axis of symmetry.
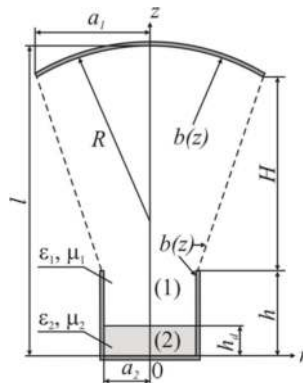

Figure 1. Geometric model of the OR with the dielectric bead.

Here, the initial problem for Maxwell equations reduces to the problem of finding the wave numbers $k = \omega/c$ (where c is the speed of the light in vacuum and ω is a circular frequency), for which exist non-zero solutions U_1 and U_2 of the two-dimensional Helmholtz's equations.

$$\Delta_{rz}U_1 + \left(k^2\varepsilon_1\mu_1 - (1/r^2)\right)U_1 = 0, \quad h_d < z < l, \ 0 < r < b(z), \tag{1}$$

$$\Delta_{rz}U_2 + \left(k^2\varepsilon_2\mu_2 - (1/r^2)\right)U_2 = 0, \quad 0 < z < h_d, \ 0 < r < b(z), \tag{2}$$

which meet boundary conditions.

$$\begin{cases} U_1(r,l) = 0, \ \ U_2(r,0) = 0, \\ U_{1,2}(b(z),z) = 0, \quad U_{1,2}(0,z) = 0, \end{cases} \tag{3}$$

and conditions of the fields matching at the medium interface $z = h_d$.

$$U_1(r,h_d) = U_2(r,h_d), \quad \frac{1}{\mu_1}\frac{\partial U_1(r,h_d)}{\partial z} = \frac{1}{\mu_2}\frac{\partial U_2(r,h_d)}{\partial z}. \tag{4}$$

Here, $U_{1,2}(r,z)$ is azimuthal component of electric field of a mode; function $b(z)$ is boundary surface, which is supposed to be piecewise-differentiable in the interval $0 < z < l$, where l is the length of the cavity; h_d is the thickness of the dielectric bead; $\Delta_{rz} = \partial^2/\partial r^2 + \partial/r\partial r + \partial^2/\partial z^2$ is the two-dimensional Laplace's operator; (r,φ,z) are cylindrical coordinate system with axis z, coinciding with the axis of the cavity symmetry.

Equations (1)-(4) with application of Bubnov-Galerkin's method reduced to the system of the linear algebraic equations.

$$\mathbf{AC} + k^2\mathbf{BC} = 0, \tag{5}$$

where $\mathbf{C} = (c_n)_{n=1}^N$ is the vector-column of unknown factors, $\mathbf{A} = \|a_{mn}\|_{m,n=1}^N$ and $\mathbf{B} = \|b_{mn}\|_{m,n=1}^N$ are matrixes with matrix elements, which are prescribed by the formulas presented in [5, 6].

An approximate solution of the initial task [Equations (1)-(4)] could be represented as follows:

$$E(r,z) = \begin{cases} \displaystyle\sum_{i=1}^I J_1\left(\frac{\mu_i r}{b}\right)\sum_{p=1}^P c_p^i \frac{\mu_1\varepsilon_2}{\mu_2\varepsilon_1}\frac{\cos\lambda_p\sqrt{\varepsilon_1\mu_1}h_d}{\cos\lambda_p\sqrt{\varepsilon_1\mu_1}(l-h_d)}\sin\lambda_p\sqrt{\varepsilon_1\mu_1}(l-z), \ h_d < z < l, \\ \displaystyle\sum_{i=1}^I J_1\left(\frac{\mu_i r}{b}\right)\sum_{p=1}^P c_p^i \sin\lambda_p\sqrt{\varepsilon_1\mu_1}z, \ 0 < z < h_d, \\ \qquad\qquad \omega = kc, \end{cases} \tag{6}$$

where k and $\mathbf{C}^N$ are solutions of the [Eq. (5)]; $\omega = \omega' + i\omega''$ is complex frequency of the natural oscillation and $N = P \cdot I$ is the dimension of the algebraic task.

For measurement of the loss-angle tangent when a sample is placed in a cavity, both the resonator frequency shift and energy characteristic of the resonator are necessary to calculate [7]. For such calculations, it is required to find the following: the resonator Q-factor Q_ε caused by losses in the sample; energy factor K_E of the resonator filling by electric field [7] and an ohmic Q-factor $Q_{R\varepsilon}$ of the resonator with the sample [8]. At the same time,

$$Q_\varepsilon = -2\omega'/\omega'', \tag{7}$$

$$K_E(\varepsilon) = -2\frac{\varepsilon_2'}{\omega'}\frac{\partial\omega'}{\partial\varepsilon_2'}, \tag{8}$$

$$Q_{R\varepsilon} = (2/\Delta_R)\left(\int_V |\mathbf{H}|^2 dV \Big/ \int_S |\mathbf{H}_\tau|^2 dS\right), \tag{9}$$

where Δ_R is depth of the electromagnetic field penetration into metal; V, S are volume and area of the resonator surface; $\mathbf{H}$ is the distribution of the magnetic components of the electromagnetic field in the volume of the resonator; $\mathbf{H}_\tau$ is the tangent component of the magnetic. Distributions of magnetic components $\mathbf{H}$ and $\mathbf{H}_\tau$ of the electromagnetic field are calculated with the application of the developed theoretical model.

2.2. Numerical results

Dimensions of the considered cavity at numerical simulations have been chosen equal to sizes of the hemispherical OR used in the experiment. The curvature radius of the spherical mirror is $R = 39$ mm, aperture of this mirror is $2a_1 = 38$ mm, the diameter of the circular waveguide segment and dielectric samples is $2a_2 = 18$ mm, the length of the resonator cylindrical part is $h = 12.434$ mm, the resonator length is $l = 35.295$ mm (**Figure 1**).

The developed algorithm was validated using rigorous formulas for the spectrum of empty spherical volume resonator [9]. Evaluation of the algorithm convergence, related to the increase of the algebraic equation dimension, was carried out as well (Eq. (5)). As a result of such evaluation matrixes' dimensions, **A** and **B** from (Eq. (5)) have been chosen to be equal to $N = P \cdot I$ at $P = 60$, $I = 6$ (Eq. (6)). Further increase of P and I does not provide considerable changes at calculation results of the eigenvalues and the eigenvectors of the task (Eq. (5)). The examples of distribution for an electric field component of the TE_{0116} mode in the cavity with the bead made of Plexiglas having thickness $h_d = 2.75$ mm and permittivity $\varepsilon_2' = 2.61$ are presented in **Figure 2a** [10]. At the same time, frequency of resonant oscillation decreases down to the value $f = 68.052$ GHz as compared to $f = 71.382$ GHz for an empty resonator with the same dimensions. For the bead made of Plexiglas having the same diameter, but thickness $h_d = 3.6$ mm, resonance frequency decreases down to the value $f = 66.999$ GHz (**Figure 2b**). As can be seen in **Figure 2a** and **b**, depending on the dielectric bead thickness, its top boundary can coincide both with the node and the antinode of the electric field component of the standing wave in the cavity. Therefore, it should be expected that in such a resonant

system, dependence of the oscillation frequency TE_{0116} on the thickness of the dielectric bead will have a quasi-periodic behavior [11].

Dependencies of the frequency shift in the cavity on the thickness of the sample located at the bottom of cylindrical part are presented in **Figure 2c**. Curve 1 in **Figure 2c** corresponds to the bead made of Teflon ($\varepsilon_2' = 2.07$ [10]), and curve 2 is for the bead made of Plexiglas. Marks (a) and (b) at the curve 2 show the beads' thicknesses, corresponding to the field distributions presented in **Figure 2a** and **b**. Apparently, the weak dependence of the frequency and the bead thickness takes place in the case, when the node of the electric field component of the standing wave in the cavity is located near to the top of the sample (**Figure 2a**, point (a) in **Figure 2c**). In the case of electric components antinode location near to the top of the sample (**Figure 2b**,

a) $h_d = 2.75$ mm, $f = 68.052$ GHz;   b) $h_d = 3.6$ mm, $f = 66.999$ GHz.

c)
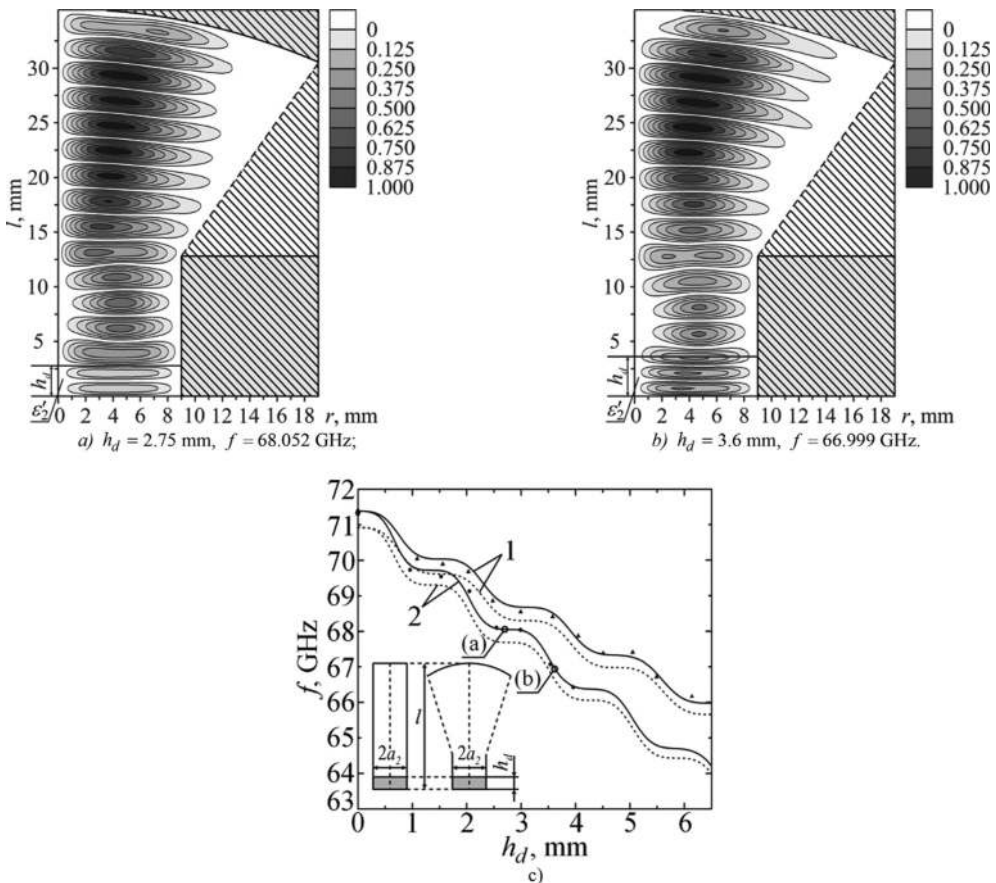
Figure 2. The examples of distribution of the electric field component of the TE_{0116} mode in the cavity with beads from Plexiglas (a, b) and dependencies of the resonance frequency on the thickness of beads made of different material for two types of cavities (c).

point (b) in **Figure 2c**), the dispersive dependence of the resonator frequency on the sample thickness has a large steepness.

Dependencies of the resonance frequency on the thickness of the dielectric beads made of Teflon (curve 1) and Plexiglas (curve 2) for cylindrical cavity are shown in **Figure 2c** by dotted curves. The diameter of this cavity is equal to the diameter of the cylindrical part of the studied cavity, and lengths of both the cavities coincide (**Figure 2c**). At the same time, a rigorous solution for a cylindrical cavity was obtained by application of the method based on the separation of variables. As it follows from **Figure 2c**, curves corresponding to both resonant systems qualitatively agree and have a quasi-periodic character. At the same time, a difference of eigen-frequencies of these cavities with beads having the same thickness and permittivity does not exceed 500 MHz.

Experimentally measured values of resonance frequency are shown in **Figure 2c** at the place-ment of beads made of Teflon (triangular marks) and Plexiglas (round marks) on the bottom of cylindrical part of the hemispherical OR [12]. The difference of experimentally obtained values of resonance frequency from calculated by using developed electrodynamic model of the OR does not exceed 50 MHz, and an error of the frequency measurement by using a resonant wavemeter in the considered frequency bandwidth is about 37 MHz [13]. With regard to the above, we can state validity of the proposed electrodynamic model of the cavity (**Figure 1**) to measure the electromagnetic parameters of substances using the OR [3].

As shown earlier (**Figure 2c**), dependencies of the resonance frequency on the bead thickness at the constant value of its permittivity have a quasi-periodic character. Dependencies of the resonance frequency on permittivity of the bead having constant thickness should look similar. It can be explained by the fact that with a change of ε'_2 the nodes and antinodes of electric component of the standing wave appear periodically on the top of the bead, and condition of equality of the bead thickness to integer number of half wavelength does not hold for any ε'_2.

Dependencies of TE_{0116} mode resonant frequency f of the considered cavity (**Figure 1**) on the permittivity of the beads having various thicknesses h_d are presented in **Figure 3a**. Curve 1 is for the bead, with thickness of 2.99 mm, and curve 2 is for the bead having thickness 3.58 mm.
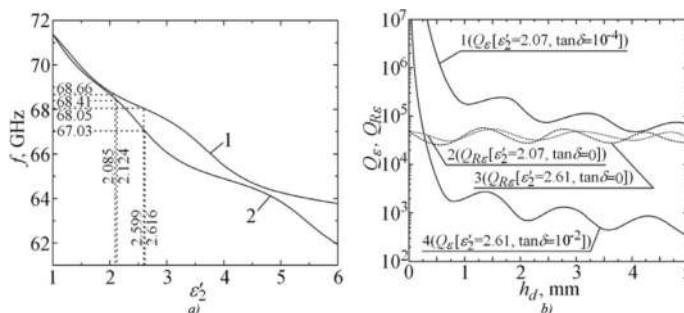
Figure 3. Dependencies of the resonant frequency on the permittivity of the beads having a various thickness (a) and of the Q-factor of the cavity on the thickness of the sample (b).

Dependencies in **Figure 3a** corresponding to the sample thickness provide defining a real part of the samples permittivity ε'_2 by resonant frequency shift.

As an important characteristic, needed for valuation of the loss-angle tangent for dielectric samples, is Q-factor Q_ε, caused by losses in a dielectric (Eq. (7)), and an ohmic Q-factor $Q_{R\varepsilon}$ of the resonator with (Eq. (9)). Dependencies of Q_ε (solid curves) and $Q_{R\varepsilon}$ (dotted curves) on the beads' thickness h_d calculated using the developed theoretical model are presented in **Figure 3b**. The Q-factor Q_ε of the cavity with the sample of the shape of a bead, permittivity of which $\varepsilon'_2 = 2.07$ and loss-angle tangent $\tan\delta = 10^{-4}$, is designated by the curve 1. The curves 2 and 3 correspond to the ohmic Q-factor of the resonator $Q_{R\varepsilon}$ with the samples having $\varepsilon'_2 = 2.07$ and $\varepsilon'_2 = 2.61$ in the absence of losses ($\tan\delta = 0$). The Q-factor Q_ε, caused by losses in the dielectric is designated by the curve 4, for the sample having parameters: $\varepsilon'_2 = 2.61$ and $\tan\delta = 10^{-2}$.

From the presented diagram one can see that behavior of the Q-factors Q_ε and $Q_{R\varepsilon}$ is quasi-periodic with the increase of the thickness h_d of the samples. Such behavior, as shown earlier, is related to distribution of the electric component of the standing wave in the resonator in the plane of the top of the samples located on the bottom of the cylindrical part. The ohmic Q-factor Q_{R0} of the "empty" resonator ($h_d = 0$) is equal to 48,005. As one can see in **Figure 3b**, the Q-factor $Q_{R\varepsilon}$ can essentially differ from the Q-factor Q_{R0}. For example, for the sample with $\varepsilon'_2 = 2.07$ and $\tan\delta = 10^{-4}$ at $h_d = 2.48$ mm, $Q_{R\varepsilon} = 33,493$, that is, the ratio $Q_{R0}/Q_{R\varepsilon} = 1.43$. Therefore, in order to understand how it can affect the calculation of $\tan\delta$, we write down an expression, determining its natural Q-factor $Q_{0\varepsilon}$ of the cavity with the sample, which expresses as $1/Q_{0\varepsilon} = (1/Q_\varepsilon) + (1/Q_{R\varepsilon})$. For samples with low losses, the Q-factor $Q_{0\varepsilon}$ is entirely determined by the resonator ohmic Q-factor $Q_{R\varepsilon}$, since at the same time, $Q_\varepsilon >> Q_{R\varepsilon}$ (**Figure 3b**, curves 1 and 2). Hence, in the calculation of the resonator Q-factor, it is inadmissible to replace the ohmic Q-factor of the resonator with the sample by the ohmic Q-factor of the "empty" resonator, as it can result in significant errors in the valuation of $\tan\delta$ for the samples having low losses ($< 10^{-4}$). Obtained conclusion is well in agreement with the results in [7].

In the investigation of the samples with high losses, a Q-factor of the resonator $Q_{0\varepsilon}$ will already be determined by the Q-factor Q_ε, since here $Q_\varepsilon << Q_{R\varepsilon}$ (**Figure 3b**, curves 3 and 4). If the sample thickness $h_d < 0.25$ mm, then, its placement on the bottom of the resonator cylindrical part can also lead to an incorrect result in measurement of dielectric samples losses, since in this case $Q_\varepsilon >> Q_{R\varepsilon}$. Just noted means that the natural Q-factor of the resonator should be defined by the Q-factor $Q_{R\varepsilon}$, instead of Q_ε (**Figure 3b**, curves 3 and 4). Hence, at the measurement of losses in thin samples ($h_d << \lambda$), they should be located not on the bottom of the cylindrical part, but at the antinode of the standing wave electric field in the resonator [7].

Here, it should be noted that in the Q-factor calculations, the conductivity of the cylindrical part surface of the considered cavity was as for aluminum, and of the spherical mirror was as for brass (**Figure 1**). Choice of the metals depends upon the necessity to carry out an evaluation of the losses in the samples of the known dielectrics using the hemispherical OR with the segment of the oversized circular waveguide, the mirrors of which are made of the specified metals. Conical cavity surface is considered as a perfectly conducting one.

Thus, analysis of the energy and spectral characteristics of the cavity with dielectric inclusions, formed by the cylindrical, conical and spherical surfaces was carried out in this subsection. As a result of the performed study, it was found out that physical processes in the considered cavity and in the hemispherical OR with the segment of the oversized circular waveguide having dielectric beads are identical. It allows to conclude that the proposed model is valid for the resonator to measure electromagnetic parameters of substances in the millimeter range of the wavelengths.

2.3. Measurement of the permittivity and losses in the samples

The hemispherical OR, formed by the spherical 13 and flat 14 mirrors having diameter 38 mm (**Figure 4**) [6, 14], is used for measurements. Short-segment of the oversized circular waveguide 15 of diameter 18 mm is located in the center of the flat mirror. The studied sample 17 having the shape of a bead is placed at the waveguide plunge 16. Distance from the flat mirror to the plunge is equal to $3\lambda_w$ (λ_w-waveguide wavelength). On the surface of the spherical mirror of curvature radius $R = 39$ mm, two coupling slots are located, through one of which the signal with 1 kHz amplitude modulation from the high-frequency generator G4–142 inputs into the resonant volume, and through the second one, signal outputs to the load. The slots are tapering from the main section of the rectangular waveguide 3.6×1.8 mm into the narrow one 3.6×0.14 mm. Both coupling elements are oriented in such a manner that the vector of TE_{10} mode electric field in the rectangular waveguides is orthogonal to the plane of **Figure 4**.
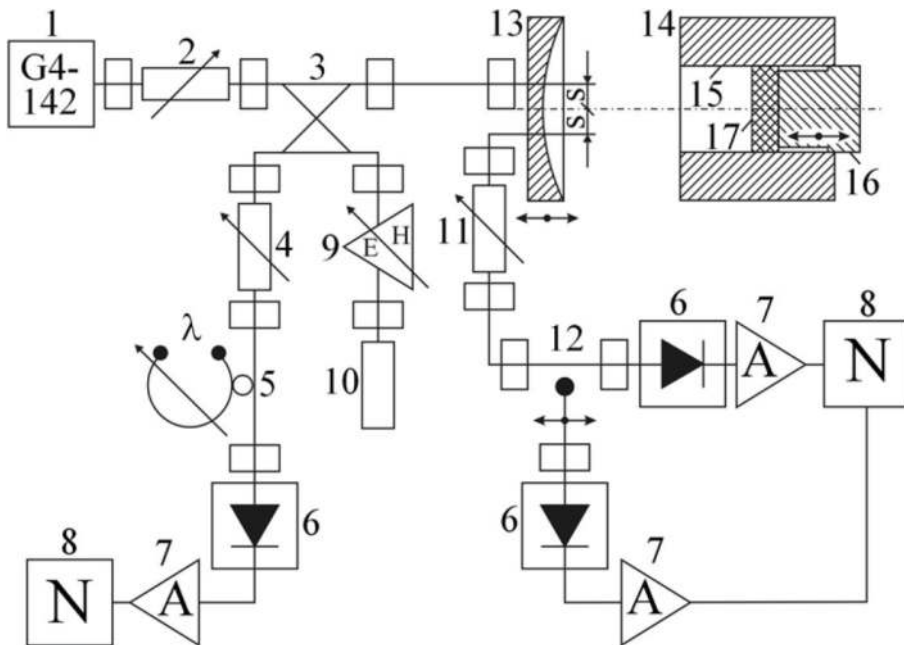

Figure 4. Block diagram of the experimental setup.

The distance s from the resonator axis, on which coupling elements are located, is defined by the peak value of the electric field of TEM_{01q} mode in the plane of the spherical mirror and is equal to 5.5 mm (**Figure 4**). In this case TE_{01q} mode is excited with maximal efficiency. For the isolation of the generator G4–142 and the resonator, additional setting attenuator 2 is included in the circuit. The tuning to the resonance is implemented by moving the spherical mirror 13 along the resonator axis. The distance between the reflectors is evaluated by using a measuring projective device having accuracy of ±0.001 mm. The signal extraction from the OR is performed by using the second slot coupling element, which, as stated earlier, is on the spherical mirror and has the same dimensions as the first one, and is located at the distance 11 mm from it.

In the circuit, an additional receiving transmission line is included for the measurement of the reflection coefficient from the resonator. This transmission line comprises a directional coupler 3, a measuring polarizing attenuator 4, a crystal detector 6, a resonant amplifier 7, tuned to the frequency of modulating voltage and an oscillograph 8. Reflectivity is measured in the plane, in which input impedance of the resonator with a certain part of the waveguide is purely active [15].

The reflectivity factor on voltage is defined by the formula $|\Gamma| = 10^{-A/20}$ [16]. Here, A is the difference in dB between data from the measuring polarizing attenuator 4 at the arrangement of the shorting plug in the coupling element plane and at the point of resonance. A resonant wavemeter 5 is included in the transmission line for more accurate measurement of the generator G4–142 frequency. A double-stub matcher 9 is placed into the branch of the directed coupler 3, containing a matched load 10, enabling compensation of the possible reflected from the waveguide junction signals, which can affect the accuracy of the resonance tuning. The signal, which passed through the OR, inputs into the receiving transmission line, consisting of a measuring polarizing attenuator 11, a measuring line 12, a detector 6, a resonant amplifier 7 and an oscillograph 8. The signal, proportional to the amplitude of the standing wave voltage in the waveguide, is registered by the measuring line 12 and enters into the receiving transmission line, consisting of a crystal detector 6, a resonant amplifier 7 and oscillograph 8. $VSWR$ of the studied OR is calculated using the formula $VSWR = 10^{B/20}$. Here, B is the difference in dB between the maximum and minimum attenuation values of the polarizing attenuator 11 during the probe movement along the waveguide. The photo of the experimental unit and the OR is given in **Figure 5**.

The validation of the proposed method of permittivity measurement using the considered OR (**Figure 4**) was performed with the samples having the shape of dielectric beads made of Teflon and Plexiglas. The diameter of the beads was equal to $2a_2 = 18$ mm, and it was equal to the diameter of the resonator cylindrical part. Values ε'_2 of the samples have been measured using the described experimental setup and curves presented in **Figure 3a**.

We measured ε'_2 for two beads having the diameter equal to 18 mm and the thickness 2.99 and 3.58 mm both made of Teflon, and two beads with the same dimensions but made of Plexiglas. Originally, the plunge is installed flush with the flat mirror, and in the hemispherical OR, the axial-asymmetric mode TEM_{0110} is excited. At the same time, the distance between the mirrors is equal to 22.139 mm ($f = 71.372$ GHz). The modes have been identified by application of the
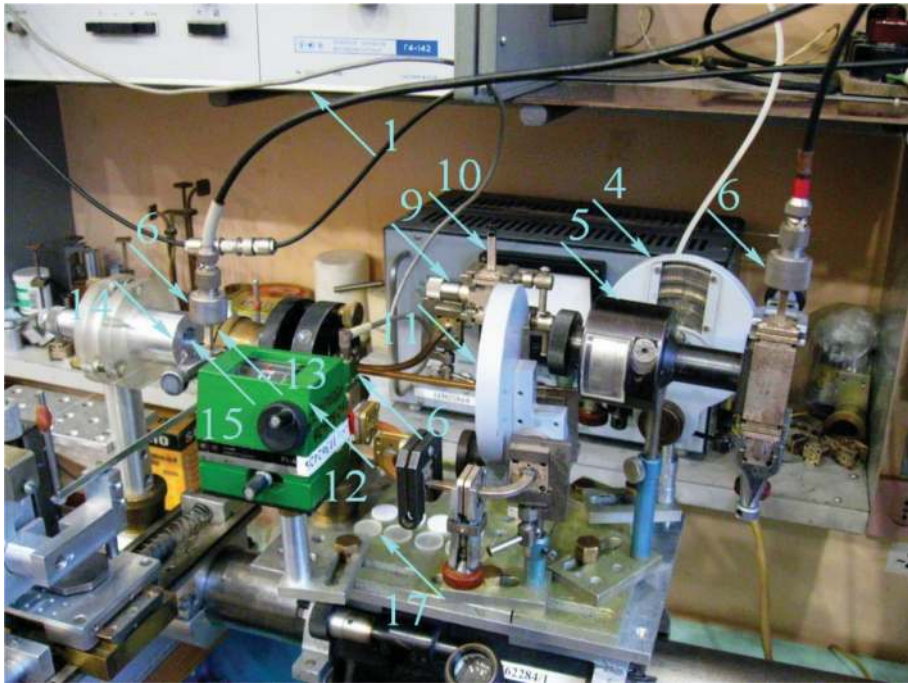
Figure 5. The photo of the OR with the segment of circular waveguide and the experimental unit.

perturbation technique [13]. In the next step, the plunge 16 is moved inside the oversized circular waveguide 15 on the distance $h = 3\lambda_w = 13.172$ mm (calculated value of $h = 3\lambda_w = 13.155$ mm). Availability of the oversized circular waveguide segment results in the fact that the TEM_{0110} mode of the hemispherical OR is being transformed into the axial-symmetric mode TE_{0116} [3]. In that case, the resonant distance is being measured already between the spherical OR mirror and the plunge.

Now the bead 17 having the thickness 2.99 mm and the diameter 18 mm, made of Teflon is placed on the plunge 16. Frequency of the generator G4–142 is tuned to achieve the resonant response. In the resonator at the frequency $f = 68.667$ GHz, which is measured by the wavemeter 5, again TE_{0116} mode is excited, that is identified by using the perturbation technique. Getting the value of the resonance frequency, we can evaluate permittivity of the studied sample with thickness 2.99 mm. For that purpose, we use the curve 1 (**Figure 3a**). In a similar way, we evaluate permittivity of the sample made of Teflon and having thickness 3.58 mm. In this situation, placing the sample on the plunge located in the cylindrical part of the OR, we got the value of the resonance frequency 68.410 GHz for the TE_{0116} mode. In order to value ε'_2 in that case, we use the curve 2 (**Figure 3a**). Measurement of two samples with the same dimensions and made of Plexiglas are carried out similarly. The mode TE_{0116} is excited but now only at the frequencies 68.051 GHz and ($h_d = 2.99$ mm) and $f = 67.030$ GHz ($h_d = 3.58$ mm). Using the curves 1 and 2 (**Figure 3a**), we evaluate permittivity of the samples having

Material	Thickness of the sample h_d (mm)	The measured value ε_2'	Literary value ε_2'	The difference	Δf, GHz
Teflon	2.99	2.085 ± 0.020	2.07 ± 0.04 [10]	0.7%	2.705
Teflon	3.58	2.124 ± 0.020	2.07 ± 0.04 [10]	2.5%	2.962
Plexiglas	2.99	2.599 ± 0.026	2.557 ± 0.026 [10]	1.6%	3.321
Plexiglas	3.58	2.616 ± 0.026	2.557 ± 0.026 [10]	2.2%	4.342

**Table 1.** Measured values of ε_2' for beads made of Teflon and Plexiglas.

various thickness and made of Plexiglas. Frequency measurement error in the millimeter range using a resonant wavemeter is $\pm 0.05\%$ [13]. Taking into account the frequency valuation by using the calibration curves of the resonant wavemeter, the total error of the samples permittivity measurement by application of the considered resonant cell is about ≈1%. The measurements results of the permittivity real part for considered samples are shown in **Table 1**.

In **Table 1**, Δf designates the frequency shift of TE_{0116} mode when putting the sample into "empty" OR.

In the next step, the dielectric losses in the samples made of Teflon and Plexiglas and having thickness 2.99 mm are evaluated. For their finding, one should calculate the energy filling factor of the resonator with the sample on the electric field K_E [7]. For its calculation, we use (Eq. (8)). The advantage of that expression is in the possibility to find the energy factor K_E not by the calculations of stored energies W_{1E} and W_{2E} in the volumes $V^{(2)}$ and $V^{(1)}$ (**Figure 1**), but by differentiation of the resonant circular frequency $\omega'(\varepsilon_2')$ dependence, presented in **Figure 3a**.

The behavior $K_E = \psi(\varepsilon_2')$ for the samples having the shape of beads and diameter 18 mm made of Teflon ($\varepsilon_2' = 2.07$) located on the bottom of the cylindrical part of the electrodynamic model of the OR and having various thickness h_d is shown in **Figure 6** as an example.

In calculations, we assume that there is nondegenerate mode TE_{0116} exists in the considered resonator. In **Figure 6**, also by dotted shows the factor K_E for a cylindrical resonator, in which
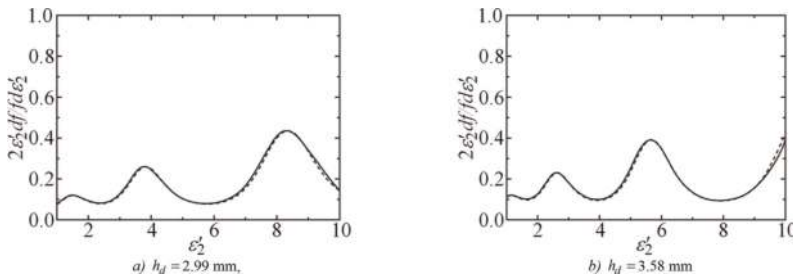
Figure 6. Dependencies of the energy filling factor of the resonator K_E on permittivity of the samples having various thicknesses.

the same axial-symmetric mode TE_{0116} is excited. The diameter of the resonator is equal to the diameter of the cylindrical part of the considered OR model, and lengths of the both resonators coincide (**Figure 2c**). The studied samples of various thicknesses are placed on the end cover of the cylindrical cavity. From the presented dependencies one can see that for the samples of various thickness in whole range of the ε_2' change, the behavior of the resonators energy filling factor on the electric field, both in cavity formed by cylindrical, conical and spherical surfaces, and in cylindrical, one almost coincide. In turn, it confirms the conclusion, noted earlier, that the patterns of the electromagnetic field both in the OR with the segment of the oversized circular waveguide and in the cylindrical cavity of same length l and diameter $2a_2$ (**Figure 1**) are identical.

Now let us evaluate losses in the bead having diameter 18 mm and $h_d = 2.99$ mm thickness made of Teflon ($\varepsilon_2' = 2.07$). The sample is placed on the plunge located in the waveguide part of the OR, the length of which h = 13.172 mm. Measurements are carried out using the experimental setup, a block diagram of which is presented in **Figure 4**, and its photo is shown in **Figure 5**. Eigen Q-factor of the resonant system with the measured sample $Q_{0\varepsilon}$ is defined by the relation [7, 17]

$$\frac{1}{Q_{0\varepsilon}} = \frac{1}{Q_{R\varepsilon}} + \frac{1}{Q_{rad\varepsilon}} + K_E \tan\delta, \tag{10}$$

where $Q_{R\varepsilon}$ is an ohmic Q-factor of the resonator with the sample; $Q_{rad\varepsilon}$ is a diffraction Q-factor of the OR with the sample caused by the coupling of the mode with free space.

In the absence of the measured sample in the resonant volume, unloaded Q_{00} of the OR with the same mode can be presented as follows:

$$\frac{1}{Q_{00}} = \frac{1}{Q_{R0}} + \frac{1}{Q_{rad0}}, \tag{11}$$

where Q_{R0} is an ohmic Q-factor of the resonator without the sample; Q_{rad0} is a diffraction Q-factor of the OR without the sample.

If we deduct (Eq. (11)) from (Eq. (10)), then, we will get the relation determining the dielectric losses tangent in the sample.

$$\tan\delta = \frac{1}{K_E}\left[\left(\frac{1}{Q_{0\varepsilon}} - \frac{1}{Q_{00}}\right) - \left(\frac{1}{Q_{R\varepsilon}} - \frac{1}{Q_{R0}}\right) - \left(\frac{1}{Q_{rad\varepsilon}} - \frac{1}{Q_{rad0}}\right)\right]. \tag{12}$$

Diffraction Q-factors, which are rather difficult to measure, enter into (Eq. (12)). The resonant system under consideration (**Figures 4** and **5**) allows to do the assumptions, which provide definition of tg δ without measuring $Q_{rad\varepsilon}$ and Q_{rad0}. Since the sample is placed into the resonator cylindrical part, then it should not disturb strongly the electromagnetic field in the open part of the OR (**Figure 5a** and **b**). Therefore, we can assume that the diffraction Q-factor of the resonator with the sample is equal to the diffraction Q-factor of the "empty" resonator,

that is, $Q_{rad\varepsilon} \approx Q_{rad0}$. Taking this into account, we write down in the final form an expression determining the tangent of dielectric losses in the sample as follows:

$$\tan\delta = \frac{1}{K_E}\left[\left(\frac{1}{Q_{0\varepsilon}} - \frac{1}{Q_{00}}\right) - \left(\frac{1}{Q_{R\varepsilon}} - \frac{1}{Q_{R0}}\right)\right]. \tag{13}$$

For the samples under studies having the shape of the bead ($2a_2 = 18$ mm, $h_d = 2.99$ mm) made of Teflon and Plexiglas, Q-factors $Q_{0\varepsilon}$ and Q_{00} are evaluated from the experiment, energy filling factor of the OR on the electric field K_E is calculated (**Figure 6a**), $Q_{R\varepsilon}$ and Q_{R0} are also calculated theoretically (**Figure 3b**, curve 2, 3).

In order to find $\tan\delta$ of the samples, loaded Q-factors $Q_{L\varepsilon}$ and Q_{L0} of the resonator with the sample and without were measured in the case of TE_{0116} mode excitation. At the same time, loaded and unloaded Q-factors of the OR are related as follows [9]:

$$\begin{cases} Q_{L\varepsilon} = \eta Q_{0\varepsilon}/(1 + \beta_{1\varepsilon} + \beta_{2\varepsilon}), \\ Q_{L0} = \eta Q_{00}/(1 + \beta_1 + \beta_2), \end{cases} \tag{14}$$

where $\beta_{1\varepsilon}$ and β_1 are coupling coefficients of the resonator with the receiving waveguide line with the sample on the bottom of the cylindrical part and without it; $\beta_{2\varepsilon}$ and β_2 are coupling coefficients of the resonator with a load with the measured sample and without it, η is efficiency of excitation of the TE_{0116} mode in the considered resonator.

Based on the research carried out with a metal screen covering the OR [18], it was determined that excitation efficiency of the considered TE_{0116} mode can be accepted to be equal to 0.96. Since two slot coupling elements having identical dimensions (3.6 × 0.14 mm) and located symmetrically to the resonator axis are used for excitation of the resonator and the signal output into the load, the input and output coupling should be the same, that is, $\beta_{1\varepsilon} = \beta_{2\varepsilon} = \beta_\varepsilon$ and $\beta_1 = \beta_2 = \beta$. In the paper [19], it was shown that the transmission type resonator having equal input and output coupling cannot be recoupled. It indicates that β_ε and β in this case are less than 1 and therefore are equal to $1/VSWR$ and the reflection coefficient $\Gamma > 0$. Measurements of Γ performed using the directional coupler ($VSWR = (1 + \Gamma)/(1 - \Gamma)$) and of $VSWR$ using the measuring line (**Figures 4** and **5**) confirmed the conclusion made of an equality of the input and output coupling. In such a way, from (Eq. (14)) we can find out the natural Q-factors of the resonant cell with the sample $Q_{0\varepsilon}$ and without it Q_{00} in (Eq. (13)).

$$\begin{cases} Q_{0\varepsilon} = Q_{L\varepsilon}(1 + 2\beta_\varepsilon)/0.96, \\ Q_{00} = Q_{L0}(1 + 2\beta)/0.96. \end{cases} \tag{15}$$

For calculation of the loss-angle tangent in the samples made of Teflon and Plexiglas, we calculate the ohmic Q-factor of the "empty" resonator using (Eq. (13)). As seen from **Figure 3b** (curves 2, 3, $h_d = 0$), $Q_{R0} = 48,005$. Experimentally measured loaded Q-factor Q_{L0} of the "empty" resonator, in which the mode TE_{0116} exists, turned out to be equal to 1660. At the same time, the coupling coefficient $\beta = 0.310$ ($\Gamma = 0.527$). Now from (Eq. (15)), we can calculate that the natural Q-factor of the "empty" resonator equals to $Q_{00} = 2801$.

Material	K_E	$Q_{R\varepsilon}$	$Q_{L\varepsilon}$	β_ε	$Q_{0\varepsilon}$	tg δ
Teflon	0.0886	49,339	1588	0.287	2604	$(3.117 \pm 0.034) \times 10^{-4}$
Plexiglas	0.0878	48,005	502	0.181	712	$(1.192 \pm 0.035) \times 10^{-2}$

Table 2. Measurement results of tanδ for the studied samples.

Taking into account the obtained values Q_{R0} and Q_{00}, the results of tanδ measurements for the samples having the shape of the beads made of Teflon and Plexiglas of thickness equal to 2.99 mm are shown in **Table 2**. Measuring error of the losses in dielectric sample amounts (1÷3)% and is defined, mainly, by the errors of the frequency measurement using a wavemeter and loaded Q-factors $Q_{L\varepsilon}$ and Q_{L0} of the resonator with the sample and without it.

As may be seen in **Table 2**, measured values of tanδ for the samples made of Teflon and Plexiglas coincide with the results obtained by other authors. For material with low losses (Teflon), we got tanδ = (2.8 ± 0.2) × 10^{-4} [20], which differs by 11% from the result obtained using the proposed resonator cell. In the case of the material with high losses (Plexiglas), obtained value of tanδ differs by 1.7% from the results of other authors (tanδ = (1.1 ± 0.06) × 10^{-2}) [20].

The authors did not aim to get high accuracy ε_2' and tanδ measurements. We showed only the basic possibility to measure electromagnetic parameters of materials with high losses in millimeter and sub-millimeter ranges by means of the resonant system consisting of the hemispherical OR with the segment of the oversized circular waveguide. In order to increase the accuracy of ε_2' and tanδ measurements for dielectric materials with low losses, it is necessary to increase the Q-factor of the resonant system. To achieve it, the resonator with the mirrors of large aperture and the spherical reflector of the larger curvature radius should be used.

3. Open resonator with a dielectric rod

3.1. Resonator model

The method of the solution is based on the same physical principles as in the case of the resonator with the dielectric bead. Thus, we consider a resonance cavity with the boundary made of a spherical, a conical and a cylindrical perfectly conducting surface. There is a cylindrical rod extended all along the resonator axis (area 2 in **Figure 7**), which is assumed to be a homogeneous and isotropic medium, having the material parameters ε_2, μ_2. The rest part of the resonator is assumed to be empty $\varepsilon_1 = \mu_1 = 1$ (area 1).

We confine ourselves to the analysis of axially-symmetric oscillations of TE-mode, which in cylindrical coordinates, where axis z coincides with the resonator symmetry axis, have E_φ-, H_r- and H_z- components of the electromagnetic field. For this oscillation mode, the initial problem for the Maxwell equations can be reduced to finding wave numbers k, for which there exist the nontrivial solutions U_1 and U_2 of the two-dimensional Helmholtz equations.
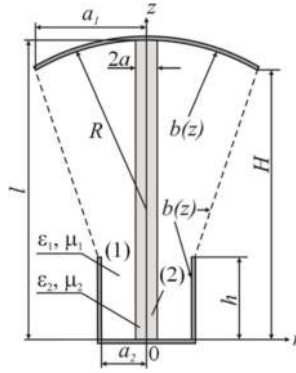
Figure 7. Geometric model of OR with the dielectric rod.

$$\Delta_{rz}U_1 + \left(k^2\varepsilon_1\mu_1 - (1/r^2)\right)U_1 = 0, \;\; 0 < z < l, \;\; a < r < b(z), \tag{16}$$

$$\Delta_{rz}U_2 + \left(k^2\varepsilon_2\mu_2 - (1/r^2)\right)U_2 = 0, \;\; 0 < z < l, \;\; 0 < r < a, \tag{17}$$

that satisfy the boundary conditions:

$$U_{1,2}(r,0) = 0, \;\; U_{1,2}(r,l) = 0, \;\; U_1(b(z),z) = 0, \;\; U_2(0,z) = 0, \tag{18}$$

and the field matching conditions at $r = a$:

$$U_1(a,z) = U_2(a,z), \;\; \frac{1}{\mu_1}\frac{\partial U_1(a,z)}{\partial z} = \frac{1}{\mu_2}\frac{\partial U_2(a,z)}{\partial z}. \tag{19}$$

Here, a is the rod radius.

The numerical algorithm for the problem (Eq. (16))÷(Eq. (19)) utilizes the method of Bubnov-Galerkin, as described earlier, and the problem is reduced to the system of linear algebraic equalizations (Eq. (5)) [6, 21].

The developed algorithm, as in the case with the dielectric bead, was tested by the passing to the limit from the geometry of the considered resonator to the cavities of spherical and cylindrical shape. The correctness of the described approach is validated by the papers [9, 12, 22]. Moreover, the algorithm convergence rate was estimated numerically for the growing dimensional representation of the algebraic problem (Eq. (5)).

3.2. Numerical and experimental results

The computations have been carried out for a resonator having the same dimensions, as in the case of the resonator with the bead. In **Figure 8**, the lines of equal amplitudes E_φ - components of the electromagnetic field of the mode TE_{0115} in the resonator with rods, made of Teflon and having permittivity $\varepsilon_2' = 2.07$, are presented. Apparently, the rod with relatively low
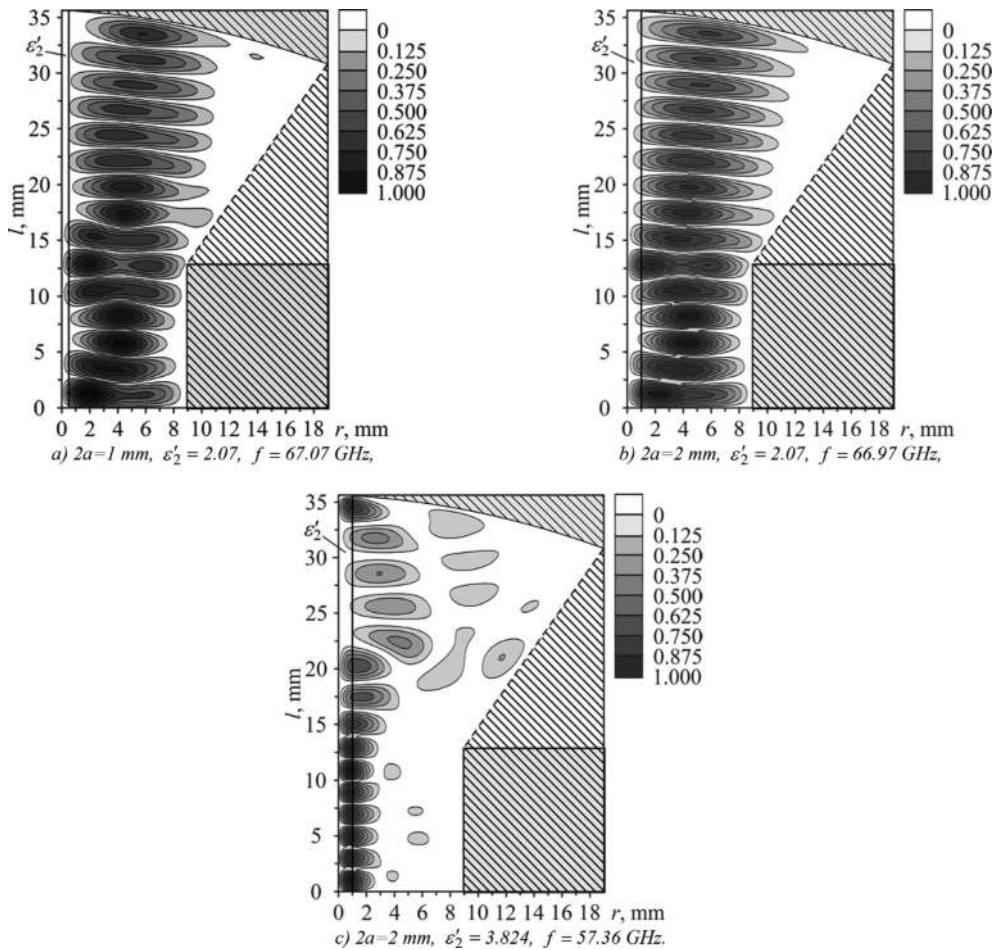
Figure 8. Field distribution in a cavity with a dielectric rod.

permittivity placed in the resonator weakly influences the distribution of the standing wave electric field components. The structure of the field at the placing of the rod, having diameter $2a = 1$ mm, into the resonator is shown in **Figure 8a**. Actually, it is identical to the structure of the field in the resonator at the rod diameter $2a = 2$ mm (**Figure 8b**). The difference of the TE_{0115} mode resonance frequencies at the increase of the rod diameter from 1 to 2 mm is insignificant. It decreases from $f = 67.07$ to $f = 66.97$ GHz. That says about the low correlation of the resonant oscillation with the dielectric rod and could be explained by the fact that the sample is located in the area with low electric field intensity. An increase of the sample permittivity can change the situation. In **Figure 8c**, the lines of equal amplitudes of E_φ - electromagnetic field component for the same mode TE_{0115} in the resonator with the same dimensions for the rod, made of fused quartz ($\varepsilon_2' = 3.824$) at the diameter $2a = 2$ mm, are

presented. It could be seen that, in this case, the field is located in the rod, and resonance frequency decreases down to $f = 57.36$ GHz. Particularly, it concerns the cylindrical part of the considered resonator. For the resonator without a rod, as shown earlier, frequency $f = 71.382$ GHz.

The carried out research (**Figure 8c**) demonstrated the strong relation of the sample with the electromagnetic field and high filling coefficient of the resonator on the electromagnetic field [7]. In such a manner, for qualitative control of liquid samples, the composition of which includes water, it is necessary to use pipes, made of the material having low permittivity, as compared with the substance under studies.

On the basis of the analysis carried out, one can say that sensitivity of the resonant cell is defined by the diameter of the cylindrical sample and by the value of its permittivity. From **Figure 8**, one can see that electric field intensity near the conical metal surface (dotted lines) is low. It indicates that the considered cavity is equivalent to the OR, in which high-Q oscillations, having low diffraction losses, are excited.

The experimental measurements of the permittivity of materials can be performed with the aid of calibration curves, that is, the dependencies of the resonator frequency shift on the permittivity of the cylindrical samples of various diameters, introduced into the resonance cavity. These characteristics are shown in **Figure 9**.

The upper part of the figure presents the series of curves plotted for the TE_{0116} mode by the rod diameter of 2 mm (curve 1) and 1.5 mm (curve 2). The bottom part presents the TE_{0115} mode by the same diameters of the samples, that is, 2 mm (curve 3) and 1.5 mm (curve 4). The dotted lines in the same figure, being almost parallel to those described earlier, illustrate similar dependencies for TE_{0116} and TE_{0115} modes in a cylindrical resonance cavity, with the cylindrical test pieces of the said size arranged along the resonator axis.

The length and the diameter of the cylindrical resonator were chosen to be equal to the length of the considered resonator and to the diameter of its cylindrical part (**Figure 7**). The problem
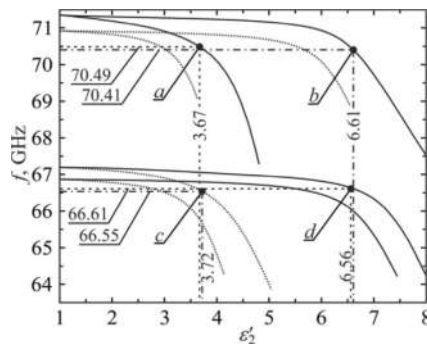
Figure 9. Dependencies of the resonance frequency on the permittivity of cylindrical bodies of various diameter for TE_{0116} and TE_{0115} modes.

considering the cylindrical resonator with a rod was solved using the method of variable separation, that is, its resonance frequency was described by rigorous formulas.

The figure demonstrates an obvious similarity of the curves for the both resonator types. It proves the fact that the nature of the physical processes occurring in the resonators is similar, both in the resonator under consideration and in the cylindrical cavity. It is an indirect evidence of the adequacy of our theoretical considerations. One more point to emphasize is that small changes of samples' permittivity, which are critical for oil quality control and food stuff, should be measured at the areas with a stronger disperse dependence, which is solely up to the diameter of the cylindrical sample (**Figure 9**).

The block diagram of the experimental unit used in the research is given in **Figure 4**. Only, in this case, the cylindrical sample was used. A sample was inserted into the cavity through a hole in the middle of the waveguide plunger, with a guide for the precise alignment of the sample along the resonator axis.

The measured shift of the resonance frequencies and the calibration curves in **Figure 9** were used to determine the dielectric permittivities of two cylindrical samples made of fused quartz and silicate glass. The value of resonance frequencies obtained in the experimental measurements for TE_{0116}- and TE_{0115}-modes for the case when the cylindrical samples of 2 mm and 1.5 mm in diameter were located along the resonator axis are marked with squares at the calculated curve. The results of measuring the dielectric permittivity of cylindrical samples of various diameters are listed in **Table 3**.

Material	Sample diameter (mm)	Mode	Measured value of ε'_2	Reference value of ε'_2	Difference (%)
Fused quartz	2	TE_{0116}	3.67 (a)	3.8 [7]	3.4
Fused quartz	2	TE_{0115}	3.72 (c)	3.8 [7]	2.1
Soda-lime glass	1.5	TE_{0116}	6.61 (b)	6.7 [23]	1.3
Soda-lime glass	1.5	TE_{0115}	6.56 (d)	6.7 [23]	2.0

Table 3. The measured values of the ε'_2 of the dielectric rods from Teflon and Plexiglas.

4. Open resonator with a segment of rectangular waveguide

A new electrodynamic system appears when inserting the segment of the short-circuited rectangular waveguide in the center of one of the OR mirrors [24]. Cross-section sizes of the waveguide $a \times b$ are chosen by the condition of the peak efficiency of the TE_{10} mode excitation by the fundamental mode TEM_{00q}. One can consider such OR as a resonant cell for measurement of composite materials and biological liquids electromagnetic specifications as well as to control the quality of food stuff in millimeter and in sub-millimeter ranges.

We consider the hemispherical OR with a rectangular waveguide located in the center of the flat mirror. Reflection from the waveguide horn is neglected. We consider the resonator

mirrors apertures as an infinite one. Omitting intermediate computations, we write down in the final form the expression, determining efficiency of the TE_{10} mode excitation in the waveguide of the OR, using the TEM_{00q} mode [25].

$$\eta = \frac{16\pi}{\tilde{a}\tilde{b}} \Phi^2 \left(\frac{\tilde{b}}{2}\right) \left\{ e^{-(\pi/2\tilde{a})^2} + \frac{e^{-(\tilde{a}/2)^2}}{2} \left[W^*\left(\frac{\pi}{2\tilde{a}} + i\frac{\tilde{a}}{2}\right) - W\left(\frac{\pi}{2\tilde{a}} + i\frac{\tilde{a}}{2}\right) \right] \right\} \qquad (20)$$

Here, w_0 is the radius of the beam waist of the fundamental mode on the flat mirror, $\tilde{a} = a/w_0$; $\tilde{b} = b/w_0$; $\Phi\left(\tilde{b}/2\right)$ is the probability integral and $W[(\pi/2\tilde{a}) + i(\tilde{a}/2)]$ is the integral of the complex argument probability.

Dependence $\eta\left(\tilde{a}, \tilde{b}\right)$, computed using (Eq. (20)), is presented in **Figure 10**. As evident from the figure, at $\tilde{a} = 2.844$ and $\tilde{b} = 1.98$, the efficiency of the TE_{10} mode excitation in the rectangular waveguide, located in the center of the OR flat mirror, using TEM_{00q} fundamental mode of the resonator is maximal and equal to 0.881.

As a result of the theoretical analysis, it was demonstrated that the efficiency of the TE_{10} mode excitation in the segment of the rectangular waveguide, using the OR fundamental oscillation, can amount the value about 90%. Therefore, such resonant system should have good selective properties, that is, an advantage for the analysis of dielectric samples with high losses. Besides, since the cross-section sizes $a \times b$ of the rectangular waveguide segment several times exceed radius of the beam waist w_0 of the fundamental OR mode, then it should be considered as the oversized.

In such OR, losses should increase, since ohmic losses in the walls of the rectangular waveguide segment are added. It would result in a decrease of the loaded Q-factor Q_L and, as a consequence, in decrease of the resonant system sensitivity. Therefore, in order to understand how the rectangular waveguide segment will influence the spectral and energy specifications of the considered resonant system, experimental researches have been carried out [25].
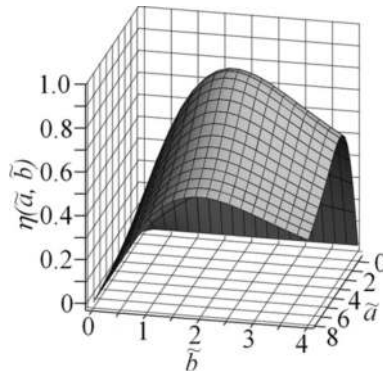
Figure 10. Efficiency of TE_{10} mode excitation in the rectangular waveguide versus its cross-section sizes.
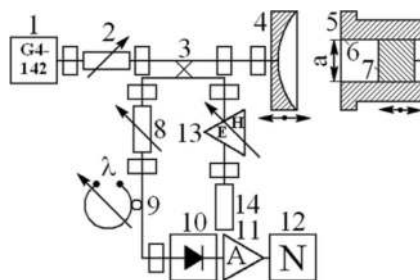
Block diagram of the experimental setup, which was used for carrying out research of the hemispherical OR with the segment of the oversized rectangular waveguide, is shown in **Figure 11**.

The OR is formed by the flat mirror 5, having aperture 60 mm, and by the spherical focusing mirror 4, having a curvature radius $R = 113$ mm and aperture 60 mm. In the center of the flat mirror, the segment of the oversized short-circuited rectangular waveguide 6 is located, cross-section sizes of which have been chosen in such a way to meet the peak efficiency condition of the TE_{10} mode excitation by the fundamental mode of the OR. As it turned out, for presented dimensions of the resonator at $\lambda = 4.203$ mm and $L/R = 0.7$ (in [26] it was shown that approximately at such normalized distance between mirrors of the hemispherical OR, the Q-factor of the fundamental mode is maximal) such condition corresponds to $a = 23.7$ mm and $b = 16.5$ mm. The plunge 7 provides changing the waveguide length. Taking into account the dimensions of the waveguide, such OR will be the most promising in the short-wave part of millimeter and sub-millimeter ranges.

The resonator is excited by the slot coupling element, having sizes 3.6×0.16 mm, located in the center of the spherical mirror. The adjusting attenuator 2 is included into the setup for decoupling of the frequency generator and the resonator. Alignment to resonance is implemented by moving the spherical mirror 4 with the elements of the waveguide along the resonator axis. The input waveguide is oriented in such a manner that the vector **E** of the fundamental mode TE_{10} is orthogonal to the plane of the drawing (**Figure 11**).

Receiving transmission line consists of the auxiliary line of the directional coupler 3, measuring polarizing attenuator 8, detector 10, resonant amplifier 11 and oscillograph 12. The resonant wavemeter 9 is included into the setup for monitoring frequency of the high-frequency generator 1. A double-stub matcher 13 is installed in the branch of the matched load 14 of the directional couplers 3. The photo of the OR and the experimental unit is represented in **Figure 12**.

The above-described procedure is used for computing of the resonant reflection factor of the resonator. Results of measurement of the resonant reflection factor on the distance between mirrors L/R of the hemispherical OR, are shown in **Figure 13** (curve 1). In the resonator, TEM_{00q} mode is excited. Identification of the oscillations modes was performed using the

Figure 11. Block diagram of the experimental unit.

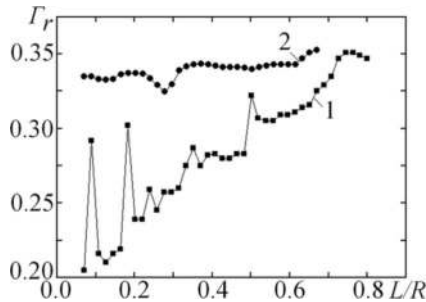**Figure 12.** The photo of the OR with the segment of rectangular waveguide and the experimental unit.

Figure 13. Dependencies of the reflection factor Γ_r on the distance between mirrors L/R for TEM_{00q} mode in the hemispherical OR (1) and resonator with the segment of the oversized waveguide (2).

perturbation technique [13]. The technique described in [27] was applied for the definition of the Γ_r character.

As can be seen from **Figure 13**, with decrease of the distance between mirrors, the reflection factor from the resonator diminishes. It is related to the reduction of the diffraction and ohmic losses in the resonant system. Exceptions include the cases of the interaction of the considered oscillation with other oscillations excited in the OR ($L/R = 0.351$, $L/R = 0.259$, $L/R = 0.183$, $L/R = 0.089$), that it is displayed in the stepwise change of the Γ_r. The distance between mirrors $L/R = 0.501$ corresponds to the semi-confocal geometry of the resonator. The increase of the number of modes interacting with TEM_{00q} mode at $L/R < 0.3$ is related with reduction of the losses in the resonant system for specified distances between reflectors.

Dependence $\Gamma_r = \psi(L/R)$ for the same mode when segment of the oversized rectangular waveguide is located in the center of the flat resonator mirror (curve 2) is shown in that diagram. The segment length is $S = 19.082$ mm, which accounts nine-and-half waveguide wavelengths. At the same time, calculated value $S = 9\lambda_w/2 = 18.988$ mm ($\lambda_w = \lambda / \sqrt{1 - (\lambda/2a)^2}$). Apparently, the difference does not exceed 0.5%. The length of the waveguide is chosen in such a way to remove reflection of the Gaussian beam from the plunge, like from the second resonator mirror. Presence of the rectangular waveguide segment results in an increase of the OR ohmic losses in the whole tuning range. It is seen from a comparison of the reflection factors of the considered resonator (curve 2) and the empty hemispherical OR (curve 1). In whole range of the resonator tuning, only the fundamental mode is excited, which confirms the above-stated conclusion about selective properties of the OR with the segment of the oversized rectangular waveguide. That conclusion follows from the absence of the Γ_r abrupt changes, which, as stated earlier, are caused by interaction of the considered mode with other modes of the resonant system. From the presented diagram one can see, that at the distance between resonator mirrors $L/R = 0.276$, the reflection factor has the minimum value equal to 0.329. When defining cross-section sizes of the rectangular waveguide, we supposed $L/R = 0.7$. The radius of the fundamental mode beam waist on the flat mirror of the hemispherical OR is defined by the expression $w_0 = \sqrt{\frac{\lambda}{\pi} R \sqrt{\frac{L}{R}\left(1 - \frac{L}{R}\right)}}$ [28].

From this relation, it follows that w_0 will be the same at $L/R = 0.7$ and $L/R = 0.3$. Since in such resonator only the fundamental mode exists, then such behavior of Γ_r is related to efficiency of the waveguide mode TE_{10} excitation by means of the OR mode TEM_{0015}. The difference between experimentally obtained value $L/R = 0.276$ and the computed one (in this case $L/R = 0.3$), at which η should have its peak value, and accounts for 8%. Such difference shows good agreement between computational and experimental results.

The dependence of reflection from resonator on the length of the oversized rectangular waveguide segment for certain mode is of the practical interest as well. We assume that in the hemispherical OR (**Figure 13**, curve 1), a no degenerate mode should exist. Moreover, the distance between resonator mirrors should correspond to low diffraction losses. Therefore, in terms of the diagram presented in **Figure 13**, we choose the mode TEM_{0016} ($L/R = 0.295$, $\Gamma_r = 0.257$). Results of the measurements are presented in **Figure 14**.
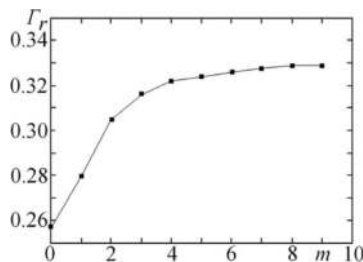

Figure 14. Dependence of reflectivity factor Γ_r on the waveguide half waves number m, keeping along rectangular waveguide for TEM_{0016} mode of the OR oscillations.

From the figure, one can see that moving the plunge from the surface of the flat mirror ($m = 0$) along the waveguide up to four waveguide half waves ($m = 4$), increase of the losses is observed (Γ_r growth from 0.257 till 0.322). This apparently could be explained by transformation of the Gaussian beam into the waveguide wave. At further increasing of the waveguide segment length up to $m = 8$, there is insignificant growth of loss is caused by ohmic losses in the waveguide segment. If m is increased further, then, Γ_r is almost without any change.

Considered here resonant system will be the most promising at the analysis of biological liquids and food stuffs, the basic element of which is water (wines, juices, drinks). For carrying out measurements, a pipe made of the material of lower permittivity than that of a sample is placed into the oversized rectangular waveguide parallel to the vector of electric field intensity of the TE_{10} mode. For reduction of losses inserted to the resonant system, it can be displaced to one of the side walls of the waveguide.

5. Conclusions

The cavity with the dielectric layer, being an electrodynamic model of the hemispherical OR with the segment of the oversized circular waveguide and dielectric bead, is considered in this chapter. As a result of the carried out theoretical analysis, it is shown that dependence of the frequency upon the thickness of the dielectric bead, located on the bottom of the cavity cylindrical part, has a quasi-periodic behavior. Such behavior is related to the amplitude distribution of E_φ - component of the axial-symmetric mode at the top of the sample. The weak dependence of frequency on the bead thickness takes place when near to the sample top the node of the electric field component of the standing wave in the resonator is located. If in the area of the sample top, there is an antinode of the electric field component of the standing wave in the resonator, and the frequency dependence on the sample thickness is strong. Measurement of the permittivity and the tangent losses of beads, made of Teflon and Plexiglas at the various thickness, using the OR with the segment of the oversized circular waveguide, have been carried out. Obtained values ε_2' and $\tan\delta$ are in agreement with the data of the other authors. In such a way, the proposed resonant system can be applied for measurement of electromagnetic parameters of substances in the short wave part of the millimeter and sub-millimeter ranges, both with high and with low losses.

The hemispherical OR, with the segment of the rectangular oversized waveguide located in the center of the flat mirror, has been considered in this chapter as well. As a result of the carried out theoretical analysis, it was demonstrated that efficiency of the TE_{10} mode excitation in such waveguide by means of the resonator fundamental mode TEM_{00q} can reach the value ~90% at cross-section sizes $\tilde{a} = 2.844$ and $\tilde{b} = 1.98$. The experimental researches carried out by the authors made it possible to establish that the resonator with the segment of the rectangular oversized waveguide possesses selective properties in the wide band of frequencies. Presence of the rectangular waveguide segment does not result in an abrupt increase of the ohmic losses. These circumstances are advantageous for application of such resonator for research of dielectric substances. Considered resonant system will be the most promising for the analysis

of liquid dielectrics. For performing measurements, the pipe with the sample is placed into the oversized rectangular waveguide parallel to the vector of electric intensity of the TE_{10} mode. For reduction of the losses inserted into the resonant system, it can be displaced to one of the side walls of the waveguide.

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