

# Influence of Greenhouse Gases to Global Warming on Account of Radiative Forcing

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## 1. Introduction

Radiative forcing is a measure of the size of a greenhouse gas's contribution to global warming. Radiative forcing values are estimated by a numerical process using radiative transfer schemes for terrestrial radiation and data from general circulation models. However, the estimation is complex and difficult to understand for non-specialists, including researchers in other fields. Understanding the essence of the Earth system is important for correctly discussing global environmental issues. Accordingly, in this chapter, radiative forcing values are calculated from a simple, intuitive radiative transfer model using the absorption spectra of greenhouse gases and the Planck formula for terrestrial radiation. [1]

### 1.1. Radiative forcing

The global atmospheric carbon dioxide (CO<sub>2</sub>) concentration has increased from about 278 ppmv in pre-industrial times (defined as 1750) to 390.5 ppmv in 2011. During the same period, the concentrations of methane (CH<sub>4</sub>) and nitrous oxide (N<sub>2</sub>O) have also increased from about 0.722 ppmv to 1.803 ppmv and about 0.270 ppmv to 0.324 ppmv, respectively (Table 1). [2]

Greenhouse gas	Concentration (ppmv)		Radiative forcing (W m <sup>-2</sup> )	Global warming potential	
	1750	2011		20 yr	100 yr
Carbon dioxide	278	390.5	1.82 (1.63 to 2.01)	1	1
Methane	0.722	1.803	0.48 (0.43 to 0.53)	84	28
Nitrous oxide	0.270	0.324	0.17 (0.14 to 0.20)	264	265

**Table 1.** Concentrations in 1750 and 2011, radiative forcing, and global warming potential for each greenhouse gas. [2]

Radiative forcing is often referred to as an index of the size of a greenhouse gas's contribution to global warming. When the Earth system is at radiative equilibrium, the energy flux reaching the top of the Earth's atmosphere exactly balances with the outgoing energy flux from the Earth to outer space. However, increasing the concentration of greenhouse gases decreases the energy flux to outer space and changes the energy flux to the Earth into a surplus. Consequently, the Earth's surface temperature and atmospheric temperature rise, causing the outgoing flux to increase, and the Earth system shifts to a new equilibrium. Radiative forcing is defined as the imbalance of the energy flux density caused by these perturbations. The Intergovernmental Panel on Climate Change (IPCC) has estimated the following radiative forcing values due to increased greenhouse gas concentrations in 2011 relative to their pre-industrial levels:  $\text{CO}_2$ ,  $1.82 \text{ W m}^{-2}$ ;  $\text{CH}_4$ ,  $0.48 \text{ W m}^{-2}$ ; and  $\text{N}_2\text{O}$ ,  $0.17 \text{ W m}^{-2}$ . The unit of radiative forcing is the same as that of energy flux density. The uncertainties in these values are all  $\pm 10\%$  under 90% confidence intervals. While the magnitude of the positive radiative forcing of greenhouse gases is well understood, the effects of other atmospheric constituents such as aerosols are subject to considerable uncertainty.[3] IPCC estimated values are the sum of the contributions from direct effect (via emissions of gases) and several indirect effects (via atmospheric chemistry). Radiative forcing to be compared with calculated value in this chapter is the direct contribution:  $\text{CO}_2$ ,  $1.68 \text{ W m}^{-2}$ ;  $\text{CH}_4$ ,  $0.64 \text{ W m}^{-2}$ ; and  $\text{N}_2\text{O}$ ,  $0.17 \text{ W m}^{-2}$ .

Global warming potential (GWP) is also used in comprehensive policies regarding the regulation of greenhouse gases. GWP is a measure of how much a given mass of a greenhouse gas contributes to global warming and is usually defined as the radiative forcing resulting from an instantaneous release of 1 kg of the greenhouse gas into the atmosphere relative to that of  $\text{CO}_2$ . [4] The GWP values for the next 20 and 100 years are given in Table 1.

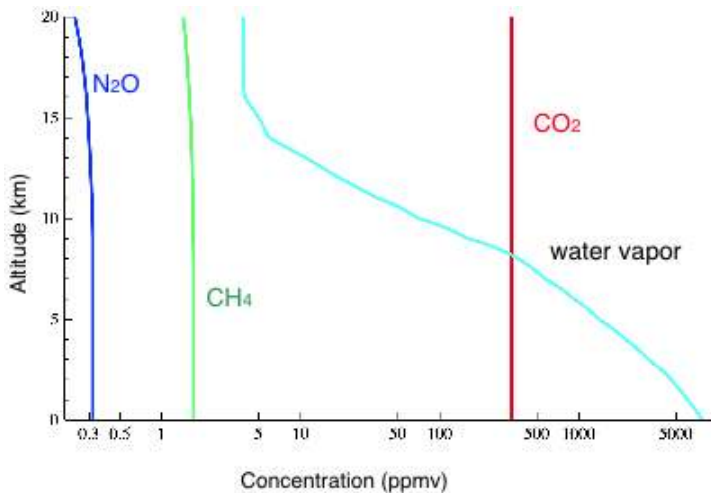
Radiative forcing is estimated by a numerical process using radiative transfer schemes for terrestrial radiation and data from models referred to as general circulation models.[5] However, since the process is difficult to understand for non-specialists, including many citizens and researchers in other fields, they simply accept the results announced by the specialists. Yet, understanding the essence of the Earth system is important for correctly discussing global environmental issues. It is therefore necessary to create models such that anyone who has acquired basic scientific knowledge can intuitively understand the Earth system as well as the essence of the calculations based on the models.

In this chapter, radiative forcing values are calculated from a simple radiative transfer model using the absorption spectra of greenhouse gases and the Planck formula for terrestrial radiation. Furthermore, the GWPs of specific greenhouse gases are derived. Finally, the increase in the Earth's surface temperature due to radiative forcing is estimated.

## 1.2. How can radiative forcing be calculated?

The mean vertical temperature of the atmosphere results from the balance between heating and cooling. The Earth's surface and the troposphere are strongly coupled by convective heat transfer processes. At the surface, solar heating is balanced by convective transport of latent and sensible heat to the troposphere. In the troposphere, radiative cooling (infrared emission by molecules) is balanced by the release of latent heat via condensation and precipitation and

by convective transport of sensible heat from the surface. This radiative-convective interaction leads to a roughly constant lapse rate in the troposphere. However, at the top of the troposphere (the tropopause), which is at an altitude of about 11 km above the Earth's surface, the temperature tends to become invariant with altitude.[6]



**Figure 1.** Vertical concentration profiles of four principal greenhouse gases. Concentrations are shown in units of ppmv. These data in the US Standard Atmosphere are obtained from the SpectralCalc website. [7]

The IPCC has defined radiative forcing as the change in net energy flux density at the tropopause. Figure 1 shows the altitude dependence of the concentrations of the four principal greenhouse gases. Water vapor is the most abundant and important greenhouse gas in the atmosphere. Nevertheless, it is excluded as an objective of the radiative forcing estimation since humans cannot directly control it. However, because of its strong absorption band, water vapor should be considered when calculating the radiative forcing of the other greenhouse gases.

The first step in building the model is to divide the atmosphere into appropriate layers, where the pressure, temperature, and concentration of each greenhouse gas are homogeneous within each layer. The concentrations of CO<sub>2</sub>, CH<sub>4</sub>, and N<sub>2</sub>O are homogeneous between the Earth's surface and the tropopause, whereas the concentration of water vapor changes considerably. Moreover, the temperature of the troposphere decreases with altitude at a roughly constant lapse rate. For intervals with a thickness of 100 m, the change in the concentration of water vapor between the adjacent layers is around 5% and the change in temperature is 0.65 K.

Therefore, up to an altitude of 11 km, the atmosphere is divided into 110 layers, each with a thickness of 100 m. The bottom layer tangent to the surface is referred to as the zeroth layer for descriptive purposes.

The terrestrial energy flux emitted from the Earth's surface enters the zeroth layer. Some of the incident flux is absorbed by the molecules of greenhouse gases in the layer, and the remainder is transmitted. Subsequently, the molecules emit radiation flux both upward and downward. Some of the combined upward flux (transmitted flux and emitted upward flux) is again absorbed in the first layer, and the layer emits radiation.[8] For simplicity, the molecules emit radiation only once. The repetition of this simple radiative transfer process leads to the outgoing flux from the tropopause to outer space.

In the first step, the outgoing energy flux density  $\bar{F}$  at the tropopause is calculated under the assumption that the concentrations of greenhouse gases in the atmosphere are equal to those in the pre-industrial era. In the next step, the flux density  $F_i$  is similarly calculated for an atmosphere in which the concentration of a specific greenhouse gas  $i$  increases up to its level in 2011, keeping everything else constant including the temperature.[4] The radiative forcing is obtained as  $\Delta F_i = \bar{F} - F_i$  for the greenhouse gas  $i$  that changes in concentration.

In these processes, the effects of the near-infrared region of incident solar radiation are ignored. Also, the intensities of absorption and emission depend on only the respective number densities of the greenhouse gases in the layer. Pressure and temperature affect these intensities through only a change in number density. Thus, for the same concentration of a given greenhouse gas, the ratio of the absorption or emission of an arbitrary layer to that of the zeroth layer is equal to the ratio of the respective number densities for that gas. When concentration is dependent on altitude, as in the case of water vapor, the number density ratio is multiplied by the factor of altitude dependence. The relation between number density and altitude is discussed in the following section.

## 2. Mathematical relation of number density of gas molecules

Assume that the atmosphere within the troposphere is composed of an ideal gas with a density  $\rho(z)$  at an altitude  $z$  and a mean molecular mass  $\mu$ . The pressure  $p(z)$  and the temperature  $T(z)$  are described using the temperature in the zeroth layer  $T_0$ , the lapse rate in the troposphere  $\Gamma$ , and the universal gas constant  $R$  as follows:

$$p = \frac{R}{\mu} \rho T, \quad (1)$$

$$T = T_0 + \Gamma z. \quad (2)$$

Using the hydrostatic equation

$$\frac{dp}{dz} + \rho g = 0, \quad (3)$$

where  $g$  is the gravitational acceleration, we arrive at the following equation for the pressure:

$$\frac{dp}{dz} = -\frac{\mu g}{R(T_0 + \Gamma z)} p. \quad (4)$$

Equation (4) can be easily integrated from a height of 0 to  $z$  to obtain:

$$\frac{p}{p_0} = \exp\left[-\frac{\mu g}{R\Gamma} \ln(1 + \Gamma z / T_0)\right]. \quad (5)$$

Again, using Eq. (1), the number density ratio of the atmosphere  $N(z)$  is as follows:

$$\frac{N}{N_0} = \frac{\rho}{\rho_0} = \frac{p}{p_0} \frac{T_0}{T_0 + \Gamma z} = \frac{T_0}{T_0 + \Gamma z} \exp\left[-\frac{\mu g}{R\Gamma} \ln(1 + \Gamma z / T_0)\right]. \quad (6)$$

The subscript 0 denotes the value for the zeroth layer. The number density of a well-mixed greenhouse gas  $n(z)$  is related by the number density of the atmosphere:

$$n = CN, \quad (7)$$

where  $C$  is the concentration (volume mixing ratio) of the greenhouse gas. For the three gases ( $\text{CO}_2$ ,  $\text{CH}_4$ , and  $\text{N}_2\text{O}$ ) that have concentrations roughly independent of altitude, the ratio of number density is as follows:

$$\frac{n}{n_0} = \frac{T_0}{T_0 + \Gamma z} \exp\left[-\frac{\mu g}{R\Gamma} \ln(1 + \Gamma z / T_0)\right]. \quad (8)$$

In the case of water vapor where the concentration changes with the altitude, Eq. (8) is multiplied by the factor of altitude dependence.

### 3. Simple radiative transfer

To obtain the change of outgoing energy flux density at the tropopause, it is necessary to calculate the radiative transfer of terrestrial radiation.[9] For a wavelength in the range between  $\lambda$  and  $\lambda+d\lambda$ , the net change in flux density passing through a layer with a thickness  $\Delta z$  is described as follows:

$$\Delta I d\lambda = -\sigma_a n \Delta z I d\lambda + \sigma_a n \Delta z B d\lambda, \quad (9)$$

where  $I(\lambda, z)$  is the intensity, which is defined as the amount of radiant energy leaving a unit area of a body per unit time per unit spectral interval  $d\lambda$ ; in other words, intensity is the energy flux density per unit spectral interval.

The first term on the right-hand side of Eq. (9) uses the Beer–Lambert law of absorption;  $\sigma_a(\lambda)$  is the absorption cross section per unit greenhouse gas molecule and  $n(z)$  is the number density of the greenhouse gas in the layer. The second term is blackbody radiation based on the Kirchhoff law, which states that emittance and absorption have identical values.  $B(\lambda, z)$  is the intensity of the Planck blackbody function in the troposphere:

$$B = \frac{2\pi hc^2}{\lambda^5 \left( \exp \left[ \frac{hc}{\lambda k(T_0 + \Gamma z)} \right] - 1 \right)}, \quad (10)$$

where  $h$  is the Planck constant,  $k$  is the Boltzmann constant, and  $c$  is the speed of light.

Scattering by molecules is ignored since the reference radiation is located in the infrared region. The absorption of the zeroth layer  $a_0(\lambda)$  is defined as follows:

$$a_0 = \sigma_a n_0 \Delta z. \quad (11)$$

As a result, we can assume that the absorption of an arbitrary layer is described by  $a_0(\lambda)$  and the ratio of number density is written as

$$\sigma_a n \Delta z = a_0 \frac{n}{n_0}. \quad (12)$$

Therefore, the net change in flux density is expressed as a function of altitude as follows:

$$\Delta I d\lambda = \frac{a_0 n}{n_0} (B - I) d\lambda = \frac{a_0 T_0}{T_0 + \Gamma z} \exp \left[ -\frac{\mu g}{R \Gamma} \ln(1 + \Gamma z / T_0) \right] (B - I) d\lambda. \quad (13)$$

Taking into account the absorption by the four kinds of molecules, the actual change of the flux density passing through the layer is

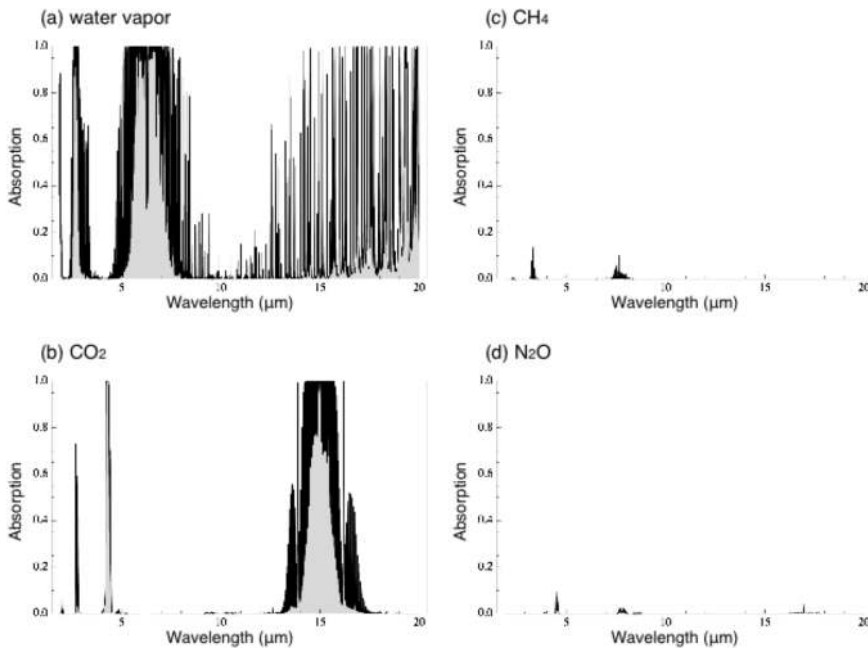
$$\Delta I d\lambda = \left( w_0 f_w + \sum_{i=1}^3 a_0^i \right) \frac{T_0}{T_0 + \Gamma z} \exp \left[ -\frac{\mu g}{R \Gamma} \ln(1 + \Gamma z / T_0) \right] (B - I) d\lambda, \quad (14)$$

where  $a_0^i$  and  $w_0$  represent the absorption spectra of anthropogenic greenhouse gas and that of water vapor of the zeroth layer, respectively.  $f_w(z)$  is the factor of altitude dependence for water vapor, which is obtained by normalizing the concentration of an arbitrary layer by 7750 ppmv, which is the value for the zeroth layer.

The absorption spectra of greenhouse gases are discussed in the following section. The outgoing flux density at the tropopause with a wavelength from  $\lambda$  to  $\lambda+d\lambda$  can be obtained by the repeated use of Eqs. (14) and (10) with an initial value of  $I(\lambda, 0)=B(\lambda, 0)$ . They are summed up for the reference range of wavelengths to give the total flux density. Of course, the total absorption through any layers must not be greater than one.

### 3.1. Estimation of radiative forcing

The calculations of radiative forcing require the absorption spectra of the four greenhouse gases for the zeroth layer in 1750.[10]



**Figure 2.** Absorption spectra of atmospheric greenhouse gases in 1750 for thickness of 100 m obtained from the SpectralCalc website: (a) water vapor, 7750 ppmv; (b) CO<sub>2</sub>, 278 ppmv; (c) CH<sub>4</sub>, 0.722 ppmv; and (d) N<sub>2</sub>O, 0.270 ppmv. The concentration of water vapor is assumed to be the same as the present value. Absorption is the intensity ratio of absorbed radiation to total radiation incident on the zeroth layer.

Figure 2 shows the absorption spectra of the zeroth layer for 1750, as calculated using the SpectralCalc database. For selected gases, the SpectralCalc website provides transmittance spectra at an arbitrary pressure, temperature, and thickness of a gas layer; the concentration of the gas in the layer; and the range of wavelengths.[7] The SpectralCalc uses a line-by-line model called LINEPAK to accurately model molecular absorption line spectra.[11] These are based on the HITRAN database,[12] which is a compilation of spectroscopic parameters widely used to simulate the gases' transmission and emission of radiation into the atmosphere. The concentrations are 7750 ppmv for water vapor, 278 ppmv for CO<sub>2</sub>, 0.722 ppmv for CH<sub>4</sub>, and 0.270 ppmv for N<sub>2</sub>O. The pre-industrial concentration of water vapor is assumed to be the same as the present value.

In the zeroth layer, the parameters are

$$T_0 = 288 \text{ K}, \quad p_0 = 1.013 \times 10^5 \text{ Pa}, \quad \Delta z = 100 \text{ m}, \quad \text{and} \quad \lambda = 1.85 - 20.0 \mu\text{m}. \quad (15)$$

The unit spectral interval is set to 1 nm. The numerical values used for the calculations are as follows:

$$\begin{aligned} R &= 8.3144 \text{ J K}^{-1}, \quad \mu = 28.964 \times 10^{-3} \text{ kg}, \quad \Gamma = -6.5 \times 10^{-3} \text{ K m}^{-1}, \quad g = 9.8 \text{ m s}^{-2}, \\ h &= 6.6261 \times 10^{-34} \text{ J s}, \quad k = 1.3806 \times 10^{-23} \text{ J K}^{-1}, \quad c = 2.9979 \times 10^8 \text{ m s}^{-1}. \end{aligned} \quad (16)$$

Figure 3 shows the calculated outgoing spectra in the pre-industrial era at altitudes of 3 km, 6 km, 9 km, and 11 km (the tropopause). It is clear that terrestrial radiation is strongly absorbed at wavelengths of 5–8  $\mu\text{m}$  and 13–17  $\mu\text{m}$ . The former is due to water vapor and the latter is due to CO<sub>2</sub>. In both these ranges, the absorption is nearly one. However, the outgoing flux in these ranges is not zero because of emission from the greenhouse gases. Moreover, the intensity of the flux decreases with altitude. The red area under the spectrum at 11 km, 191.60 W m<sup>-2</sup>, corresponds to the total outgoing flux density  $\bar{F}$ .

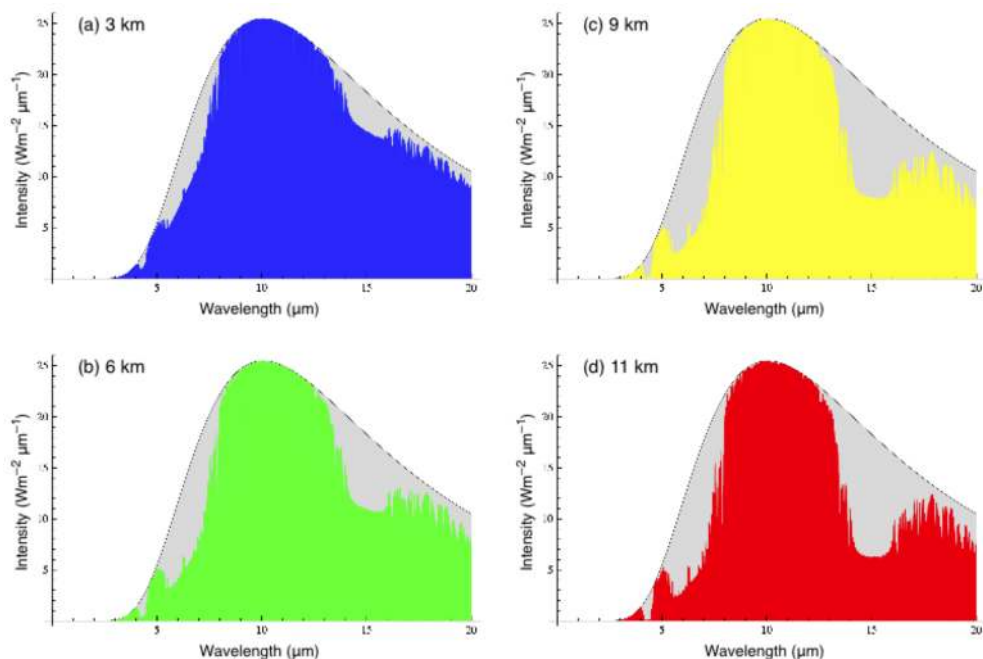
The growth rates of greenhouse gas concentrations from the pre-industrial era to 2011 are given as follows: CO<sub>2</sub>, 1.40; CH<sub>4</sub>, 2.50; and N<sub>2</sub>O, 1.20. For simplicity, we assume that the absorption spectrum in 2011 may be approximated by multiplying the spectrum in 1750 by the growth rate. The absorption values never exceed one.

A similar calculation in which the absorption spectrum of only 278 ppmv CO<sub>2</sub> is replaced with that of 390.5 ppmv CO<sub>2</sub> gives  $F_{\text{CO}_2} = 189.26 \text{ W m}^{-2}$ . Therefore, the radiative forcing of CO<sub>2</sub> for the period between 1750 and 2011 is

$$\Delta F_{\text{CO}_2} = \bar{F} - F_{\text{CO}_2} = 2.34 \text{ W m}^{-2}. \quad (17)$$

Similarly, the outgoing flux density  $F_i$  for the spectrum of greenhouse gas  $i$  when its concentration in 1750 is replaced with its concentration in 2011, is  $F_{\text{CH}_4} = 190.74 \text{ W m}^{-2}$  and  $F_{\text{N}_2\text{O}} = 191.41$





**Figure 3.** Outgoing terrestrial radiation spectrum at 3 km (blue), 6 km (green), 9 km (yellow), and 11 km (tropopause; red) in 1750, as calculated using the spectra shown in Figure 2.

$\text{W m}^{-2}$ . The radiative forcing of each greenhouse gas is listed in Table 2. Despite of the simplicity of the radiative transfer model used here, the calculated values of  $\Delta F$  are close to those in the IPCC estimations, but are relatively higher:  $\text{CO}_2$ , 39%;  $\text{CH}_4$ , 33%; and  $\text{N}_2\text{O}$ , 12%. The 2011 spectrum used in this calculation was approximated by multiplying the greenhouse gas's growth rate by the spectrum in 1750. Nevertheless, the differences between the approximated radiative transfer values and the precise values based on the real spectrum in 2011 in SpectralCalc are very small except for  $\text{CO}_2$ :  $\text{CO}_2$ , 12%;  $\text{CH}_4$ , 1.4%; and  $\text{N}_2\text{O}$ , 0.5%.

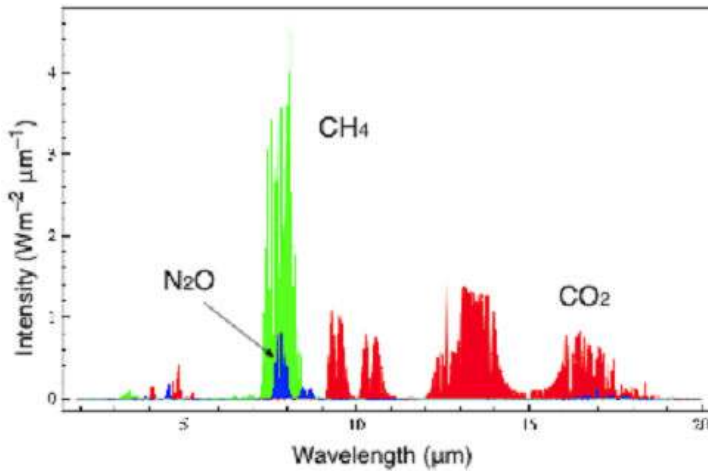
Greenhouse gas	Outgoing flux density		Radiative forcing	
	1750	2011	$\Delta F$	IPCC*
Carbon dioxide		189.260	2.34	1.68
Methane	191.597	190.743	0.85	0.64
Nitrous oxide		191.407	0.19	0.17

\* These estimates are direct contributions through the emissions of the gases.

**Table 2.** Calculated outgoing flux density in 1750 and 2011, radiative forcing  $\Delta F$ , and IPCC estimates for each greenhouse gas.

Figure 4 shows the distribution of radiative forcing. We can see that the saturated absorption regions, 5–7  $\mu\text{m}$  for water vapor and 16–18  $\mu\text{m}$  for  $\text{CO}_2$ , do not affect the radiative forcing, whereas the unsaturated area does. Nevertheless, the  $\text{CO}_2$  absorption in the range 9–11  $\mu\text{m}$  is negligibly small (Figure 2(b)), contributing 10% of the radiative forcing because terrestrial radiation is maximal in this range.

The robustness of the proposed model must be confirmed by testing the sensitivity of the results to changes in certain parameters. A change in water vapor concentration does not affect the radiative forcing considerably. The increase in radiative forcing is around 1% even when water vapor concentration decreases by 5%. Moreover, the outgoing flux density decreases as the lapse rate increases. For a change of 10% in the lapse rate, the radiative forcing values change between about 3% and 6%.



**Figure 4.** Radiative forcing spectra calculated with the precise absorption spectra:  $\text{CO}_2$  (red),  $\text{CH}_4$  (green), and  $\text{N}_2\text{O}$  (blue).

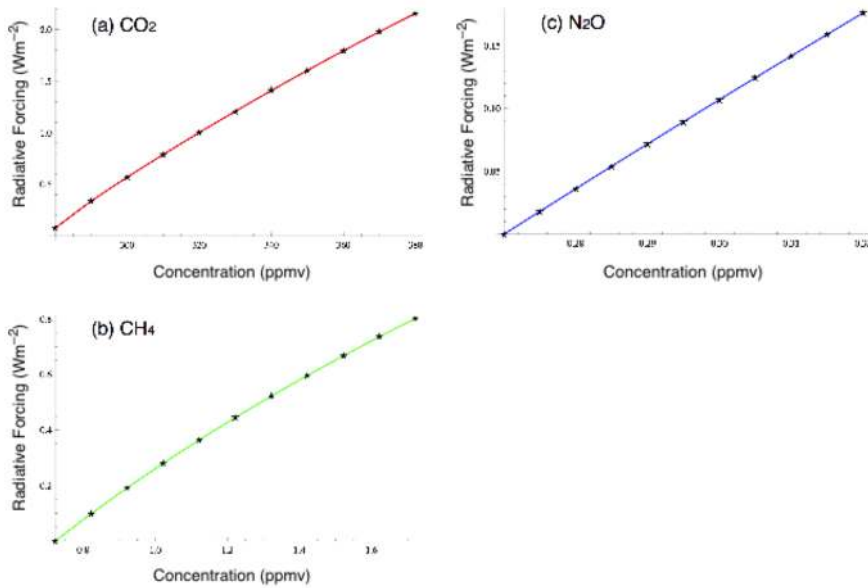
There are some sources of the uncertainties in this calculation. The first is assuming that the absorption spectra of greenhouse gases in each layer are proportional to only their number density. However, the absorption spectrum also changes with pressure and temperature in the layer: absorption line width broadening is caused by the thermal motion of the molecules and the collisions between them, both of which depend on pressure and temperature.[13,14]

This effect is captured by  $a_0(\lambda)$  provided by SpectralCalc. However, the effects of the other layer are approximated as the effects of the zeroth layer for simplicity. These differences in the spectra in each layer will produce uncertainties in the outgoing fluxes.

The second source of uncertainty is ignoring the radiative transfer of incident solar radiation in the near-infrared region. The absorption and emission at 2.6–3.5  $\mu\text{m}$  may not be negligible since the solar intensity in the region is 5–8  $\text{W m}^{-2} \mu\text{m}^{-1}$  (the maximum intensity of the terrestrial

radiation is about  $25 \text{ W m}^{-2} \mu\text{m}^{-1}$ , as shown in Figure 3). Therefore, the outgoing flux contains some uncertainties.

Figure 5 shows the calculated radiative forcing for each increasing greenhouse gas from the pre-industrial era to the present age. This result implies that each radiative forcing becomes slightly rounded due to saturation over the specific wavelength areas and it is linearly approximated by the gas's concentration: the radiative forcing per ppmv of  $\text{CO}_2$  is  $0.025 \text{ W m}^{-2}$ ;  $\text{CH}_4$ ,  $0.7 \text{ W m}^{-2}$ , and  $\text{N}_2\text{O}$ ,  $3.5 \text{ W m}^{-2}$ .



**Figure 5.** Calculated radiative forcings for increasing concentrations of greenhouse gases: (a)  $\text{CO}_2$ , for every 20 ppmv from 280 ppmv to 380 ppmv; (b)  $\text{CH}_4$ , for every 0.1 ppmv from 0.7 ppmv to 1.7 ppmv; (c)  $\text{N}_2\text{O}$ , for every 0.005 ppmv from 0.270 ppmv to 0.320 ppmv.

### 3.2. Estimation of global warming potential

GWP is defined as the radiative forcing per unit mass of released greenhouse gas  $i$  relative to that of  $\text{CO}_2$ . It is necessary to integrate the radiative forcing for a reference time since the number of gas molecules decreases with time. The Kyoto Protocol is based on GWPs from pulse emissions over a 100-year time frame. For simplicity, the GWP of greenhouse gas  $i$  for its 100-year effect is approximated as

$$\text{GWP}_i = \frac{(\Delta F / \Delta n)_i / M_i \times \int_0^{100} e^{-t/\tau_i} dt}{(\Delta F / \Delta n)_{\text{CO}_2} / M_{\text{CO}_2} \times \int_0^{100} e^{-t/\tau_{\text{CO}_2}} dt}, \quad (18)$$

where  $\Delta F / \Delta n$  is the radiative forcing per unit increased concentration and  $M$  is the molecular mass of the greenhouse gas. The mean residual life of a greenhouse gas over 100 years is given by

$$\int_0^{100} e^{-t/\tau} dt, \quad (19)$$

where  $\tau$  is its lifetime.

The lifetimes of  $\text{CH}_4$  and  $\text{N}_2\text{O}$  are 12.4 and 121 years, respectively, according to the IPCC Fifth Assessment Report. [2] For  $\text{CO}_2$ , the mean residual life is given as follows:

$$\int_0^{100} (0.217 + 0.259 e^{-t/172.9} + 0.338 e^{-t/18.51}) dt. \quad [15] \quad (20)$$

The GWPs of  $\text{CH}_4$  and  $\text{N}_2\text{O}$  for the 100-year effect are calculated as 27.24 and 242.2, respectively, using  $\Delta F$  as obtained in Section 3.1. They are listed in Table 3 along with the IPCC estimates. The GWP of  $\text{CH}_4$  is very close to the IPCC estimate whereas the radiative forcing value is not.

Greenhouse gas	$\Delta n$ (ppmv)	$\tau$ (years)	GWP	
			This work	IPCC
Carbon dioxide	112	*	1	1
Methane	1.08	12.4	27.2	28
Nitrous oxide	0.054	121	242	265

\* For carbon dioxide, the IPCC uses the parameters in Eq. (20) which represent the mean residual life, without defining its lifetime. [15]

**Table 3.** Increased concentration  $\Delta n$  from 1750 to 2011, lifetime  $\tau$ , calculated GWP, and IPCC estimate of GWP over an interval of 100 years for each greenhouse gas.

### 3.3. Radiative forcing and surface temperature

Finally, we estimate the increase in the Earth's temperature by using radiative forcing. In the pre-industrial era, the incoming solar radiation flux at the tropopause exactly balanced the outgoing solar radiation flux reflected by the Earth system and the outgoing terrestrial radiation.

Let the outgoing terrestrial radiation be approximated roughly as

$$\bar{F} = (1 - f) \sigma T_0^4, \quad (21)$$

where  $f$  is the mean absorption of the terrestrial radiation at the tropopause,  $T_0$  is the surface temperature and  $\sigma$  is the Stephan-Boltzmann constant.

Increasing abundance of a greenhouse gas (e.g., CO<sub>2</sub>) decreases the mean absorption by  $\Delta f$ .

$$F_{\text{CO}_2} = (1 - f - \Delta f) \sigma T_0^4. \quad (22)$$

The perturbation  $\Delta f$  is related to radiative forcing by using Eqs. (21) and (22):

$$\Delta f = \frac{\Delta F_{\text{CO}_2}}{\sigma T_0^4}. \quad (23)$$

Assuming  $\Delta f$  is maintained for some time, a new equilibrium state will develop in which the surface temperature has increased by  $\Delta T$ . At the new radiative equilibrium, the outgoing terrestrial flux at the tropopause does not change if the incoming solar flux and the outgoing solar flux reflected by the Earth system are unchanged. Therefore,

$$(1 - f) \sigma T_0^4 = (1 - f - \Delta f) \sigma (T_0 + \Delta T)^4. \quad (24)$$

Ignoring the second-order and higher terms in Eq. (24), we obtain

$$\Delta T = \frac{\Delta f}{4(1 - f)} T_0. \quad (25)$$

Substituting Eqs. (21) and (23) into Eq. (25) gives the relationship between  $\Delta T$  and  $\Delta F_{\text{CO}_2}$ :

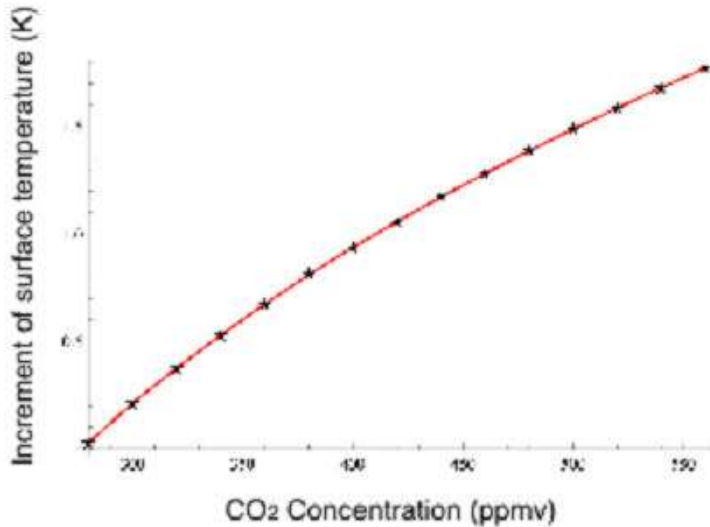
$$\Delta T = \frac{\Delta F_{\text{CO}_2}}{4\bar{F}} T_0. \quad (26)$$

By substituting the numerical values in Table 2, we can estimate that the Earth's surface temperature has risen by  $\Delta T=0.88$  K as a result of the CO<sub>2</sub> increase since 1750. Then, in the new equilibrium state, the surface temperature is  $T_0'=T_0+\Delta T=288.9$  K and the lapse rate should be  $-6.6 \times 10^{-3}$  K m<sup>-1</sup> due to the temperature invariance at the tropopause. Next, the outgoing flux density at the tropopause with the surface temperature  $T_0'$  is calculated. This value is close to  $\bar{F}$ .

The radiative forcing of CO<sub>2</sub> is dominant, and the radiative forcings of other greenhouse gases nearly balance with negative radiative forcings (e.g., of aerosols). As a result, the influence of all radiative forcings is almost equivalent to the influence of CO<sub>2</sub>.

Figure 6 shows the surface temperature increase due to CO<sub>2</sub> considering radiative forcing when the atmospheric CO<sub>2</sub> concentration increases to twice that in the pre-industrial era. In the future

(when the CO<sub>2</sub> concentration exceeds 390 ppmv), the surface temperature increase per 20 ppmv CO<sub>2</sub> is about 0.1 K.



**Figure 6.** Surface temperature increase due to the radiative forcing of CO<sub>2</sub> when the atmospheric CO<sub>2</sub> concentration increases to twice that in the pre-industrial era.

## 4. Conclusions

The one-dimensional radiative equilibrium model can estimate the radiative forcing and global warming potential of anthropogenic greenhouse gases by using their absorption spectra at Earth's surface and number densities. Under some approximations, radiative forcing values are calculated higher than the IPCC estimates; CO<sub>2</sub>, 39%; CH<sub>4</sub>, 33%; and N<sub>2</sub>O, 12%. Calculated values of global warming potential are very similar to IPCC estimates. Furthermore, increases in Earth's surface temperature due to radiative forcing can be easily estimated by the model.

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## References

- [1] This chapter is based on the author's paper: A. Tomizuka, "Estimation of the power of greenhouse gases on the basis of absorption spectra," *Am. J. Phys.* 78, 359-366 (2010).
- [2] "Climate Change 2013: The Physical Science Basis" (2014), <http://www.ipcc.ch/report/ar5/wg1/>.
- [3] M. D. Mastrandrea and S. H. Schneider, "Resource Letter GW-2: Global Warming," *Am. J. Phys.* 76, 608-614 (2008).
- [4] D. J. Jacob, *Introduction to Atmospheric Chemistry* (Princeton University Press, Princeton, NJ, 1999).
- [5] G. Myhre, E. J. Highwood, K. P. Shine, and F. Stordal, "New estimates of radiative forcing due to well mixed greenhouse gases," *Geophys. Res. Lett.* 25, 2715-2718 (1998).
- [6] *Radiative Forcing of Climate Change: Expanding the Concept and Addressing Uncertainties*, edited by D. J. Jacob, J. Antonio J. Busalacchi, and R. J. Serafin (National Academies Press, Washington, DC, 2005).
- [7] "Spectral Calculator" (GATS Inc Atmospheric Science, Newport News, VA), <http://spectralcalc.com/calc/spectralcalc.php>.
- [8] J. Harte, "Earth's Surface Temperature," in *Consider a Spherical Cow: a course in environmental problem solving* (University Science Books, California, 1988), pp. 160-171.
- [9] F. W. Taylor, "Radiative Transfer," in *Elementary Climate Physics* (Oxford University Press, New York, 2005), pp. 67-103.
- [10] J. Barrett, "Greenhouse molecules, their spectra and function in the atmosphere," *Energy & Environment* 16, 1037-1045 (2005).
- [11] L. L. Gordley, B. T. Marshall, and D. A. Chu, "LINEPAK: Algorithm for Modeling Spectral Transmittance and Radiance," *J. Quant. Spectrosc. Radiat. Transfer* 52, 563-580 (1994).
- [12] L. S. Rothman, "The HITRAN Database", 2009, <http://www.cfa.harvard.edu/HITRAN/>.
- [13] R. H. Dicke, "The Effect of Collisions upon the Doppler Width of Spectral Lines," *Phys. Rev.* 89, 472-473 (1953).
- [14] G. Visconti, "Molecular Spectra," in *Fundamentals of Physics and Chemistry of the Atmosphere* (Springer-Verlag, Berlin, 2001), pp. 324-328.
- [15] *Climate Change 2007: The Physical Science Basic: Contribution of Working Group I to the Fourth Assessment*, edited by S. Solomon, D. Qin, M. Manning, Z. Chen, M. Marquis,

K. B. Averyt, M. Tignor, and H. L. Miller (Cambridge University Press, Cambridge, 2007).