Chapter

Application of Onsager and Prigozhin Variational Principles of Nonequilibrium Thermodynamics to Obtain MHD-Equation Dissipative System in Drift Approximation

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Abstract

Electromagnetic phenomena in plasma are easier to describe in terms of fields, expressing the electric current through the rotor of the magnetic field. But the approach that ignores the corpuscular aspect of the electric current, as noted by H. Alfven, does not allow describing many processes in space plasma. Indeed, relying on the concept of continuity, it is impossible in the mechanics of continuous media to take into account the fluctuations of hydrodynamic functions formed due to the molecular structure of the medium. At the hydrodynamic level of description, taking into account the structure leads to the Langevin equation. Therefore, to describe processes in a magnetized plasma, it is of certain interest to obtain MHD equations in the drift approximation not from the Vlasov equations, but based on the principles of Onsager and Prigogine, combined by Gyarmati into one variational principle and obtaining a one-liquid plasma model in the drift approximation. Fluctuations are taken into account by introducing an additional term in the expression for pressure, written in the drift approximation, which is similar to the postulation of the Langevin source for describing Brownian motion. The obtained fluctuating-dissipative system differs from the reversible one-liquid approximation of the two adiabatic invariants of Chu, Goldberger, Low.

Keywords: nonequilibrium thermodynamics, variational principles of Onsager and Prigogine, the combined Gyarmati principle, collisionless plasma, drift approximation

1. Introduction

Alfven in his work [1] noted that the approach which does not take into account the corpuscular aspect of the electric current does not allow to fully describe many processes in the cosmic plasma. Relying on the concept of continuity, it is impossible in continuum mechanics to take into account fluctuations of hydrodynamic functions formed due to the molecular structure of the medium. It is known that at the hydrodynamic level of description, taking into account the corpuscular structure leads to the Langevin equation, in which the parameters of the medium are described by random sources [2]. These sources are responsible for fluctuations of density, velocity, temperature and, being the unavoidable properties of the medium, cannot be excluded. In turn, the model of "collisionless" plasma based on the Vlasov equations, in principle, does not contain fluctuations, since it is collisions that lead to fluctuations and, as a consequence, to dissipation. Naturally, magnetohydrodynamic equations (MHD equations) obtained in the drift approximation from the Vlasov equation through the moments of the distribution function also do not take into account dissipative processes (see, for example, [3]). In a magnetized plasma, the distribution of electrons and ions can have axial symmetry with respect to the magnetic field. In the absence of heat flux along the magnetic field lines (or it can be neglected), slow plasma motions obey MHD equations with anisotropic pressure. In a number of interesting cases, the description of the plasma behavior without collisions in the hydrodynamic approximation can be used as a heuristic tool for obtaining qualitatively correct results [3]. It should be noted that a significant part of the work on the macroscopic description of plasma behavior is devoted to clarifying the question of how much a real plasma can differ from its ideal twin under the assumption, for example, of an ideally conducting liquid [4].

Therefore the problem arises to try to obtain the MHD equations not from the Vlasov equations, but on the basis of another approach, in which the drift equations themselves, in the conclusion of which the perturbation theory lies [1], are the initial ones. Such a possibility opens in the case of application of the principles of the least dissipation of energy of Onsager [5] and the least production of entropy of Prigozhin [6], combined by Gyarmati into one variational principle [7]. In this case, the fluxes corresponding to the observed transport processes in a magnetized plasma are represented in the drift approximation. In turn, the drift approximation, being one-particle, simultaneously admits fluctuations within the accuracy of this approximation $T_L/H|dH| < < 1$, where T_L is the period of the Larmorian rotation.

The application of variational principles allows one to obtain a hydrodynamic system of equations, which in the linear approximation describes in the drift approximation the dynamics of a collisionless plasma located near the equilibrium state. Unlike the Vlasov equation and the equations of hydrodynamics that follow from it (or postulated on the basis of known conservation laws), the resulting system of equations is completely self-consistent and takes into account the fluctuation interaction of local currents with electric and magnetic fields within the accuracy of the used drift approximation. Fluctuations are taken into account by introducing an additional term in the expression for the pressure, which is responsible for its nonequilibrium part, which is analogous to the postulation of a Langevin source in describing Brownian particles in hydrodynamics.

2. Statement of the problem

Difficulties that arose from the very beginning after obtaining kinetic equations and introducing the terms collisional and collisionless plasma [8, 9] are associated in the physics of open systems with the concept of continuous medium. In this case, it becomes important to determine the physically infinitesimal scale corresponding to the point of the "continuous medium".

Indeed, the concept of continuous medium, depending on the chosen model for describing the behavior of an ionized gas (kinetic, diffusion, or hydrodynamic), implies the choice of a scale characterizing a physically infinitesimal point of a continuous medium ℓ_f for which differential equations are written. However, in this case, information is lost inside these points, since the large number of particles filling their volume ($g^{-1} = n\lambda_D^3 > > 1, g$ is the plasma parameter, λ_D is Debye radius) is not taken into account, which ultimately determines the internal openness of the chosen level of description [2]. Therefore, taking into account the structure due to the "artificial" introduction of an additional collision integral into the dissipative Vlasov equation when calculating the Landau collisionless damping coefficient leads to the appearance of dissipation and, as a consequence, to nonequilibrium. The need to take into account the structure of a physical "point" is one of the main provisions that determine the substantive part of fundamental works [2, 10, 11]. This position sets the direction of the search for the possibility of describing nonequilibrium processes on the kinetic and hydrodynamic scales from a single point of view, and will be used in this work.

It is known that the description of the dynamics of an ionized gas is also possible at the hydrodynamic level. Indeed, the kinetic method for some practical problems may turn out to be too detailed and mathematically complex. At the same time, without being interested in the motion and interaction of a large number of particles, one can significantly simplify the problem associated with the study of collective processes occurring in a plasma. Considering such macroscopic quantities as the average velocity of motion of a medium $\overleftarrow{V}$, pressure P, density of particles n and currents $\overleftarrow{j}$, and so on, postulating then the basic equations of hydrodynamics of continuous media, based on the laws of conservation of mass, momentum, energy and charge, together with Maxwell's equations, we can reduce the problem to the problems of magnetohydrodynamics (MHD). The system of MHD equations has the simplest form in the case of a one-fluid approximation for scalar (see, for example, [12, 13]) or tensor pressure (quasi-hydrodynamic approximation of Chu, Goldberger, and Lowe (ChGL) [14]).

At the same time, the Lorentz force acting on charged particles in a magnetic field twists them around the lines of force, preventing movement across the lines of force, and in this regard, the action of the field is similar to the effect of collisions, limiting the movement of the particle by the value of the Larmor radius. Consequently, the drift approximation shows how, in the absence of collisions, the order inherent in "collisional" continuous media and practically sufficient for describing the dynamics of a "collisionless" plasma at the hydrodynamic level is provided by a magnetic field. ("Practical sufficiency", from the point of view of the kinetic description, is achieved by neglecting the third moments in the equations, which corresponds to the not entirely justified neglect of the heat flux along the lines of force. Experimentally, this is realized in closed axial plasma systems or under real conditions, for example, in the region capture of the Earth's magnetosphere). Consequently, in a magnetized plasma, the role of the mean free path is played by the Larmor radius of ions $\rho_{L_i}\left(\rho_{L_i} > > \rho_{L_e}\right)$, and the condition for the applicability of the continuous medium approximation takes the form $L > > \rho_{L_i}$, where L is the characteristic size in the plasma. As for the frequency dependence, which makes it possible to consider a collisionless plasma as a continuous medium during the propagation of a wave process in it, it has the form: $\omega < < \omega_{L_i} < < \omega_{0_i}$ for a not too discharged ionized gas and a weak magnetic field (hot plasma) and $\omega < < \omega_{0_i} < < \omega_{L_i}$ for a magnetized plasma satisfying the drift approximation (cold plasma). Moreover, the possibility of describing the behavior of a collisionless plasma using a pressure gradient is associated with the mechanism of pressure transfer not through collisions, but through the interaction of currents flowing in the plasma drift currents and magnetizing currents. In addition, a large role in the processes occurring in a collisionless plasma is played by self-consistent fields that bind particles and prevent them from scattering.

For physically small linear and time scales ℓ_f and τ_f, as well as the number of particles N_f in the volume ℓ_f^3, the inequalities are valid $\tau_f \sim (\lambda_D/V_T) < < T$, $\ell_f \sim \lambda_D$ [8]. The first inequality makes it possible to use the "continuous medium" approximation, the second - to use the concept of "collisionless plasma", and the third notes the fact that the interaction of charged particles in an ionized medium has a collective character (V_T is the thermal velocity of particles, T is the characteristic time).

However, magnetohydrodynamic equations (MHD equations) obtained in the drift approximation from the Vlasov equation through the moments of the distribution function do not take into account dissipative processes [3]. In other words, in this case, the structure of the physically small volume of the continuous medium is not taken into account, with respect to which the macroscopic equations are written. At the same time, the possibility of taking into account the drift approximation in the hydrodynamic consideration of the theory of magnetized plasma without any additional assumptions appears in the case of applying the variational principles of nonequilibrium thermodynamics of Prigogine and Onsager [5, 6], combined by Gyarmati [7]. Thus, in the mechanics of continuous media, it becomes possible to construct non-equilibrium models that describe the dynamics of continuous systems located near equilibrium (linear approximation). In turn, the construction of new models is an important section of continuum mechanics, and they are based on the search for additional relationships between the parameters that describe the state of the considered continuous medium.

With this in mind, the following provisions were the starting points for constructing a hydrodynamic model based on variational principles and drift equations [3, 7, 15]:

1. Incomplete description of plasma in the language of fields, considered as a continuous medium, which arises when currents are replaced according to Maxwell's equations by a magnetic field [1]. This leads to neglect of the corpuscular aspect of currents and, as a consequence, neglect of the fluctuation interaction, which is formed precisely due to the molecular structure of the medium. In turn, taking into account the molecular structure of a continuous medium inevitably leads to the appearance of dissipation in it.

2. The variational principle of Gyarmati [7], which combines the principles of Onsager and Prigogine [5, 6], makes it possible, within the framework of the Lagrangian formalism, to obtain the equation of motion with allowance for dissipation for a magnetized plasma (the pressure is anisotropic) in the approximation in which thermodynamic forces Xi and fluxes Ji. In this chapter, we use the drift approximation [3, 7, 15] and the approximation of two adiabatic invariants [14]. Since the ChGL approximation is holonomic, i.e. all quantities can be expressed using the displacement vector and described by the Lagrange formalism, then the dissipative approximation of the ChGl will be obtained within the framework of this formalism.

3. Equation of collisionless plasma motion considering dissipation. Anisotropic case

To describe nonequilibrium thermodynamic processes in continuous media in a linear approximation, the Hungarian physicist Gyarmati formulated a variational principle that combines the principle of the least dissipation of Onsager's energy and the principle of the least production of Prigogine's entropy. To obtain the equation of motion that takes into account dissipation, we introduce the entropy production function $\sigma = \sum_{i=1}^{n} J_i X_i$, as well as the scattering potentials $\Psi = \frac{1}{2}\sum_{i,k}^{n} L_{i,k} X_i X_k$ and $\Phi = \frac{1}{2}\sum_{i,k}^{n} R_{i,k} J_i J_k$, expressed in terms of thermodynamic forces X_i (gradients of temperature, pressure, potential, field strength, and so on), and fluxes J_i corresponding to the observed transfer processes. If we now construct a function, $L = \Psi + \Phi - \sigma$, then, as shown in [7], thermodynamic nonequilibrium processes near a steady state develop in such a way that the integral of over the volume occupied by the medium under study is minimal

$$\int_{\mho} L d\mho = \int_{\mho} [\Psi + \Phi - \sigma] d\mho = min$$

In this formulation, the Gyarmati principle is similar to Hamilton's principle in mechanics and the variation of this integral is equal to zero. Following the general provisions of [7, 11], we represent the tensor pressure of positively charged particles of an ionized gas as a sum of two parts. One part $\overleftrightarrow{P}$ depends on the state and corresponds to the equilibrium part, the other part $\overleftrightarrow{P}_d$ depends on the rate of change of this state and corresponds to the nonequilibrium part, that is

$$\overleftrightarrow{P}^{i}_{\Sigma} = \overleftrightarrow{P}^{i} + \overleftrightarrow{P}^{i}_{d} \tag{1}$$

the subscript $"i"$ denotes the ionic component of the equilibrium and nonequilibrium parts of the plasma pressure tensor. From the general provisions on the form of the explicit dependence of pressure $\overleftrightarrow{P}^{i}_{d}$, it follows that it should depend on the macroscopic velocity of the medium $\overleftarrow{V}_i$ and on the physical reasons causing the appearance of the nonequilibrium part of the pressure (for example, for viscous media with Brownian particles, this is taken into account by introducing the corresponding coefficients of viscosity and a random Langevin source). In our case, viscosity in the usual sense is absent, and the nonequilibrium part of the equation should be proportional to the flows of charged particles, which also corresponds to the general concept of pressure transfer through electromagnetic interaction, and also takes into account the discreteness of the ionized medium (its atomic-molecular structure [2]). With this approach, the ionic component of the pressure tensor $\overleftrightarrow{P}^{i}_{d}$ is similar to a Langevin source. According to what has been said, we represent the nonequilibrium part of the pressure in the form

$$\overleftrightarrow{P}^{i}_{d} = -m_i \left(\overleftarrow{V}^{i}_{k} \cdot \overleftarrow{J}^{i}_{n} \right) \overleftrightarrow{I} \tag{2}$$

where $\overset{\leftrightarrow}{I}$ is the unit tensor, and for the equilibrium part we write out the standard representation of this part of the pressure [12]

$$\left(\overset{\leftrightarrow i}{P}\right)_{kn} = p^i_{II}\overset{\leftarrow}{e}_k\overset{\leftarrow}{e}_n + p^i_{\perp}\left(\delta_{kn} - \overset{\leftarrow}{e}_k\overset{\leftarrow}{e}_n\right), \overset{\leftarrow}{e}_1 = \frac{\overset{\leftrightarrow}{H}}{H}. \tag{3}$$

Spatial heterogeneity and concentration n are taken into account in the explicit form of the flow $\overset{\leftarrow}{J}_i$.

We represent the Gyarmati principle in the form [13],

$$\delta\int_{\mho}(\sigma_d - \Psi_d)d\mho = 0 \tag{4}$$

where $\sigma_d = \sum_{j=1}^{f} J_j X_j$ and $\Psi_d = \frac{1}{2}\sum_{j,k=1}^{f} L_{jk} X_j X_k$. The integral in (4) is taken over the entire volume $\mho$ occupied by the plasma. Since in a collisionless plasma there are no chemical reactions and sources of death and production of particles, and the interaction of currents leads to dissipative phenomena, then according to the general principles of construction σ_d and Ψ_d [7] we have for the positive plasma component

$$\sigma^i_d = -\overset{\leftrightarrow i}{P}_d : \left(\overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V}\right), \Psi^i_d = \frac{1}{2}m_i\left(\overset{\leftarrow i}{V}\cdot\overset{\leftarrow i}{J}\right)\left(\overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow}{V}_i\right) = -\frac{1}{2}\overset{\leftrightarrow i}{P}_d : \left(\overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V}\right).$$

Considering σ_d and Ψ_d values and on the basis of (4), we obtain

$$\delta\int_{\mho}\left(-\overset{\leftrightarrow i}{P}_d : \overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V} + \frac{1}{2}\overset{\leftrightarrow i}{P}_d : \overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V}\right)d\mho = -\frac{1}{2}\delta\int_{\mho}\overset{\leftrightarrow i}{P}_d : \left(\overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V}\right)d\mho. \tag{5}$$

To calculate the integrand, we use the equation of balance of translational kinetic energy [7]

$$\rho_i\frac{d}{dt}\frac{\left(\overset{\leftarrow i}{V}\overset{\leftarrow i}{V}\right)}{2} + \overset{\leftarrow}{\nabla}\cdot\left(\overset{\leftrightarrow i}{P}_{\Sigma}\cdot\overset{\leftarrow i}{V}\right) = \rho_i\left(\overset{\leftarrow i}{V}\cdot\overset{\leftarrow i}{F}_{ext}\right) + \overset{\leftrightarrow i}{P}_{\Sigma} : \left(\overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V}\right) \tag{6}$$

where $\overset{\leftrightarrow i}{P}_{\Sigma}$ is total pressure, determined by (1), $\overset{\leftarrow i}{F}_{ext}$ is external and internal forces per mass unit, ρ_i is ion component density. If we now express $\overset{\leftrightarrow i}{P}_d : \left(\overset{\leftarrow}{\nabla}\cdot\overset{\leftarrow i}{V}\right)$ in (5) on the basis of (6), we obtain

$$-\frac{1}{2}\delta\int_{\mho}\overset{\leftarrow i}{V}\left(\rho_i\frac{d\overset{\leftarrow i}{V}}{dt} + Di\nu\overset{\leftrightarrow i}{P} - m_i\overset{\leftarrow i}{V}\overset{\leftarrow i}{\nabla}\cdot\overset{\leftarrow i}{J} - \rho_i\overset{\leftarrow i}{F}_{ext}\right)d\mho = -\delta\int_{\mho}L_i d\mho = 0,$$

where $L_i = \frac{1}{2}\overset{\leftarrow i}{V}\left(\rho_i\frac{d\overset{\leftarrow i}{V}}{dt} + Di\nu\overset{\leftrightarrow i}{P} - m_i\overset{\leftarrow i}{V}\overset{\leftarrow i}{\nabla}\cdot\overset{\leftarrow i}{J} - \rho_i\overset{\leftarrow i}{F}_{ext}\right)$ is Lagrange density, which satisfy the general equation

$$\frac{\partial L}{\partial V_\beta} - \sum_{\alpha=1}^{3} \frac{\partial}{\partial X_\alpha} \frac{\partial L}{\partial(\partial V_\beta/\partial X_\alpha)} = 0, \tag{7}$$

which is also valid for electronic component. Substituting the value L_i into Eq. (7) and performing differentiation, we obtain the equation of motion for the ionic component "i"

$$\rho_i \frac{d\overleftarrow{V}^i}{dt} = -Div\overleftrightarrow{P}^i + \rho_i \overleftarrow{F}^i_{ext} + 2m_i\left(\overleftarrow{V}^i \cdot \overleftarrow{\nabla}^i \overleftarrow{J}^i\right) \tag{8}$$

where Div the operator denotes tensor divergence. Repeating the same procedure for the plasma negative component, which is near the thermodynamic equilibrium ($T_e \approx T_i$), a similar equation may be obtained for electronic e component. Adding the obtained equation for electrons to (8) and considering $\overleftarrow{V}^e \approx \overleftarrow{V}^i = \overleftarrow{V}$, $\overleftarrow{F}^i_{ext} \approx \overleftarrow{F}^e_{ext} = \overleftarrow{F}_{ext}$, and $m_e + m_i = m_i(1 + m_e/m_i) \approx m_i = m, \rho_i = m_i n_i \approx \rho$ we obtain the following equation of motion:

$$\rho \frac{d\overleftarrow{V}}{dt} = -Div\overleftrightarrow{P}^i + \rho\overleftarrow{F}^i_{ext} + 2m\overleftarrow{V}\left[\overleftarrow{\nabla} \cdot \overleftarrow{J}^i + \frac{m_e}{m_i}\frac{\partial}{\partial t}(n_e - n_i)\right] \tag{9}$$

where $\overleftrightarrow{P} = \overleftrightarrow{P}^e + \overleftrightarrow{P}^i = \overleftrightarrow{P}^{e,i}_\perp + \overleftrightarrow{P}^{e,i}_{II}$. In (9) $\nabla \cdot \overleftarrow{J}^i$ value is expressed through $\frac{\partial}{\partial t}(n_e - n_i)$, considering the violation of quasi-neutrality and condition $1 > > m_e/m_i$.

Taking into account the structure of a physically infinitesimal element of the medium, it must be remembered that it has linear dimensions of the order of the Debye radius, within which the condition of quasineutrality, due to fluctuations, can be violated. This is of fundamental importance, since it is the fluctuations that determine the character of the development of possible instabilities in the plasma. Therefore, in the last expression, the partial derivative of the difference between the concentrations of the electronic and ionic components is multiplied by a small value $(m_e/m_i) \approx (m_e/m)$.

Since in (9) the total flux is determined through the sum of fluxes of positively charged particles $\overleftarrow{J} \approx \overleftarrow{J}^i = \sum_k \overleftarrow{J}^i_k$, then after simple ones associated with calculating the corresponding divergences in the drift approximation for fluxes [1, 16] (see appendix), we have

$$\overleftarrow{J}_1 = \frac{cn}{eH} rot\left[\frac{p_\perp}{n}\frac{\overleftarrow{H}}{H}\right], div\overleftarrow{J}_1 = 0, \tag{10}$$

$$\overleftarrow{J}_2 = nc\frac{\left[\overleftarrow{E}, \overleftarrow{H}\right]}{H^2}, div\overleftarrow{J}_2 = \frac{2}{m\nu_\perp^2}\left(\overleftarrow{E}\overleftarrow{j}_m\right) - \frac{2}{m\nu_\perp^2}\left(\overleftarrow{E}\overleftarrow{j}_{gr}\right), \tag{11}$$

$$\overleftarrow{J}_3 = n\frac{mc\nu_\perp^2}{2eH^3}\left[\overleftarrow{H}, \overleftarrow{\nabla}H\right], div\overleftarrow{J}_3 = \frac{2}{me\nu_\perp^2}\left(\overleftarrow{F}_m\overleftarrow{j}_m\right), \tag{12}$$

$$\overleftarrow{J}_4 = \frac{nc}{eH^2}\left[\overleftarrow{H}, \overleftarrow{\nabla}\frac{p_\perp}{n}\right], div\overleftarrow{J}_4 = \frac{2}{me\nu_\perp^2}\left(\overleftarrow{F}_m\overleftarrow{j}_m\right) - \frac{2}{me\nu_\perp^2}\left(\overleftarrow{F}_m\overleftarrow{j}_{gr}\right), \tag{13}$$

$$\overleftarrow{J}_5 = n\frac{mc\nu_{II}^2}{eH^2R^2}\left[\overleftarrow{R},\overleftarrow{H}\right], div\overleftarrow{J}_5 = \frac{2}{me\nu_\perp^2}\left[\left(\overleftarrow{F}_c\overleftarrow{j}_m\right) + e\frac{\nu_\perp^2}{\nu_{II}^2}\left(\overleftarrow{E}\overleftarrow{j}_c\right) + \frac{\nu_\perp^2}{\nu_{II}^2}\left(\overleftarrow{F}_m\overleftarrow{j}_c\right)\right], \quad (14)$$

$$\overleftarrow{J}_6 = \frac{nc}{eH^2}\left[\overleftarrow{H},\nabla\left(\frac{p_{II}}{n}\right)\right], div\overleftarrow{J}_6 = -\frac{2}{me\nu_\perp^2}\left[2e\left(\overleftarrow{E}\overleftarrow{j}_m\right) + 2\left(\overleftarrow{F}_m\overleftarrow{j}_m\right) - 2\left(\overleftarrow{E}\overleftarrow{j}_{gr}\right) - 2\left(\overleftarrow{F}_m\overleftarrow{j}_c\right)\right], \quad (15)$$

$$\overleftarrow{J}_7 = n\nu_{II}\overleftarrow{e}_1, div\overleftarrow{J}_7 = \frac{1}{me\nu_{II}^2}\left(\overleftarrow{F}_M\overleftarrow{j}_{II}\right) + \frac{1}{me\nu_{II}^2}\left(\overleftarrow{E}\overleftarrow{j}_{II}\right). \quad (16)$$

Flows $\overleftarrow{J}_{2,3}$ arise due to electric and gradient drifts. Accounting for fluxes $\overleftarrow{J}_{4,6}$ is associated with the interdependence of magnetic pressure and plasma pressure observed in the quasi-hydrodynamic approximation, since the pressure of charged particles in the absence of collisions is transferred by currents. In addition, the fluxes $\overleftarrow{J}_{4,6}$ also take into account thermal diffusion, which is associated with the temperature gradient ($p_\perp \sim T_\perp$ and $p_\perp \sim T_\perp$). The flow $\overleftarrow{J}_5$ is associated with centrifugal forces due to the curvature of the lines of force, $\overleftarrow{J}_7$ - the flow of charged particles along the line of force. Opposite the corresponding values of the fluxes, their divergences are presented, in the derivation of which the invariance n/H and μ (first adiabatic invariant) with the accuracy of the drift approximation were taken into account and the following designations were adopted [16]:

$\overleftarrow{j}_{gr} = \frac{nc}{H^2}\mu\left[\overleftarrow{H},\overleftarrow{\nabla H}\right]$ is gradient drift current; $\overleftarrow{j}_m = -\frac{nc}{H}\mu rot\overleftarrow{H}$ is magnetizing current \$;

$\overleftarrow{j}_c = \frac{m\nu_{II}^2}{R^2}\left[\overleftarrow{R},\overleftarrow{H}\right]$ is centrifugal drift; $\overleftarrow{F}_m = -\mu\overleftarrow{\nabla}H$ is magnetic force;

$\overleftarrow{F}_c = \frac{m\nu_{II}^2}{R^2}\overleftarrow{R} = 2\frac{\varepsilon_{II}}{H}\overleftarrow{\nabla}_\perp H$ is a force, affecting a charged particle in inhomogeneous magnetic field (centrifugal). It is clear that in this case the divergence of the flow of particles $\overleftarrow{J}_1$ is equal to the divergence of the flow of leading centers, since in an ionized medium the motion of non-interacting particles differs from the motion of leading centers only by vortex terms, therefore $div\overleftarrow{J}_1 = 0$. In addition, in deriving (10), the change in the average kinetic energy along the magnetic field line was neglected. Let us consider the second term in square brackets of (9), associated with the violation of the quasi-stationarity condition. Fluctuational charge separation in plasma leads to the appearance of an alternating electric field, which is responsible for the onset of polarization drift, which, in turn, leads to the formation of a drift polarization current $\overleftarrow{j}_p$. The magnitude of the drift current arising from the separation of charges is proportional to the rate of change in the electric field strength. This allows us to consider it as a displacement current that occurs during the polarization of dielectrics. Having carried out the appropriate calculations, and without limiting the generality of the proposed approach, we consider a special case when an alternating electric field is perpendicular to the magnetic field, we obtain (see Appendix)

$$\frac{\partial}{\partial t}(n_e - n_i) = -\frac{4H^2}{\left(H^2 + 4\pi nmc^2\right)}\frac{\overleftarrow{F}_m\overleftarrow{j}_p}{em\nu_\perp^2}. \quad (17)$$

Substituting divergence values (9), calculated from the corresponding fluxes (10-16), and expression (17) into the equation of motion (9), we obtain

$$\frac{d\overleftarrow{V}}{dt} = -\frac{1}{\rho}Di\nu\overleftrightarrow{P} + \overleftarrow{F}_{ext} + \frac{2\overleftarrow{V}}{n\mu H}\left[\overleftarrow{E}\left(\overleftarrow{j}_{gr} - \overleftarrow{j}_{m} + \frac{\nu_\perp^2}{\nu_{II}^2}\overleftarrow{j}_{c} + \frac{\nu_\perp^2}{\nu_{II}^2}\overleftarrow{j}_{II}\right) + \right.$$
$$\left. + \frac{\nu_\perp^2}{e\nu_{II}^2}\overleftarrow{F}_m\left(\overleftarrow{j}_{c} + +\overleftarrow{j}_{II}\right) + \frac{1}{e}\left(\overleftarrow{F}_c\overleftarrow{j}_{m}\right)\right] + \tilde{\varepsilon}\frac{2V}{n\mu H}\frac{2H^2}{H^2 + 4\pi nmc^2}\left(\overleftarrow{F}_m\overleftarrow{j}_{p}\right), \tilde{\varepsilon} = (m_e/m_i) < < 1. \tag{18}$$

Since the derived equation uses macroscopic quantities $n, \overleftrightarrow{P}, \overleftarrow{E}, \overleftarrow{H}, \overleftarrow{V}$ as the main parameters, there is no need for additional assumptions about the form of the distribution function associated with the termination of the chain of moments and the transition to hydrodynamic equations from the Vlasov kinetic equation. However, the most important thing in Eq. (18) is that it takes into account small dissipative and fluctuation processes arising due to the interaction of drift currents with inhomogeneous electric and magnetic fields. The reason for the smallness of the fluctuations taken into account in (18) is the condition of applicability of the leading center approximation and is a consequence of the perturbation theory, which is valid up to the constancy of the first adiabatic invariant ($\mu = const$)and therefore allows the parameters to vary within this accuracy. At the same time, it is known that fluctuations in plasma are responsible for the appearance of local currents, which are determined by space-time inhomogeneities in the distribution of the field and plasma. In turn, the interaction of these currents with forces, also associated with inhomogeneities in the spatial distribution of magnetic and electric fields, determines the further development of the resulting fluctuations, as well as the nature of the possible instability.

Eq. (18) under the assumption of quasineutrality ($n_e = n_i$) and infinite conductivity along the field line is greatly simplified ($\overleftarrow{E}_{II} = 0$). In addition, if we consider a closed axially symmetric system, then the inhomogeneity in the plasma distribution along the drift trajectory may be absent and the current intensity j_{II} proportional to this inhomogeneity tends to zero. Finally, instead of (18), we obtain a simplified, but not changing the physical essence, equation

$$\frac{d\overleftarrow{V}}{dt} = -\frac{1}{\rho}Di\nu\overleftrightarrow{P} + \overleftarrow{F}_{ext} + \frac{2\overleftarrow{V}}{n\mu H}\left[\frac{\nu_\perp^2}{e\nu_{II}^2}\left(\overleftarrow{F}_m \cdot \overleftarrow{j}_{c}\right) + \frac{1}{e}\left(\overleftarrow{F}_c \cdot \overleftarrow{j}_{m}\right)\right] = \frac{1}{\rho}di\nu\overleftrightarrow{P} + \overleftarrow{F}_{ext} + \overleftarrow{f}_{dis}\left(\overleftarrow{F}, \overleftarrow{j}\right) \tag{19}$$

Eqs. (18) and (19) differ from generally used equations of motion by the third term in the right part, which describes dissipative interaction of drift currents with $e\overleftarrow{E}, \overleftarrow{F}_m, \overleftarrow{F}_c$, forces. This additional part evidently take into account magnetization of physically infinitesimal element of a continuum, since besides the dependence on drift current $\overleftarrow{j}_{gr}, \overleftarrow{j}_{c}, \overleftarrow{j}_{II}$ and $\overleftarrow{j}_{m}$, it is proportional to $1/\mu$. We should note, that in the case with axial-symmetrical plasma system, currents $\overleftarrow{j}_{m}$ and $\overleftarrow{j}_{c}$ constantly flow in it. Nevertheless, they do not break freezing-in, as $\overleftarrow{j}_{m}$ and $\overleftarrow{j}_{c}$ are directed along the azimuth and $\overleftarrow{F}_m \perp \overleftarrow{j}_{c}$, $\overleftarrow{F}_c \perp \overleftarrow{j}_{m}$. At the same time the appearance of fluctuations may cause azimuthal inhomogeneity and, consequently, coincidence of $\overleftarrow{F}_m$ and $\overleftarrow{j}_{c}$, $\overleftarrow{F}_c$ and $\overleftarrow{j}_{m}$ components. Moreover, $\overleftarrow{F}_{ext} \sim \left[\overleftarrow{j}, \overleftarrow{H}\right]$ force in the Eqs. (18) and (19) is expressed through drift current explicit values, not through $rot\overleftarrow{H}$, which is within the framework of general conception of this chapter: consideration of current corpuscular structure.

4. Dissipative system of equations in the approximation of two adiabatic invariants of Chu, Goldberger, Low in the drift approximation.

In order to obtain a complete system of hydrodynamic equations in the drift approximation, it is necessary to add Maxwell's equations to the equation of motion (19), and to close the system, add two equations of state for the parallel ρ_{II} and perpendicular $\rho_{\perp}$ components of the pressure tensor, as is done in the approximation of two adiabatic invariants of the ChGL [14]. If one equation for is a consequence of the applicability of the drift approximation and corresponds to the constancy of the first adiabatic invariant $(d\mu/dt) = 0$, then the second equation can be obtained on the basis of the energy conservation law in the drift approximation [17]

$$\frac{d\varepsilon}{dt} = e\left(\overleftarrow{E}\cdot\overleftarrow{U}_{dr}\right) + \mu\frac{\partial H}{\partial t}, \tag{20}$$

where $\varepsilon = \varepsilon_{II} + \varepsilon_{\perp} = \left(m\nu_{II}^2/2\right) + \left(m\nu_{\perp}^2/2\right)$ is particle mean energy, U_{dr} is drift velocity. From (20) we obtain

$$\frac{d\varepsilon_{II}}{dt} = e\left(\overleftarrow{E}\cdot\overleftarrow{U}_{dr}\right) + \mu\frac{\partial H}{\partial t} - \frac{d\varepsilon_{\perp}}{dt} = e\left(\overleftarrow{E}\cdot\overleftarrow{U}_{dr}\right) - \mu\left(\overleftarrow{U}_{dr}\cdot\overleftarrow{\nabla}\right)H. \tag{21}$$

Since

$$\frac{d(n\varepsilon_{II})}{dt} = \varepsilon_{II}\frac{dn}{dt} + n\frac{d\varepsilon_{II}}{dt}$$

and $p_{II} = 2n\varepsilon_{II}$, $p_{\perp} = n\varepsilon_{\perp}$ and $\dot{n} = -ndiv\overleftarrow{U}_{dr}$ are valid, than from (20,21) and the latest expression we obtain

$$\frac{dp_{II}}{dt} = 2ne\left(\overleftarrow{E}\cdot\overleftarrow{U}_{dr}\right) - \frac{2p_{\perp}}{H}\left(\overleftarrow{U}_{dr}\cdot\overleftarrow{\nabla}\right)H - p_{II}div\overleftarrow{U}_{dr}$$

or

$$\frac{dp_{II}}{dt} + p_{II}div\overleftarrow{U}_{dr} = 2ne\left(\overleftarrow{E}\cdot\overleftarrow{U}_{dr}\right) - \frac{2p_{\perp}}{H}\left(\overleftarrow{U}_{dr}\cdot\overleftarrow{\nabla}\right)H. \tag{22}$$

Relation (22) is a substantial balance equation in the drift approximation for the pressure tensor component ρ_{II} with a nonzero right-hand side (the presence of a source). We multiply the left-hand side of (22) by H^2/ρ^3 and, taking into account that $\rho div\overleftarrow{U}_{dr} = -(d\rho/dt)$, we obtain after transformations

$$\frac{H^2}{\rho^3}\left(\frac{dp_{II}}{dt} - \frac{p_{II}}{\rho}\frac{d\rho}{dt}\right) = \frac{H^2}{\rho^3}\frac{dp_{II}}{dt} + \frac{p_{II}H^2}{\rho^2}\frac{d}{dt}\left(\frac{1}{\rho}\right) = \frac{H^2}{\rho^3}\frac{dp_{II}}{dt} + p_{II}\frac{d}{dt}\left(\frac{H^2}{\rho^3}\right) = \frac{d}{dt}\left(\frac{p_{II}H^2}{\rho^3}\right).$$

Now, after multiplying the right part (22) by $\left(H^2/\rho^3\right)$, we equate this product to the latest equation. Finally, we obtain

$$\frac{d}{dt}\left(\frac{p_{II}H^2}{\rho^3}\right)=\frac{H^2}{\rho^3}\left[2ne\left(\overleftarrow{E}\cdot\overleftarrow{U}_{dr}\right)-\frac{2p_\perp}{H}\left(\overleftarrow{U}_{dr}\cdot\overleftarrow{\nabla}\right)H\right]. \tag{23}$$

The condition

$$\frac{d}{dt}\left(\frac{p_\perp}{\rho H}\right)=0, \tag{24}$$

equivalent to the condition of the first adiabatic invariant conservation, (since $\overline{v}_\perp^2\approx p_\perp/\rho$, where $\overline{v}_\perp$ is a perpendicular component of particle mean velocity), together with (23) are two condition equations for the parallel p_{II} and $p_\perp$ perpendicular components of pressure tensor, which close the dissipative system of equations in drift approximation.

Now, the first part of (23) is under analysis. Since we consider plasma systems in axial-symmetrical magnetic fields with potential electric field equal to zero, than in the stationary case $E=0$, $\left(\overleftarrow{U}_{dr}\cdot\overleftarrow{\nabla}\right)H\sim U_\varphi(\partial H/\partial\varphi)=0$ and the right part (23) identically vanish. In variable fields, in our case $\left(\overleftarrow{E}\overleftarrow{U}_{dr}\right)=E_\varphi U_\varphi$, since for electric field $\overleftarrow{E}_\varphi=-\frac{1}{c}\frac{\partial\overleftarrow{A}_\varphi}{\partial t}$ and

$$\left(\overleftarrow{U}_{dr}\cdot\overleftarrow{\nabla}\right)H\approx\overleftarrow{U}_R\frac{\partial H}{\partial R}=\frac{U_RH}{R_{cr}}=\frac{U_RH}{R}k,$$

where k is a coefficient of proportionality between field line curvature radius R_{cr} and guiding center radius-vector R [18, 19], $U_R=c(E/H)$ is electric drift velocity. In the result, for the right part of (23) we obtain

$$2\frac{H^3}{\rho^3}n\cdot U_\varphi\cdot U_R\left(\frac{e}{c}n-\frac{\mu\cdot k}{R\cdot U_\varphi}\right).$$

According to the results of the papers [18, 19], we have

$$\frac{RU_\varphi}{k}=\frac{c}{e}\mu+\frac{\nu_{II}^2}{c\omega_L}=\frac{c}{enH}\left(p_\perp+p_{II}\right)$$

and, finally, for (23)) we may write

$$\frac{d}{dt}\left(\frac{p_{II}H^2}{\rho^3}\right)=\left(\frac{p_{II}H^2}{\rho^3}\right)\cdot\frac{2nU_\varphi}{p_\perp+p_{II}}\cdot eE_\varphi, \tag{25}$$

The total equation system of two adiabatic invariant approximations, considering $\overleftarrow{f}_{dis}$ in the approximation of ideal conductivity $E_\varphi\sim\left[\overleftarrow{V},\overleftarrow{H}\right]$, is written as follows:

$$\begin{gathered}\frac{d\overleftarrow{V}}{dt}=-\frac{1}{nm}Div\overleftrightarrow{P}+\overleftarrow{F}_{ext}+\overleftarrow{f}_{dis}\left(\overleftarrow{F},\overleftarrow{j}\right),\ \frac{\partial n}{\partial t}=-\overleftarrow{\nabla}\left(n\overleftarrow{V}\right),\ \frac{d}{dt}\left(\frac{p_\perp}{\rho H}\right)=0,\\ \frac{d}{dt}\left(\frac{p_{II}H^2}{\rho^3}\right)=\left(\frac{p_{II}H^2}{\rho^3}\right)\cdot\frac{2nU_\varphi}{p_\perp+p_{II}}\cdot eE_\varphi,\ \frac{\partial\overleftarrow{H}}{\partial t}=rot\left[\overleftarrow{V},\overleftarrow{H}\right],\ \overleftarrow{E}_\varphi=\frac{1}{c}\left[\overleftarrow{V},\overleftarrow{H}\right],\end{gathered} \tag{26}$$

where - $(P)_{kn} = p_{II}\overleftarrow{e}_k\overleftarrow{e}_n + p_\perp\left(\delta_{kn} - \overleftarrow{e}_k\overleftarrow{e}_n\right), \overleftarrow{F}_{ext} = \frac{1}{H}\left[-p_\perp\overleftarrow{\nabla}_{II}H + (p_{II} - p_\perp)\overleftarrow{\nabla}_\perp H\right]$, $\overleftarrow{f}_{dis} = \frac{2c\overleftarrow{V}}{enH^3}\left[2\frac{p_\perp}{R^2}\left(\overleftarrow{\nabla}H\left[\overleftarrow{R}\overleftarrow{H}\right]\right) - p_{II}\left(rot\overleftarrow{H}\cdot\overleftarrow{\nabla}_\perp H\right)\right]$.

Let us multiply the first equation in the system (26) scalarly by V

$$\left(\overleftarrow{V}\cdot\frac{d\overleftarrow{V}}{dt}\right) = -\frac{1}{nm}\left(\overleftarrow{V}\cdot Div\overleftrightarrow{P} + \left(\overleftarrow{V}\cdot\overleftarrow{F_{ext}}\right) + \left(\overleftarrow{V}\cdot\overleftarrow{f}_{dis}\left(\overleftarrow{F},\overleftarrow{j}\right)\right)\right.$$

Since we are interested in the influence of the dissipative term on the character of motion of a plasma element with macroscopic velocity $\overleftarrow{V}$, let us assume for simplicity that the scalar product of the first two terms is early to zero, then, given the explicit form $\overleftarrow{f}_{dis}$, we obtain

$$\frac{1}{2}\frac{dV^2}{dt} = \frac{2cV^2}{enH^3}\left[2\frac{P_\perp}{R^2}\left(\overleftarrow{\nabla}H\left[\overleftarrow{R},\overleftarrow{H}\right]\right) - p_{II}\left(rot\overleftarrow{H}\cdot\overleftarrow{\nabla}_\perp H\right)\right].$$

The last expression shows that in the case of fluctuations, azimuthal inhomogeneity may appear and, as a consequence, to the coincidence of the direction of the components $\overleftarrow{F}_m$ and $\overleftarrow{j}_c$, and $\overleftarrow{j}_m$ (see (19) and explanations to it), then, depending on the sign of the term in square brackets, the energy of the plasma element will increase or change.

5. Conclusions

The right parts of the functions $\overleftarrow{F}_{ext}$ and $\overleftarrow{f}_{dis}$ are expressed through drift current explicit values separating the components of pressure tensor $p_\perp$ and p_{II}. In the system (26) the unknown values are $p_\perp$, p_{II}, $\overleftarrow{H}$, $\overleftarrow{E}_\varphi$, n and $\overleftarrow{V}$.

Obtaining theoretical models describing the motion of continuous systems is an important branch of continuum mechanics. The construction of these models is based both on the use of experimental data and on the application of the well-known principles of mechanics, thermodynamics, physics, and they are based on the search for additional relationships between the parameters describing the state of the considered continuous medium. It is known that the basic equations of mechanics, electrodynamics, hydrodynamics, and so on are derived on the basis of the variational Lagrange equation. The corresponding analysis shows that with the help of variational principles it is possible to construct any physical models describing both reversible and non-reversible processes. Therefore, the application of the principles of Prigogine and Onsager, combined by Gyarmati, to obtain the equation of motion of a magnetized plasma at the hydrodynamic level of description seems to be quite promising. And here the following should be noted.

In the hydrodynamic approximation, fluctuations are not taken into account, since in continuum mechanics it is assumed to be continuous. The Navier-Stokes equation, in contrast to the Euler equation, already takes into account dissipative phenomena, but does not contain fluctuation interactions (without additional assumptions about the form of the stress tensor that takes into account the molecular structure), which describe Brownian motion. Relying on the concept of continuity, as already noted, it is impossible in the mechanics of continuous media to take into account the fluctuations of the hydrodynamic functions formed due to the molecular structure of the medium. At this level of description, taking into account the atomic-molecular structure leads to the Langevin equation, in which the parameters of the medium are described by random sources. These sources are responsible for fluctuations $\rho, \overleftarrow{V}, T$ and being unavoidable properties of the medium, cannot be excluded. Therefore, postulating Langevin sources in hydrodynamics brings the corresponding equations as close as possible to describing the behavior of a real medium.

In turn, the possibility of taking into account the structure of a physically infinitesimal plasma element in this work was achieved, on the one hand, by using the variational methods of Prigogine and Onsager, combined by Gyarmati, and making it possible to obtain a completely self-consistent equation with the accuracy of the chosen drift approximation. On the other hand, this approximation, being single-particle, initially takes into account the discreteness of the considered ionized medium ("atomic-molecular" structure). In addition, it admits small perturbations within the limits of its accuracy, that is, within the limits of the constancy of the first adiabatic invariant μ.

1. The application of the methods of nonequilibrium thermodynamics based on the combined principles of Prigogine and Onsager for the description in the linear approximation of transport processes in a collisionless plasma and taking into account the structure of a physically infinitesimal element of the medium ($\ell_f \sim \lambda_D$) makes it possible to obtain the equation of motion of an electron-ion plasma in the drift approximation. This equation takes into account fluctuation-dissipative processes, which are determined by the interaction of local drift currents and forces. The expediency of an approach in which the influence of local currents is taken into account in describing the behavior of a nonequilibrium plasma was noted in [20]. The resulting fluctuations lead to the formation of spatial inhomogeneities in the distribution of the field and plasma and to the coincidence of the components of the current and forces and "turn on" the dissipative source, which determines the further development of possible instabilities.

2. The transition from an arbitrary to an axially symmetric magnetic system greatly simplifies the equation of motion, but retains the basis associated with taking into account the structure of a physically infinitesimal element. This is fundamental in comparison with the usual nondissipative Euler equation, which is used in a one-fluid hydrodynamic system of equations and in the system of equations of two adiabatic invariants of Chu, Goldberger, and Low.

3. The possibility of kinetic foundation of the postulated in the paper random source in pressure nonequilibrium part appears, when small scale initial correlations, which are superimposed for derivation of Landau and Vlasov equations, partially decreasing.

4. The obtained equation of motion can be used when taking into account the scattering of charged particles by electromagnetic fluctuations. This determines an additional mechanism for the regularization of particle motion in a magnetized plasma, which automatically implies a revision of the scale associated with the path length determined by the Coulomb collision, since it may turn out to be much

larger than the distance between collisions on fluctuations. For example, in the problem of plasma flow around the solar wind of the Earth's magnetosphere, the characteristic size of the latter is much less than the mean free path corresponding to Coulomb collisions. This, proceeding from rigorous considerations, indicates the inadmissibility of using the hydrodynamic approximation to describe the processes in this problem. However, the experimental data are in good agreement with the results that follow for this problem from the solution of hydrodynamic equations, which indicates the presence of an effective particle scattering mechanism, which leads to a significant decrease in the mean free path in comparison with Coulomb collisions [12].

Abbreviations

T_L	the period of Larmor's rotation
ℓ_f	scale, characterizing a physically small "point" of a solid medium
g	plasma parameter
n	particle concentration
λ_D	Debye radius
H	magnetic field strength
$\overleftarrow{V}$	macroscopic average plasma velocity
P	pressure tensor
$\overleftarrow{j}$	current density
ρ_{L_i}	Larmor's ion radius
ρ_{L_e}	Larmor's electron radius
L	lagrangian
ω	frequency
ω_{L_i}	Larmor frequency of the ion
ω_{0_i}	plasma ion frequency
τ_f	characteristic time
N_f	number of particles in volumel_f^3
V_T	thermal velocity of the particle
σ	entropy production function
J_i	fluxes corresponding to the observed transfer processes
X_i	thermodynamic forces
Ψ	scattering potentials
Φ	scattering potentials
$L_{i,k}$	Onsager's reciprocity coefficient
$R_{i,k}$	inverse Onsager reciprocity coefficient
U	volume
$\overleftrightarrow{P}_d$	nonequilibrium part of the pressure tensor
$\overleftrightarrow{P}^{i,e}$	equilibrium part of the pressure tensor of the ionic and electron components
$\overleftrightarrow{P}_d^{i,e}$	nonequilibrium part of the pressure tensor of the ion and electron components
$\overleftrightarrow{P}_\Sigma^{i,e}$	total pressure for the electron and ion component

$m_{i,e}$	mass of an ion or electron
$\overleftrightarrow{I}$	unit tensor
$\overleftarrow{V}_{i,e}$	macroscopic velocity of the ion and electron components
$p^{i}_{II,\perp}$	parallel and perpendicular pressure components
$\overleftarrow{e}_{k}$	unit vectors
$\overleftarrow{e}_{n}$	unit vectors
$\overleftarrow{e}_{1} = \overleftarrow{H}/H$	a unit vector pointing along the field
$\rho_{i,e}$	density of the ion and electron components of the plasma
$\overleftarrow{F}^{i,e}_{ext}$	external force acting on electrons and ions
$T_{i,e}$	temperature of the ion and electron plasma components
$\overleftarrow{V}_{i,e}$	average velocity of the ionic and electronic components
$\nu_{II,\perp}$	perpendicular and parallel components of the particle velocity
$\overleftarrow{j}_{m}$	magnetizing current
$\overleftarrow{j}_{gr}$	gradient drift current
$\overleftarrow{R}$	radius-vector of the particle
$\overleftarrow{F}_{m}$	magnetic force
$\overleftarrow{F}_{c}$	centripetal force
$\overleftarrow{j}_{c}$	centripetal current
$\overleftarrow{j}_{II}$	parallel current
μ	magnetic moment magnetic moment
$\overleftarrow{j}_{p}$	polarizing current
ε	particle energy
U_{dr}	drift velocity
E_{φ}	azimuthal component of the electric field
U_{φ}	azimuthal drift velocity
k	coefficient of proportionality between the radius of curvature of the force line and the radius vector
R_{cr}	radius of curvature of the force line
U_{R}	radial drift velocity
$\overleftarrow{f}_{dis}$	dissipative force
$\tilde{\varepsilon}$	order of smallness
$\overleftarrow{A}_{\varphi}$	azimuthal component of the vector potential

Appendix

Let us calculate the divergences from each of the fluxes $J_2 - J_7$, whose explicit form is represented by expressions (10a)–(10g) $(divJ_1 = 0)$. In the preconversion process we will take into account the invariance of n/H and the first adiabatic invariant $/mu = \left(m\nu_{\perp}^{2}/2H\right)$, as well as the corresponding preconversion of the divergence from the vector product $div[A, B] = (B \cdot rotA)(A \cdot rotB)$.

For the flow divergence J_2 we obtain

$$div\overleftarrow{J}_2 = div\left(nc\frac{\left[\overleftarrow{E},\overleftarrow{H}\right]}{H^2}\right) = \frac{nc}{H^2}div\left[\overleftarrow{E},\overleftarrow{H}\right] + \left[\overleftarrow{E},\overleftarrow{H}\right]div\left(\frac{nc}{H^2}\right) = -\frac{nc}{H^2}\left(\overleftarrow{E}rot\overleftarrow{H}\right) -$$
$$-\frac{nc}{H}\left(\left[\overleftarrow{E},\overleftarrow{H}\right]\overleftarrow{\nabla}\frac{1}{H}\right) = \frac{2}{mv_\perp^2}\left(\overleftarrow{E}\overleftarrow{j}_m\right) - \frac{nc}{H^3}\overleftarrow{E}\left[\overleftarrow{H},\overleftarrow{\nabla}H\right] = \frac{2}{mv_\perp^2}\left(\overleftarrow{E}\overleftarrow{j}_m\right) - \frac{2}{mv_\perp^2}\left(\overleftarrow{E}\overleftarrow{j}_{gr}\right), \tag{27}$$

where

$$\overleftarrow{j}_m = -\frac{nc}{H}\mu rot\overleftarrow{H}, j_{gr} = \frac{nc}{H^2}\mu\left[\overleftarrow{H},\overleftarrow{\nabla}H\right].$$

Let's calculate the divergence from the J_3 flow:

$$div\overleftarrow{J_3} = div\left(n\frac{mcv_\perp^2}{2eH^3}\left[\overleftarrow{H},\overleftarrow{\nabla}H\right]\right) = n\frac{mcv_\perp^2}{2eH^3}div\left[\overleftarrow{H},\overleftarrow{\nabla}H\right] + \left[\overleftarrow{H},\overleftarrow{\nabla}H\right]\overleftarrow{\nabla}n\frac{mcv_\perp^2}{2eH^3} =$$
$$= -\frac{p_\perp}{eH^3}\overleftarrow{\nabla}Hrot\overleftarrow{H} - \left[\overleftarrow{H},\overleftarrow{\nabla}H\right]n\frac{mcv_\perp^2}{2eH^2}\overleftarrow{\nabla}\frac{1}{H} = \tag{28}$$
$$= \frac{\left(\overleftarrow{j}_m\overleftarrow{\nabla}\right)H}{eH} + n\frac{mcv_\perp^2}{2eH^4}\left[\overleftarrow{H},\overleftarrow{\nabla}H\right] = \frac{2}{mev^2}\left(\overleftarrow{F}_m\overleftarrow{j}_m\right),$$

The final result in formulas (27) and (28) correspond to formulas (11) and (13). where $\overleftarrow{F}_m = -\mu\overleftarrow{\nabla}H$.

Similarly, transform the divergences from the fluxes $J_4 - J_7$, we obtain

$$div\overleftarrow{J}_4 = -\frac{2}{mev_\perp^2}\left(\overleftarrow{j}_m\overleftarrow{\nabla}\frac{p_\perp}{n}\right) + \frac{2}{mev_\perp^2}\left(\overleftarrow{j}_{gr}\overleftarrow{\nabla}\frac{p_\perp}{n}\right). \tag{29}$$

$$div\overleftarrow{J}_5 = -\frac{2}{mev_\perp^2}\left(\overleftarrow{F}_c\overleftarrow{j}_m\right) + \frac{2}{mev_{II}^2}\left(\overleftarrow{j}_c\overleftarrow{\nabla}\frac{p_{II}}{n}\right), \tag{30}$$

$$div\overleftarrow{J}_6 = -\frac{2}{mev_\perp^2}\left(\overleftarrow{j}_m\overleftarrow{\nabla}\frac{p_{II}}{n}\right) + \frac{2}{mev_\perp^2}\left(\overleftarrow{j}_{gr}\overleftarrow{\nabla}\frac{p_{II}}{n}\right), \tag{31}$$

$$div\overleftarrow{J}_7 = -\frac{n}{H}\left(\overleftarrow{H}\overleftarrow{\nabla}v_{II}\right) = \left(\overleftarrow{e_1}\overleftarrow{\nabla}v_{II}\right) \tag{32}$$

In (29)-(32) it is necessary to transform the gradient terms from and. To do this, we use the invariance of $(p_\perp/n)$, (p_{II}/n) and v_{II}. Since

$$\overleftarrow{\nabla}\frac{p\perp}{n} = \overleftarrow{\nabla}\frac{\overleftarrow{m}v_\perp^2}{2} = \overleftarrow{\nabla}\frac{\overleftarrow{m}v_\perp^2}{2H}H = \mu\overleftarrow{\nabla}H = -\overleftarrow{F}_m,$$

Then

$$\left(\overleftarrow{j}_m\overleftarrow{\nabla}\frac{p_\perp}{n}\right) = -\left(\overleftarrow{j}_m\overleftarrow{F}_m\right) \tag{33}$$

and

$$\left(\overleftarrow{j}_{gr}\overleftarrow{\nabla}\frac{p_{\perp}}{n}\right) = -\left(\overleftarrow{j}_{gr}\overleftarrow{F}_{m}\right). \tag{34}$$

In a stationary magnetic field it is true with the accuracy of the drift approximation [21]

$$\frac{d\nu_{II}}{dt} = \frac{e}{m}\left(\overleftarrow{E}\overleftarrow{e}_{1}\right) + \frac{\nu_{\perp}^{2}}{2}div\overleftarrow{e}_{1}. \tag{35}$$

Assume that the first term in (35) is zero (the electric field is perpendicular to the magnetic field). Convert the second term in (35)

$$\frac{\nu_{\perp}^{2}}{2}div\overleftarrow{e}_{1} = \frac{\nu_{\perp}^{2}}{2}div\frac{\overleftarrow{H}}{H} = \frac{\nu_{\perp}^{2}}{2}div\overleftarrow{H} - \frac{\nu_{\perp}^{2}}{2H^{2}}\left(\overleftarrow{H}\overleftarrow{\nabla}H\right) = -\frac{\mu}{m}\left(\overleftarrow{e}_{1}\overleftarrow{\nabla}H\right) = -\frac{1}{m}\left(\overleftarrow{e}_{1}\overleftarrow{F}_{m}\right).$$

Given this transformation (35) will take the form

$$\frac{d\nu_{II}}{dt} = \frac{1}{m}\left(\overleftarrow{e}_{1}\overleftarrow{F}_{m}\right). \tag{36}$$

In addition, for the constant magnetic field in the drift approximation it is true

$$\frac{d\nu_{II}}{dt} = \frac{\partial\nu_{II}}{\partial t} + \left(\overleftarrow{U}\overleftarrow{\nabla}\right)\overleftarrow{\nabla}\nu_{II} = \left(\nu_{II}\overleftarrow{e}_{1} + \overleftarrow{U}_{dr.}\right)\overleftarrow{\nabla}\nu_{II} \approx \nu_{II}\left(\overleftarrow{e}_{1}\overleftarrow{\nabla}\nu_{II}\right). \tag{37}$$

By equating (36) and (37), we obtain

$$\overleftarrow{e}_{1}\nu_{II}\overleftarrow{\nabla}\nu_{II} = \frac{1}{m}\left(\overleftarrow{e}_{1}\overleftarrow{F}_{m}\right). \tag{38}$$

Let's write the gradient from

$$\overleftarrow{\nabla}\left(\frac{p_{II}}{n}\right) = \overleftarrow{\nabla}m\nu_{II}^{2} = 2m\nu_{II}\overleftarrow{\nabla}\nu_{II},$$

whence, taking into account, we have

$$\overleftarrow{j}_{gr.}\overleftarrow{\nabla}\frac{p_{II}}{n} = 2e\left(\overleftarrow{E}\overleftarrow{j}_{gr.}\right) + 2\left(\overleftarrow{F}_{m}\overleftarrow{j}_{gr.}\right), \tag{39}$$

$$\overleftarrow{j}_{m.}\overleftarrow{\nabla}\frac{p_{II}}{n} = 2e\left(\overleftarrow{E}\overleftarrow{j}_{m.}\right) + 2\left(\overleftarrow{F}_{m}\overleftarrow{j}_{m.}\right), \tag{40}$$

$$\overleftarrow{j}_{c.}\overleftarrow{\nabla}\frac{p_{II}}{n} = 2e\left(\overleftarrow{E}\overleftarrow{j}_{c.}\right) + 2\left(\overleftarrow{F}_{m}\overleftarrow{j}_{c.}\right), \tag{41}$$

$$\left(\overleftarrow{e}_{1}\overleftarrow{\nabla}\nu_{II}\right) = \frac{e}{m\nu_{II}}\left(\overleftarrow{E}\overleftarrow{e}_{1}\right) + \frac{\left(\overleftarrow{e}_{1}\overleftarrow{F}_{m}\right)}{m\nu_{II}}, \tag{42}$$

By substituting the values of (33), (34), (39)-(42), into (29)-(32), we obtain the expressions (13), (14), (15) and (16) presented in Section 3, respectively.

If a time-varying electric field acts in the plasma, the crossed $\overleftarrow{E}, \overleftarrow{H}$ fields produce an acceleration of electric drift

$$\frac{d\overleftarrow{V}_E}{dt} = c\frac{\left[\overleftarrow{E}, \overleftarrow{H}\right]}{H^2} \tag{43}$$

creating an inertial force $\overleftarrow{F}_{iner.} = -m\ \dot{\overleftarrow{\nu}}_E$.

In the drift approximation, the electric field $\overleftarrow{E}$ and its rate of change are limited by $(cE/H) < < V$ and $(\partial E/\partial t < < E/T_L)$ (T_L is the period of Larmor's rotation).

The force (43) causes drift with speed

$$\overleftarrow{\nu}_P = \frac{mc^2}{H^2}\dot{\overleftarrow{E}} \tag{44}$$

and leads to the occurrence of electric polarization current

$$\overleftarrow{j}_P = ne\overleftarrow{\nu}_P = \frac{nmc^2}{H^2}\dot{\overleftarrow{E}} \tag{45}$$

In (44) and (45) we took into account the equality to zero of the scalar product $\left(\overleftarrow{E} \cdot \overleftarrow{e}_1\right)$. According to (45) we have

$$\frac{\partial\overleftarrow{E}}{\partial t} = \frac{H^2}{nmc^2}\overleftarrow{j}_p. \tag{46}$$

Since

$$(n_e - n_i) = \frac{1}{4\pi e}div\ \dot{\overleftarrow{E}}\ ,$$

then

$$\frac{\partial}{\partial t}(n_e - n_i) = \frac{1}{4\pi e}div\frac{\partial\overleftarrow{E}}{\partial t}. \tag{47}$$

Substituting the values of from (46) into (47), we obtain

$$\frac{\partial(n_e - n_i)}{\partial t} = \frac{1}{4\pi e}div\left[\frac{H^2}{nmc^2}\overleftarrow{j}_P\right].$$

Calculating the divergence from the expression in square brackets of the last expression gives

$$\frac{\partial}{\partial}(n_e - n_i) = \frac{H^2}{4\pi emc^2}div\overleftarrow{j}_P - \frac{H^2}{\pi nemc^2}\frac{1}{m\nu_\perp^2}\left(\overleftarrow{j}_P\overleftarrow{F}_m\right). \tag{48}$$

In the derivation of (48) the spatial derivatives of $\overleftarrow{E}$ were neglected with the accuracy of the drift approximation. Let us now calculate the value of $div\overleftarrow{j}_P$. To do this, we substitute in Maxwell's equation

$$rot\overleftarrow{H} = \frac{1}{c}\frac{\partial \overleftarrow{E}}{\partial t} + \frac{4\pi}{c}\overleftarrow{j}_P$$

value of the current $\overleftarrow{j}_P$ from (45) and after simple transformations we obtain

$$rot\overleftarrow{H} = \frac{æ}{c}\overleftarrow{E},$$

where $æ = 1 + \frac{4\pi nmc^2}{H^2}$ [16]. From which we get

$$\frac{\partial \overleftarrow{E}}{\partial t} = \frac{c}{æ} rotH. \tag{49}$$

Substituting the value of the derivative according to (49) and (45), we obtain

$$\overleftarrow{j}_P = \frac{nmc^3}{(H^2 + 4\pi nmc^2)} rot\overleftarrow{H}. \tag{50}$$

Let's calculate the divergence from the right and left parts of (50), we get

$$div\overleftarrow{j}_P = \frac{4}{m\nu_\perp^2}\left(\overleftarrow{j}_P \cdot \overleftarrow{F}_m\right). \tag{51}$$

After substituting in (48) the value of, according to (51), we finally obtain expression (17) for the derivative of the concentration difference $(n_e - n_i)$ given in Section 3,

$$\frac{\partial}{\partial t}(n_e - n_i) = -\frac{4H^2}{(H^2 + 4\pi nmc^2)} \frac{\left(\overleftarrow{j}_P \overleftarrow{F}_m\right)}{em\nu^2}.$$

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References

[1] Alfven H. Space Plasma. M.: Mir; 1983. p. 216

[2] Klimontovich YL. Statistical Physics. New York: Harwood Acfdemic Pablishers; 1986

[3] Volkov TF. Gidrodinamicheskoe opisanie sil'no razregennoi plasmy. Voprosy fizisici plazmy. Pod red. M.A. Leontovicha. M.: Atomizdat; 1964. pp. 3-19

[4] Petschekb HE. The physics of solar flares. In: Ness WH, editor. AAS-NASA Symposiom. Vol. **SP-50**. NASA; 1964. p. 425

[5] Onsaqer L. Reciprocal relations in irreversible processes. Physics Review. 1931;**37**:405

[6] Priqoqin I. Bull. Acad. Roy. Belq. Cl. Sci. 1945;**31**:600

[7] Gyarmati I. Neravnovesnaya termodinamica. Vol. **304**. M.: Mir; 1974

[8] Landau LD. Kineticheskoe uravnenie v sluchae culonovskogo vzaimodeystviya. ZhETF. 1937;**7**:154

[9] Vlasov AA. Vibratsonnye svoystva electronnogo gaza. ZhETF. 1938;**8**:291

[10] Klimontovich YL. Statistical Theory of Open Systems. Dordrecht: Kluwer Academic pablishes; 1995. p. 1

[11] Klimontovich YL. Statisticheskaya teoriya otkrytykh sistem. Vol. **2**. Ðœ.: Yanus-K; 1999. p. 438

[12] Baranov VB, Krasnobaev KV. Gidrodinamicheskaya teoriya kosmicheckoy plazmy. M.: Nauka; 1977. p. 336

[13] Kadomtsev BB. Kollektivnye protsy v plazme. M.: Nauka; 1976. p. 340

[14] Chew G, Goldberger M, Low F. Proceedings of the Royal Society. 1956; **A236**:1204

[15] Nortrop T. The Adiabatic Motion of Charged Particles. New York-London-Sydney: Interscience Publishers, a division of John Wiley and Sons; 1963

[16] Frank-Kamenskii DA. Lectures po fizike plazmy. M.: Atomizdat; 1968. p. 288

[17] Morozov AI, Solovev LS. Dvizhenie zaryannykh chastits v elektromagnitnykh polyakh. Voprosy fiziki Plazmy. Vol. 2. Pod red. M.A. Leontovicha. M.: Atomizdat; 1963. pp. 177-261

[18] Bogdanov VV, Pletnev VD. K voprosy o tochnosti sokhraneniya tret'ego adiabaticheskogo invarianta dvizheniya zaryannykh chastits v aksial'no-simmetrichnykh polyakh. Cosmic Research. 1972;**10**(3):358-367

[19] Bogdanov VV, Pletnev VD. K voprosy o tochnosti sokhraneniya tret'ego adiabaticheskogo invarianta dvizheniya zaryannykh chastits v aksial'no-simmetrichnykh polyakh. Cosmic Research. 1972;**10**(4):528-531

[20] Fadeev VM, Kvardtskhava IF, Komarov NN. Yadernyy sintez. 1965;**5**: 202

[21] Bogolubov NN, Mitropolski UA. Asimtoticheskie metody v teorii nelineinyx colebanij. Nauka. 1974:504