

Chapter

Reliability-Based Marginal Cost Pricing Problem

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Abstract

This chapter is concerned with first-best marginal cost pricing (MCP) in a stochastic network with both supply and travel demand uncertainty and perception errors within the travelers' route choice decision processes. To account for the travelers' perception error, moment analysis is adopted in this chapter to derive the mean and variance of total perceived travel time of the network. We then developed a Perceived Risk-Based Stochastic Network Marginal Cost Pricing (PRSN-MCP) model. Furthermore, in order to illustrate the effect of incorporating both stochastic supply and demand into the PRSN-MCP model, the calculation of the PRSN-MCP model is divided up into four scenarios under different simplifications of network uncertainties. Numerical examples are also provided to demonstrate the importance and properties of the proposed model. The main finding is that ignoring the effect of stochastic travel demand, capacity degradation, and travelers' perception error may significantly reduce the performance of the first-best MCP tolls, especially under high traveler's confidence and network congestion levels.

Keywords: marginal cost pricing, moment analysis, demand uncertainty, supply uncertainty, perception error

1. Introduction

It is well known, due to stochastic variations in both supply and demand, that travel time almost always involves a measure of uncertainty. Recently, several empirical studies on the value of time and reliability revealed that travel time reliability plays an important role in the traveler's route choice decision-making process [1–3]. With these studies as a basis, the study of travel time variability (reliability) has gradually emerged as an important topic. In this context, travel time reliability pertains to the probability that a trip can be successfully completed within a specified time interval, reflecting the uncertainty in trip journey times [4, 5]. To model the characteristics of travel time reliability, the concept of TTB is commonly used. TTB is defined as the average travel time plus extra time (for a measure of the buffer time) such that the probability of completing the trip within the TTB is no less than a predefined reliability threshold α [6]. Earlier research applied the concept of effective travel time to capture the travel time reliability [7]. Recently, [6] further proposed a stochastic mean-excess traffic equilibrium model to represent both the reliability and unreliability aspects of travel time variability and travelers' route choice perception errors.

Generally speaking, uncertainties from both the demand and supply sides of a system directly lead to recurring variability and unreliability of travel times and

have an obvious impact on the traveler's route choice behavior. Supply-side sources refer to the capacity variations that can occur, due to several exogenous sources of uncertainty on the road sections or at-grade intersections concerned. These exogenous sources of uncertainty may take different forms, such as environmental conditions, traffic incidents, traffic management and control, work zones, and so on. Such stochastic link capacity degradations usually lead to non-recurrent congestion [8–10]. Demand-side sources are regarded as the travel demand fluctuations, which result from various endogenous sources. These endogenous sources can include temporal factors, special events, population characteristics, and traffic information among others. Travel demand variations usually lead to recurrent congestion [4, 11, 12].

Several stochastic traffic network (SN) modeling approaches have been proposed to represent such uncertainties. On the capacity side, [13] proposed a probabilistic approach using the concept of capacity reliability to model the uncertain characteristics of link capacities. Lo et al. [14] proposed the Probabilistic User Equilibrium (PUE) model, which takes the fact that the link capacities are subject to stochastic degradations into account. In subsequent research using the concept of Travel Time Budget (TTB), [10] further extended the PUE model to capture the route choice behaviors of travelers with heterogeneous risk aversions. On the demand side, [11] proposed a framework of the stochastic network model to represent the stochastic demand. Ref. [12] extended the TTB model and proposed a travel time reliability-based traffic assignment model to consider the effect of daily demand fluctuations. On both the demand and supply sides, [15] proposed a traffic assignment model, which considers the uncertainties of a traffic network due to adverse weather conditions. Sumalee et al. [16] proposed a stochastic network model with log-normal distributed origin–destination (OD) travel demands and link capacities. It should be noted that all of the above studies focused on the question of how to represent the travel time reliability in a traffic assignment model, but did not answer the question of how to improve the travel time reliability in a stochastic traffic network.

All the aforementioned studies discovered that travelers do indeed consider travel time variability as a risk in their route choice decisions. Nevertheless, the first-best marginal cost pricing (MCP) is commonly modeled via a deterministic approach, which assumes that both traffic supply and travel demand are known, and that the route travel times are deterministic [17]. Furthermore, travelers are assumed to know exactly the time on each available route and can always choose the least-cost routes for their trips. As indicated earlier, due to various sources of uncertainty coming from both supply-side and demand-side of road network, it is unreasonable to assume that travel times are deterministic and known perfectly by all the travelers. Though several traffic equilibrium models have been developed for environments characterized by uncertainty in the past decades, such models have not been adopted in the analysis of first-best MCP. Intuitively, the variability and unreliability of travel times caused by network uncertainties directly influence the traveler's route choice behavior, thereby negatively affecting the performance of MCP. However, there is little theoretical basis for this intuition. At least, it is not yet clear to what extent the stochastic demand and supply and the travelers' perception error affect the performance of MCP. In this context, the study of first-best MCP under an uncertain environment is a necessary and urgent theoretical task. In addition, this investigation is also practically relevant. As indicated by [18], the recent change in the Electronic Road Pricing (ERP) toll adjustment scheme in Singapore involves the consideration of the 85th-percentile traffic condition (speed) to reflect the variability of traffic conditions. This involves determining optimal tolls in a stochastic environment, where both demand and capacity are subject to uncertainty.

Although considerable research exists on congestion pricing and travel time reliability, relatively little research combines the two, especially regarding travelers' risk attitudes and/or the valuation of reliable travel [19]. Some examples of research that do combine congestion pricing and travel time reliability are included here. Li et al. [20] proposed a reliability-based optimal toll design bi-level model. On the upper level, network performance is optimized from a road authority point of view including travel time reliability, while a dynamic user-equilibrium is achieved from the viewpoint of travelers on the lower level. Boyles et al. [19] proposed a first-best congestion pricing model considering network capacity uncertainty and user valuation of travel time reliability, while [18] investigated marginal cost pricing in a stochastic traffic network in which demand uncertainty is explicitly considered. By assuming that all travelers have complete information about the road traffic condition, [18] derived an analytical function of Stochastic Network-Marginal Cost Pricing (SN-MCP) for a risk-neutral case and risk-based SN-MCP (RSN-MCP) for a risk-based case under the assumptions of lognormal demand and constant VMR across all OD pairs. Gardner et al. [21] consider the uncertainty in long-term travel demand and in day-to-day network capacity, and discuss the benefit of responsive pricing and travel information.

In the above-mentioned studies, MCP is analysis in a stochastic network, which considers either link supply uncertainty (e.g., see [19]) or stochastic travel demand (e.g., see [18]). In addition, to account for the travelers' perception error, researchers usually assume the commonly adopted Gumbel variate as the random error term and use the conventional logit-based Stochastic User Equilibrium (SUE) model. However, this approach may not reflect the travelers' perception of the random travel time exactly. Due to the variation of travel time, it is more rational to assume that the travelers' perception error is also dependent on the random perceived travel time [22]. Therefore, in order to explicitly consider both supply and demand aspects of a stochastic network and to reflect the travelers' perception error of the random travel time, this investigation extends [18] by (1) considering both the stochastic travel demand and link capacity degradation, and (2) incorporating travelers' perception error into the first-best MCP analysis.

The remainder of the chapter is organized as follows. The next section introduces the assumptions used in the analysis and presents the variational inequality (VI) formulation for different stochastic models. It also discusses the stochastic travel times under different sources of uncertainty. Then, Section 3 and Section 4

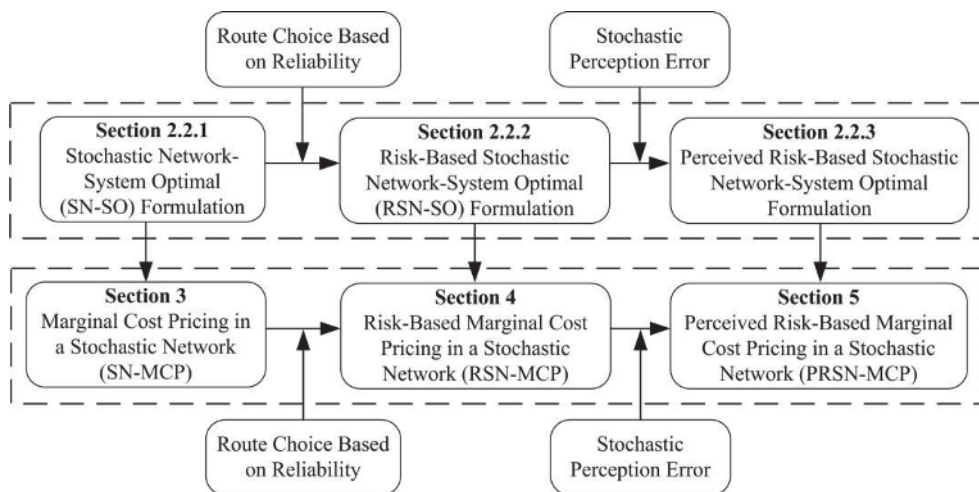


Figure 1.
 Flow chart for the research process.

derive the analytical function of SN-MCP for a risk-neutral case and RSN-MCP for a risk-based case in a stochastic network with both supply and demand uncertainty, respectively. In Section 5, the analysis for the PRSN-MCP is then described under different simplifications of network uncertainties. In Section 6, numerical examples with respect to a small-scale network and a medium-scale network (Sioux Falls network) are undertaken to demonstrate the effects of the proposed models. The final section contains some concluding remarks and recommends further research. The flow chart of the process applied in this chapter is presented in **Figure 1**.

2. Framework of stochastic network model

2.1 Notations and assumptions

Consider a strongly connected network $G = (N, A)$, where N is the set of nodes and A is the set of links in the network. Let W represent the set of OD pairs in the network and the set of routes between OD pair $w \in W$ be denoted by R_w . Random variables are expressed in capital letters and lower-case letters are used for mean values of random variables or deterministic variables.

Q^w	travel demand between OD pair $w \in W$
q^w	mean travel demand between OD pair $w \in W$
ε_q^w	variance of travel demand between OD pair $w \in W$
VMR_w	variance-to-mean ratio (VMR) of the random travel demand
F_r^w	route flow on path $r \in R_w$
f_r^w	mean traffic flow on path $r \in R_w$
$\varepsilon_f^{w,r}$	variance of traffic flow on path $r \in R_w$
\mathbf{f}	column vector of mean route flow, where $\mathbf{f} = \{f_r^w\}$
V_a	traffic flow on link $a \in A$
v_a	mean traffic flow on link $a \in A$
ε_v^a	variance of traffic flow on link $a \in A$
\mathbf{v}	column vector of mean link flow, where $\mathbf{v} = \{v_a\}$
$\delta_{a,r}^w$	link-path incidence parameter; 1 if link a on path r , zero otherwise
TT	total travel time of the system, where $TT = \sum_{a \in A} V_a T_a$
VoR	relative weight assigned to the travel time budget, that is, value of reliability
T_r^w	travel time on path $r \in R_w$
t_r^w	mean travel time on path $r \in R_w$
$\varepsilon_t^{w,r}$	variance of travel time on path $r \in R_w$
T_a	travel time on link a
t_a	mean travel time on link a
ε_t^a	variance of travel time on link $a \in A$
\tilde{T}_r^w	perceived travel time on path $r \in R_w$
\tilde{t}_r^w	mean perceived travel time on path $r \in R_w$
$\tilde{\varepsilon}_t^{w,r}$	variance of perceived travel time on path $r \in R_w$
\tilde{T}_a	perceived travel time on link a

\bar{t}_a	mean perceived travel time on link a
$\bar{\epsilon}_t^a$	variance of perceived travel time on link $a \in A$
$\bar{T}\bar{T}$	total perceived travel time of the system, where $\bar{T}\bar{T} = \sum_{a \in A} V_a \bar{T}_a$
t_a^0	free-flow travel time on link $a \in A$
C_a	capacity of link $a \in A$
	design capacity (upper bound) of link $a \in A$
	degree of worst-degraded capacity for link $a \in A$
γ_a	parameter, where $\gamma_a = \sqrt{1 + VMR/v_a}$
$\epsilon_a T_a$	travelers' perception error on link $a \in A$
$N(\chi, \varpi^2)$	perception error distribution of traveler, in this chapter $N(\chi, \varpi^2)$ follows a normal distribution with predefined and deterministic mean χ and variance ϖ^2

Before proceeding with the analysis, some assumptions are made to allow for the closed-form formulation/calculation of the PRSN-MCP model.

A1. The travel demand Q^w between each OD pair is assumed to be an independent random variable with a mean of q^w and variance of ϵ_q^w , while VMR_w is the variance-to-mean ratio (VMR) of the random travel demand in which $VMR_w = \epsilon_q^w / q^w$. Stochastic demand is further assumed to follow a lognormal distribution, which is a nonnegative, asymmetrical distribution. This has been adopted in the literature as a more realistic approximation of the stochastic travel demand, as opposed to the more commonly used normal distribution [18, 23].

A2. The route flow F_r^w , and link flow V_a are also assumed to be independent random variables that follow the same statistical distribution as OD demand. The VMRs of route flows are equal to those of the corresponding OD demand.

A3. The VMRs of travel demand are assumed to be the same for all OD pairs in order to derive the closed-form formulation of the PRSN-MCP model.

A4. The capacity degradation random variable C_a is independent of the traffic flow v_a on it and follows a uniform distribution with the design capacity \bar{c}_a of the link as its upper bound and the worst-degraded capacity as its lower bound (the lower bound would be a fraction θ_a of the design capacity).

2.2 VI formulation for different stochastic network models

2.2.1 Stochastic network-system optimal (SN-SO) formulation

According to the Assumption A1 and A2, the OD travel demand, route flow F_r^w , and link flow V_a are random variables, which consequently induce the random route/link travel times. As such, we have the following flow conservation relationships among them

$$Q^w = \sum_{r \in R_w} F_r^w, w \in W \quad (1)$$

$$V_a = \sum_{w \in W} \sum_{r \in R_w} \delta_{a,r}^w F_r^w, \forall a \in A \quad (2)$$

$$F_r^w \geq 0, w \in W, r \in R_w \quad (3)$$

where Eq. (1) is the travel demand conservation constraint, Eq. (2) is a definitional constraint that sums up all route flows that pass through a given link a ,

and Eq. (3) is a non-negativity constraint on the route flows. Let $\Delta = [\delta_{a,r}^w]$ denote the route-link incidence matrix, $\delta_{a,r}^w = 1$ if route r traverses link a , and $\delta_{a,r}^w = 0$ otherwise. Let f_r^w, v_a denote the mean route flow and link flow, respectively. From Eqs. (1) ~ (3), these route and link flows satisfy the following conservation conditions:

$$q^w = \sum_{r \in R_w} f_r^w, w \in W \quad (4)$$

$$v_a = \sum_{w \in W} \sum_{r \in R_w} \delta_{a,r}^w f_r^w, \forall a \in A \quad (5)$$

$$f_r^w \geq 0, w \in W, r \in R_w \quad (6)$$

Let $\varepsilon_f^{w,r}, \varepsilon_v^a$ be the variance of route flow and link flow, respectively. Then from the Assumption A1 and A2, we have

$$\sum_{r \in R_w} \varepsilon_f^{w,r} = \sum_{r \in R_w} f_r^w \text{VMR}_w = q^w \text{VMR}_w = \varepsilon_q^w, w \in W \quad (7)$$

$$\begin{aligned} \varepsilon_v^a &= \sum_{w \in W} \sum_{r \in R_w} (\delta_{a,r}^w)^2 \varepsilon_f^{w,r} = \sum_{w \in W} \sum_{r \in R_w} \delta_{a,r}^w \varepsilon_f^{w,r} \\ &= \sum_{w \in W} \sum_{r \in R_w} \delta_{a,r}^w f_r^w \text{VMR}_w \end{aligned} \quad (8)$$

From Eqs. (7) and (8), we know that the variances of both route flow and link flow can be determined by the means of route flows. Furthermore, the route and link flow distribution can be derived through known travel demand distributions. Next, we discuss the VI formulation for the SN-SO model. In this section, we consider all the travelers to be risk-neutral. That is, travelers are not sensitive to the travel time variations and they do not need to budget the safety margin for their trips. The system optimal assignment under the stochastic network (SN-SO) aims to minimize the expected total travel time. The VI formulation for the SN-SO model can be obtained by finding $\mathbf{v}^* \in \Omega_v$ such that for any $\mathbf{v} \in \Omega_v$,

$$(\mathbf{v} - \mathbf{v}^*)^T \nabla_v E[TT^*] \geq 0 \quad (9)$$

where $\nabla_v E[TT^*] = \{\partial E[\sum_{a \in A} V_a^* T_a^*] / \partial v_a^*\}$, $\Omega_v = \{\mathbf{v} | \mathbf{v} = \Delta \mathbf{f}, \mathbf{f} \geq 0; q^w = \sum_{r \in R_w} f_r^w, w \in W\}$. \mathbf{v} and \mathbf{f} are the column vector of mean link and route flow, respectively. T_a represents the travel time on link a . TT is the total travel time of the system, where $TT = \sum_{a \in A} V_a T_a$.

2.2.2 Risk-based SN-SO (RSN-SO) formulation

Up to this point, we have presented the risk-neutral case. However, several empirical studies reveal that travel time reliability plays an important role in the traveler's route choice decision process [1-3]. In this section, we consider the risk-based (averse or prone) case in which travelers are assumed to consider both the mean travel time and travel time variability in their route decision-making process. Researchers have used the Travel Time Budget (TTB) to represent travelers' risk-based travel behavior. Mathematically, the TTB associated with route r, b_r^w , is expressed as

$$b_r^w = E[T_r^w] + V\sigma R \cdot \varepsilon_t^{w,r}, w \in W, r \in R_w \quad (10)$$

where $\varepsilon_t^{w,r}$ is the variance of route travel time, which represents the travel time reliability (TTR) of that route, is the route travel time, and VoR is the relative weight assigned to the TTR, that is, value of reliability. Similarly, let ε_t^a be the variance of the link travel time, the TTB associated with link a , b_a , which can be described by

$$b_a = E[T_a] + VoR \cdot \varepsilon_t^a, a \in A \quad (11)$$

Based on the assumption of independent travel time on each link, we can infer the following relationship between route travel time variance and link travel time variance as shown below:

$$\varepsilon_t^{w,r} = \sum_{a \in A} \delta_{a,r}^w \varepsilon_t^a, w \in W, r \in R_w \quad (12)$$

From Eqs. (10) ~ (12), the TTB of route and link satisfy the following conservation conditions:

$$\begin{aligned} b_r^w &= E[T_r^w] + VoR \cdot \varepsilon_t^{w,r} = \sum_{a \in A} \delta_{a,r}^w E[T_a] + VoR \cdot \sum_{a \in A} \delta_{a,r}^w \varepsilon_t^a \\ &= \sum_{a \in A} \delta_{a,r}^w b_a, w \in W, r \in R_w \end{aligned} \quad (13)$$

Let $U[TT] = E[TT] + VoR \cdot Var[TT]$. With Eq. (13), the VI formulation for the link-based RSN-SO model can be expressed as

$$(\mathbf{v} - \mathbf{v}^*)^T \nabla_{\mathbf{v}} U[TT^*] \geq 0 \quad (14)$$

$$\text{where } \nabla_{\mathbf{v}} U[TT^*] = \left\{ \partial E \left[\sum_{a \in A} V_a^* T_a^* \right] / \partial v_a^* + VoR \cdot \partial Var \left[\sum_{a \in A} V_a^* T_a^* \right] / \partial v_a^* \right\}.$$

2.2.3 Perceived RSN-SO formulation

In the previous subsections, we consider that travelers can always choose the route with the minimum TTB; the resulting model is called a deterministic traffic assignment model. The main assumption underlying this kind of model is that travelers have full information about travel conditions, that is, they have perfect information about travel time and its variability. In this subsection, we relax this unreasonable assumption and include travelers' perception errors in their route choice process. The perceived TTB associated with route r , \tilde{b}_r^w is described as

$$\tilde{b}_r^w = E \left[\tilde{T}_r^w \right] + VoR \cdot \tilde{\varepsilon}_t^{w,r}, w \in W, r \in R_w \quad (15)$$

where $\tilde{\varepsilon}_t^{w,r}$ is the variance of the perceived route travel time, and \tilde{T}_r^w is the perceived route travel time. Similarly, let $\tilde{\varepsilon}_t^a$ be the variance of perceived link travel time, and \tilde{T}_a be the perceived link travel time. The perceived TTB associated with link a , \tilde{b}_a can be described by

$$\tilde{b}_a = E \left[\tilde{T}_a \right] + VoR \cdot \tilde{\varepsilon}_t^a, a \in A \quad (16)$$

Based on the assumption of independent travel time on each link, we can infer the following relationship between variances of perceived route travel time and perceived link travel time as follows:

$$\tilde{\varepsilon}_t^{w,r} = \sum_{a \in A} \delta_{a,r}^w \tilde{\varepsilon}_t^a, w \in W, r \in R_w \quad (17)$$

From Eqs. (15) ~ (17), the perceived TTB of the route and link satisfy the following conservation conditions

$$\begin{aligned} \tilde{b}_r^w &= E[\tilde{T}_r^w] + VoR \cdot \tilde{\varepsilon}_t^{w,r} = \sum_{a \in A} \delta_{a,r}^w E[\tilde{T}_a] + VoR \cdot \sum_{a \in A} \delta_{a,r}^w \tilde{\varepsilon}_t^a \\ &= \sum_{a \in A} \delta_{a,r}^w \tilde{b}_a, w \in W, r \in R_w \end{aligned} \quad (18)$$

Let $\tilde{T}\tilde{T}$ represent the total perceived travel time of the system, where $\tilde{T}\tilde{T} = \sum_{a \in A} V_a \tilde{T}_a$, and let $U[\tilde{T}\tilde{T}] = E[\tilde{T}\tilde{T}] + VoR \cdot Var[\tilde{T}\tilde{T}]$. With Eq. (18), the VI formulation for the link-based perceived RSN-SO model can be expressed as

$$(\mathbf{v} - \mathbf{v}^*)^T \nabla_{\mathbf{v}} U[\tilde{T}\tilde{T}^*] \geq 0 \quad (19)$$

$$\text{where } \nabla_{\mathbf{v}} U[\tilde{T}\tilde{T}^*] = \left\{ \partial E[\sum_{a \in A} V_a^* \tilde{T}_a^*] / \partial v_a^* + VoR \cdot \partial Var[\sum_{a \in A} V_a^* \tilde{T}_a^*] / \partial v_a^* \right\}.$$

2.3 Stochastic travel times under different sources of uncertainty

Next, we will review the commonly adopted stochastic network models and their associated corresponding derivations of stochastic travel time in the literature in order to clarify the derivation of our proposed modeling approach.

The link travel time function is assumed to be the Bureau of Public Roads (BPR) function, $T_a = t_a^0(1 + \beta(V_a/C_a)^n)$, $\forall a \in A$, where T_a, t_a^0, C_a, V_a are the travel time, free-flow travel time, capacity, and traffic flow on link a . β and n are the deterministic parameters.

2.3.1 Capacity degradation

As has been discussed in Section 1, link capacities are subject to stochastic degradations to different degrees in the forms of traffic incidents, traffic management and control, work zones, and others. These constitute one of the main sources of travel time variability. To model the characteristics of stochastic link capacity degradation, [14] proposed the Probabilistic User Equilibrium (PUE) model. By assuming the capacity degradation random variable is independent of the traffic flow on it and follows a uniform distribution with the design capacity of the link as its upper bound and the worst-degraded capacity as its lower bound (the lower bound to be a fraction of the design capacity), they derived the mean and variance of T_a as follows:

$$E[T_a] = t_a^0 + \beta t_a^0 v_a^n \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \quad (20)$$

$$Var[T_a] = \beta^2 (t_a^0)^2 v_a^{2n} \left\{ \frac{(1 - \theta_a^{1-2n})}{\bar{c}_a^{2n} (1 - \theta_a)(1 - 2n)} - \left[\frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \right]^2 \right\} \quad (21)$$

They further indicated that the uniform distribution assumption can be relaxed with respect to other probability distributions via the Mellin transform technique [14].

2.3.2 Demand fluctuation

Another main source of travel time variability, to be discussed in this section, is the stochastic travel demand. Several types of probability distributions of OD travel demand have been adopted by researchers to simulate the travel demand fluctuation, such as normal distribution [12], lognormal distribution [23], and Poisson distribution [11]. As indicated in Assumption A1, we use the lognormal distribution in this study, which is more realistic than the commonly adopted normal distribution. The probability density function of the lognormal distribution is given below

$$f(x|\mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), \forall x > 0 \quad (22)$$

where x is the random variable, μ and σ are the distribution parameters, and the mean and variance of x are $E[x] = e^{\mu + \sigma^2/2}$ and $\text{Var}[x] = e^{2\mu + \sigma^2/2}(e^{\sigma^2} - 1)$. Based on the Assumption A1 and A2, with lognormal OD demand, the link flows also follow a lognormal distribution

$$V_a \sim LN(\mu_v^a, \sigma_v^a), \forall a \in A \quad (23)$$

where $\mu_v^a = \ln(v_a) - \frac{1}{2} \ln\left(1 + \frac{\varepsilon_v^a}{(v_a)^2}\right)$, $(\sigma_v^a)^2 = \ln\left(1 + \frac{\varepsilon_v^a}{(v_a)^2}\right)$. v_a , ε_v^a are the mean and variance of link flow on link $a \in A$. All of the moments of a lognormal random variable exist and are given as follows:

$$E[X^s] = \exp(s\mu + s^2\sigma^2/2) \quad (24)$$

where $E[X^s]$ is the s th moment of X . With Eq. (8) and A3, we have

$$\varepsilon_v^a = \sum_{w \in W} \sum_{r \in R_w} (\delta_{a,r}^w)^2 \varepsilon_f^{w,r} = \text{VMR} \cdot \sum_{w \in W} \sum_{r \in R_w} \delta_{a,r}^w f_r^w = \text{VMR} \cdot v_a, a \in A \quad (25)$$

Let $y_a = \sqrt{1 + \text{VMR}/v_a}$. Then, by using Eqs. (23) ~ (25) and performing some derivations according to [18], we can obtain

$$E[V_a^n] = \exp\left(n\mu_v^a + n^2(\sigma_v^a)^2/2\right) = v_a^n \left(\sqrt{1 + \text{VMR}/v_a}\right)^{n^2 - n} = v_a^n y_a^{n^2 - n} \quad (26)$$

$$\text{Var}[V_a^n] = E[V_a^{2n}] - (E[V_a^n])^2 = v_a^{2n} y_a^{4n^2 - 2n} - v_a^{2n} y_a^{2n^2 - 2n} \quad (27)$$

Using the BPR function of link travel time, we can derive the mean and variance of the link travel time as follows:

$$E[T_a] = t_a^0 + \frac{\beta t_a^0}{C_a^n} \left(v_a^n y_a^{n^2 - n}\right) \quad (28)$$

$$\text{Var}[T_a] = \frac{\beta^2 (t_a^0)^2}{C_a^{2n}} \left[v_a^{2n} y_a^{4n^2 - 2n} - \left(v_a^n y_a^{n^2 - n}\right)^2\right] \quad (29)$$

2.3.3 Both link capacity and demand variation

From the above analysis and under the Assumption A4, we can easily derive the mean and variance of the link travel time in the case of both link capacity and demand variation as follows:

$$E[T_a] = t_a^0 + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \left(v_a^n y_a^{n^2-n} \right) \quad (30)$$

$$Var[T_a] = \beta^2 (t_a^0)^2 \left\{ \frac{(1 - \theta_a^{1-2n})}{\bar{c}_a^{2n} (1 - \theta_a)(1 - 2n)} \left(v_a^{2n} y_a^{4n^2-2n} \right) - \left[\frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \left(v_a^n y_a^{n^2-n} \right) \right]^2 \right\} \quad (31)$$

3. Marginal cost pricing in a stochastic network (SN-MCP) with both supply and demand uncertainty

3.1 Analysis of SN-MCP

In this section, we discuss the SN-MCP in the risk-neutral case. The MCP in the stochastic network aims to minimize the expected total travel time. Sumalee and Xu [18] investigated the relationship between the Stochastic Network-User Equilibrium (SN-UE) and Stochastic Network-System Optimal (SN-SO) models and established the first-marginal cost toll scheme for the SN model. They classified the marginal cost toll in the stochastic network into three forms. The first one is referred to as original marginal cost pricing, which takes the form of $E[V_a] \cdot dt(E[V_a])/dE[V_a]$; the second one is referred to as average marginal cost pricing, which takes the form of $E[V_a] \cdot dE[T_a(V_a)]/dE[V_a]$; and the third one is referred to as Stochastic Network-Marginal Cost Pricing (SN-MCP), which takes the form of $\partial E[\sum_{a \in A} V_a T_a] / \partial v_a - E[T_a]$. They further indicate that only the SN-MCP can make the traffic network achieve the optimal pattern.

Let, then, the real gap between the marginal social and marginal private costs in a stochastic network be represented by

$$SN - MCP = \partial E \left[\sum_{a \in A} V_a T_a \right] / \partial v_a - E[T_a] = \partial E[TT] / \partial v_a - E[T_a] \quad (32)$$

3.2 Calculation of SN-MCP

In this study, we attempt to compute the value of SN-MCP in the case of both link capacity and demand variation. To achieve this goal, we need to calculate $\partial E[\sum_{a \in A} V_a T_a] / \partial v_a$ and $E[T_a]$, respectively. In considering the stochasticity of both link capacity and demand, $E[T_a]$ should be determined by Eq. (30). The expected total travel time is expressed as

$$\begin{aligned} E[TT] &= E \left[\sum_{a \in A} V_a T_a \right] = \sum_{a \in A} \left\{ t_a^0 E[V_a] + \beta t_a^0 E[V_a^{n+1}] E \left[\frac{1}{C_a^n} \right] \right\} \\ &= \sum_{a \in A} \left\{ t_a^0 v_a + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \left(v_a^{n+1} y_a^{n^2+n} \right) \right\} \end{aligned} \quad (33)$$

Differentiating Eq. (33) with respect to the mean link flow v_a yields

$$\frac{\partial E[TT]}{\partial v_a} = t_a^0 + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \left[\frac{nv_a^{n-1}(1 - y_a^2)}{2y_a^2} + 1 \right] \left[(n + 1)v_a y_a^{n^2+n} \right] \quad (34)$$

By substituting Eqs. (30) and (34) into Eq. (32), the value of SN-MCP in case of Stochastic Supply and Stochastic demand (SS-SD) can be determined as follows:

$$\begin{aligned} \text{SN - MCP} &= \frac{\partial E[TT]}{\partial v_a} - E[T_a] \\ &= \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} v_a^n y_a^{n^2-n} \left\{ \left[(n + 1)y_a^{2n} + \frac{(n^2 + n)}{2} y_a^{2n-2} (1 - y_a^2) \right] - 1 \right\} \end{aligned} \quad (35)$$

Note that if we neglect the degradation of link capacity, Eq. (35) degenerates into the classical SN-MCP model proposed by [18], which considers only the stochastic travel demand. Furthermore, they pointed out that the SN-MCP toll is guaranteed to be positive when $y_a \leq 1.4$. This conclusion is also applicable in the SN-MCP proposed in this section.

4. Risk-based MCP (RSN-MCP) in a stochastic network

4.1 Analysis of risk-based SN-MCP

In the previous section, we know that the Stochastic Network-User Equilibrium (SN-UE) flow pattern can be driven toward a SN-SO flow pattern by charging a toll equal to the SN-MCP. Meanwhile, the expected total travel time can be minimized. In this section, we consider the risk-based (averse or prone) case. The objective function of the RSN-MCP model is to minimize the weighted sum of the mean and the variance of the total travel time, not simply to minimize the expected total travel time. Therefore, the RSN-MCP toll can be determined as

$$\text{RSN - MCP} = \{ \partial E[TT] / \partial v_a - E[T_a] \} + \text{VoR} \cdot \{ \partial \text{Var}[TT] / \partial v_a - \text{Var}[T_a] \} \quad (36)$$

4.2 Calculation of RSN-MCP

In this section, we discuss the most complete and realistic situation in which travelers consider both stochastic fluctuations in supply (or link capacity) and demand in their route choice decision-making process. From Eqs. (32) and (36), we can see that the difference between SN-MCP and RSN-MCP is the term in the second parentheses of Eq. (36). This second term reflects the congestion toll on travel time reliability due to travelers' risk-based behavior. Let us now turn our attention to $\partial \text{Var}[TT] / \partial v_a$. The variance of the total travel time is described by

$$\begin{aligned} \text{Var}[TT] &= E[TT^2] - (E[TT])^2 \\ &= \sum_{a \in A} \left\{ (t_a^0)^2 \cdot \text{Var}[V_a] + (\beta t_a^0)^2 \frac{\text{Var}[V_a^{n+1}]}{\text{Var}[C_a^n]} + 2\beta (t_a^0)^2 \frac{\{ E[V_a^{n+2}] - E[V_a^{n+1}]E[V_a] \}}{E[C_a^n]} \right\} \\ &= \sum_{a \in A} \left\{ (t_a^0)^2 \cdot \text{VMR} \cdot v_a + (\beta t_a^0)^2 \left\{ \frac{(1 - \theta_a^{1-2n})}{\bar{c}_a^{2n} (1 - \theta_a)(1 - 2n)} v_a^{2n} y_a^{4n^2+6n+2} - \left[\frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} v_a^{n+1} y_a^{n^2+n} \right]^2 \right\} \right. \\ &\quad \left. + 2\beta (t_a^0)^2 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} v_a^{n+2} y_a^{n^2+n} (y_a^{2n+2} - 1) \right\} \end{aligned} \quad (37)$$

Differentiating Eq. (37) with respect to the mean link flow yields

$$\begin{aligned} \frac{\partial \text{Var}[TT]}{\partial v_a} = & (t_a^0)^2 \cdot \text{VMR} + (\beta t_a^0)^2 \left\{ \frac{(1 - \theta_a^{1-2n})}{\bar{c}_a^{2n} (1 - \theta_a)(1 - 2n)} \left\{ v_a^{2n} y_a^{4n^2 + 6n} [(2n + 2)v_a - (2n^2 + n - 1) \cdot \text{VMR}] \right\} \right. \\ & \left. - \left(\frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \right)^2 \left\{ v_a^{2n} y_a^{2n^2 + 2n - 2} [(2n + 2)v_a - (n^2 - n - 2) \cdot \text{VMR}] \right\} \right\} \\ & + 2\beta (t_a^0)^2 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \left\{ \left\{ v_a^n y_a^{n^2 + 3n} \left[(n + 2)v_a - \frac{(n^2 + n - 2)}{2} \cdot \text{VMR} \right] \right\} \right. \\ & \left. - \left\{ v_a^n y_a^{n^2 + n - 2} \left[(n + 2)v_a - \frac{(n^2 - n - 4)}{2} \cdot \text{VMR} \right] \right\} \right\} \end{aligned} \quad (38)$$

By substituting Eqs. (31), (35), and (38) into Eq. (36), the value of RSN-MCP in case of SS-SD can be determined. In the same way, by neglecting the degradation of link capacity, the RSN-MCP in case of SS-SD degenerates into the classical RSN-MCP model proposed by [18], which considers only the stochastic travel demand.

5. Formulation of perceived RSN-MCP (PRSN-MCP)

5.1 Model incorporating the travelers' perception error

Up to this point, we have studied the SN-MCP model and RSN-MCP model based on the assumption that all the travelers have perfect knowledge about the network condition. However, in real life, due to the limitations of their own condition, travelers' perception errors have to be incorporated into their route choice decision process. In view of this, it is necessary to investigate the RSN-MCP model with travelers' perception errors. In order to develop such a model, we need to make some additional assumptions on the perception error as follows:

A5. The perception error distribution of an individual traveler for a segment of road with unit travel time equals $N(\chi, \varpi^2)$, where $N(\chi, \varpi^2)$ represents a normal distribution with predefined and deterministic mean χ and variance ϖ^2 .

A6. Traveler's perception errors are independent for nonoverlapping route segments.

A7. Traveler's perception errors are mutually independent over the population of travelers.

In order to compute the value of PSN-MCP of each link in the stochastic network, we need to derive the perceived link travel time, based on moment analysis. According to Assumption A5, the perception error for unit travel time, denoted by $\varepsilon|_{t=1}$, is a sample from. Besides, travel time on link a is the sum of independent unit travel times (see Assumption A6). Therefore, the conditional perception error for link with deterministic travel time t_a^0 is normally distributed as

$$\varepsilon_a |_{T_a=t_a^0} \sim N(\chi t_a^0, \varpi^2 t_a^0) \quad (39)$$

with conditional moment generating function (MGF)

$$M_{\varepsilon_a |_{T_a=t_a^0}}(s) = \exp\left(\chi t_a^0 s + \frac{\varpi^2 t_a^0 s^2}{2}\right) = \exp\left[st_a^0\left(\chi + \frac{\varpi^2 s}{2}\right)\right] \quad (40)$$

where s is a real number. Following [22], the MGF of the perceived travel time \tilde{T}_a of link for an individual traveler can be derived as follows:

$$\begin{aligned}
 M_{\tilde{T}_a}(s) &= E[\exp(s\tilde{T}_a)] \\
 &= E[\exp(s(T_a + \varepsilon_a))] \\
 &= E\left\{\exp(sT_a)E_{\varepsilon_a|T_a}[\exp(s\varepsilon_a|T_a)]\right\} \\
 &= E_{T_a}\left\{\exp(sT_a)M_{\varepsilon_a|T_a}(s)\right\}
 \end{aligned} \tag{41}$$

where $E_x[\cdot]$ denotes the expectation with respect to random variable x . Substituting Eq. (40) in Eq. (41), we can get

$$\begin{aligned}
 M_{\tilde{T}_a}(s) &= E_{T_a}\left\{\exp\left[sT_a\left(1 + \chi + \frac{\varpi^2 s}{2}\right)\right]\right\} \\
 &= M_{T_a}\left[s\left(1 + \chi + \frac{\varpi^2 s}{2}\right)\right]
 \end{aligned} \tag{42}$$

From the first derivative of the equation above and evaluating at $s = 0$, we can obtain the first moment of the perceived travel time distribution

$$E[\tilde{T}_a] = (1 + \chi)E[T_a] \tag{43}$$

where $E[T_a]$ denotes the mean of the random travel time. Likewise, the second-order moment is derived from the second derivative evaluated at

$$E[(\tilde{T}_a)^2] = (1 + \chi)^2 E[(T_a)^2] + \varpi^2 E[T_a] \tag{44}$$

The variance of the perceived travel time can be expressed as follows:

$$Var[\tilde{T}_a] = E[(\tilde{T}_a)^2] - E[\tilde{T}_a]^2 = (1 + \chi)^2 Var[T_a] + \varpi^2 E[T_a] \tag{45}$$

Using these equations, we can analyze the RSN-MCP model with travelers' perception errors. When taking travelers' perception error into consideration, the objective function of the PRSN-MCP model is to minimize the weighted sum of the mean and the variance of the total perceived travel time. Thus, the PRSN-MCP toll can be given by

$$\text{PRSN} - \text{MCP} = \left\{\partial E[\tilde{T}\tilde{T}]/\partial v_a - E[\tilde{T}_a]\right\} + VoR \cdot \left\{\partial Var[\tilde{T}\tilde{T}]/\partial v_a - Var[\tilde{T}_a]\right\} \tag{46}$$

where $\tilde{T}\tilde{T} = \sum_{a \in A} V_a \tilde{T}_a$.

According to Eq. (46), it is clear that the value of PRSN-MCP can be determined as long as $\partial E[\tilde{T}\tilde{T}]/\partial v_a$, $E[\tilde{T}_a]$, $\partial Var[\tilde{T}\tilde{T}]/\partial v_a$, and $Var[\tilde{T}_a]$ are known. From the conditional moment analysis above, we have already obtained $E[\tilde{T}_a]$ and $Var[\tilde{T}_a]$. Moreover, based on the moment analysis, we can derive the mean and variance of $\tilde{T}\tilde{T}$ (see Appendix for the derivations). Substituting Eqs. (43), (45), (A2), and (A4) into Eq. (46) and performing some derivation, we have

$$\begin{aligned}
 \text{PRSN} - \text{MCP} &= (1 + \chi)\{\partial E[TT]/\partial v_a - E[T_a]\} \\
 &\quad + VoR \cdot \left\{(1 + \chi)^2\{\partial Var[TT]/\partial v_a - Var[T_a]\} + \varpi^2\{\partial E[V_a^2 T_a]/\partial v_a - E[T_a]\}\right\}
 \end{aligned} \tag{47}$$

5.2 Calculation of PRSN-MCP

In order to illustrate the importance of incorporating both stochastic supply and demand into the proposed PRSN-MCP model, the calculation of PRSN-MCP can be separated into four scenarios based on (1) network uncertainty caused by the stochasticity of travel demand; and (2) network uncertainty induced by the stochastic supply (link capacity). Case A is the most complete situation in which both stochastic link capacity and travel demand are considered. In contrast to Case A, which describes the “true” behaviors of travelers, Case D is the simplest case, neglecting the stochasticity of traffic network. Case B and C ignore, respectively, the effect of stochastic demand and link capacity.

5.2.1 Case a: stochastic supply, stochastic demand (SS-SD)

To begin, we discuss the most complete and realistic case in which the travelers consider both stochastic fluctuations in supply (or link capacity) and demand in their route choice decision-making process. As of now, we have already obtained the values of $\partial E[TT]/\partial v_a$, $E[T_a]$, $Var[T_a]$ and $\partial Var[TT]/\partial v_a$. The only value left unknown is $\partial E[V_a^2 T_a]/\partial v_a$. With Eq. (26) we can obtain

$$\begin{aligned} E[(V_a)^2 T_a] &= t_a^0 E[V_a]^2 + \beta t_a^0 E[V_a^{n+2}] E\left[\frac{1}{C_a^n}\right] \\ &= t_a^0 v_a^2 y_a^2 + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \left(v_a^{n+2} y_a^{n^2+3n+2}\right) \end{aligned} \quad (48)$$

Differentiating Eq. (48) with respect to the mean link flow v_a and performing some simple algebraic operations we have

$$\begin{aligned} \frac{\partial E[(V_a)^2 T_a]}{\partial v_a} &= 2 \cdot t_a^0 v_a y_a^2 - t_a^0 \cdot VMR + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \\ &\quad \left[(n+2)v_a^{n+1} y_a^{n^2+3n+2} - \frac{n^2 + 3n + 2}{2} \cdot VMR \cdot v_a^n y_a^{n^2+3n} \right] \end{aligned} \quad (49)$$

Substituting Eqs. (30), (31), (34), (38), and (49) into Eq. (47), we can obtain the value of PRSN-MCP in case of SS-SD.

5.2.2 Case B: stochastic supply, deterministic demand (SS-DD)

In Case B, the effect of stochastic demand is neglected; only the effect of stochastic link capacity is considered in modeling the travelers' route choice decision-making process. Thus, the mean and variance of T_a are given by Eqs. (20) and (21), respectively. To calculate the value of PRSN-MCP in case of stochastic supply and deterministic demand, we need to recalculate $\partial E[V_a T_a]/\partial v_a$, $\partial Var[TT]/\partial v_a$, and $\partial E[V_a^2 T_a]/\partial v_a$, respectively.

The expected total travel time can be simplified to

$$\begin{aligned} E[TT] &= E\left[\sum_{a \in A} V_a T_a\right] = \sum_{a \in A} \left\{ t_a^0 E[V_a] + \beta t_a^0 E[V_a^{n+1}] E\left[\frac{1}{C_a^n}\right] \right\} \\ &= \sum_{a \in A} \left\{ t_a^0 v_a + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} v_a^{n+1} \right\} \end{aligned} \quad (50)$$

Differentiating Eq. (50) with respect to the mean link flow v_a yields

$$\frac{\partial E[TT]}{\partial v_a} = t_a^0 + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} [(n + 1)v_a^n] \quad (51)$$

The variance of the total travel time is described by

$$\begin{aligned} \text{Var}[TT] &= E[TT^2] - (E[TT])^2 \\ &= \sum_{a \in A} \left\{ (\beta t_a^0)^2 \frac{\text{Var}[V_a^{n+1}]}{\text{Var}[C_a^n]} \right\} \\ &= \sum_{a \in A} \left\{ (\beta t_a^0)^2 v_a^{2n+2} \left\{ \frac{(1 - \theta_a^{1-2n})}{\bar{c}_a^{2n} (1 - \theta_a)(1 - 2n)} - \left[\frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \right]^2 \right\} \right\} \end{aligned} \quad (52)$$

Differentiating Eq. (52) with respect to the mean link flow yields

$$\frac{\partial \text{Var}[TT]}{\partial v_a} = (2n + 2) (\beta t_a^0)^2 v_a^{2n+1} \left\{ \frac{(1 - \theta_a^{1-2n})}{\bar{c}_a^{2n} (1 - \theta_a)(1 - 2n)} - \left[\frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} \right]^2 \right\} \quad (53)$$

With Eq. (26) we have

$$E[(V_a)^2 T_a] = t_a^0 E[V_a]^2 + \beta t_a^0 E[V_a^{n+2}] E\left[\frac{1}{C_a^n}\right] = t_a^0 v_a^2 + \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} v_a^{n+2} \quad (54)$$

Differentiating Eq. (54) with respect to the mean link flow v_a we have, upon simplifying

$$\frac{\partial E[(V_a)^2 T_a]}{\partial v_a} = 2 \cdot t_a^0 v_a + (n + 2) \beta t_a^0 \frac{(1 - \theta_a^{1-n})}{\bar{c}_a^n (1 - \theta_a)(1 - n)} v_a^{n+1} \quad (55)$$

By substituting Eqs. (20), (21), (51), (53), and (55) into Eq. (47), the value of PRSN-MCP in case of SS-DD can be determined.

5.2.3 Case C: deterministic supply, stochastic demand (DS-SD)

In Case C, only the effect of stochastic travel demand is captured in modeling travelers' route choice decision process. The effect of stochastic link capacity is ignored in this case. Therefore, $E[1/C_a^n]$ and $E[1/C_a^{2n}]$ are simplified to $1/\bar{c}_a^n$ and $1/\bar{c}_a^{2n}$, respectively. Consequently, the mean and variance of T_a are given by Eqs. (28) and (29), respectively. Similar to Case B, we need to recalculate $\partial E[V_a T_a]/\partial v_a$, $\partial \text{Var}[TT]/\partial v_a$, and $\partial E[V_a^2 T_a]/\partial v_a$, respectively.

The expected total travel time is given by

$$\begin{aligned} E[TT] &= E\left[\sum_{a \in A} V_a T_a\right] = \sum_{a \in A} \left\{ t_a^0 E[V_a] + \beta t_a^0 E[V_a^{n+1}] E\left[\frac{1}{C_a^n}\right] \right\} \\ &= \sum_{a \in A} \left\{ t_a^0 v_a + \frac{\beta t_a^0}{\bar{c}_a^n} (v_a^{n+1} \gamma_a^{n+1}) \right\} \end{aligned} \quad (56)$$

Differentiating Eq. (56) with respect to the mean link flow v_a yields

$$\frac{\partial E[TT]}{\partial v_a} = t_a^0 + \frac{\beta t_a^0}{\bar{c}_a^n} \left[\frac{nv_a^{n-1}(1-y_a^2)}{2y_a^2} + 1 \right] \left[(n+1)v_a y_a^{n^2+n} \right] \quad (57)$$

The variance of the total travel time is expressed as

$$\begin{aligned} \text{Var}[TT] &= E[TT^2] - (E[TT])^2 \\ &= \sum_{a \in A} \left\{ (t_a^0)^2 \cdot \text{Var}[V_a] + \left(\frac{\beta t_a^0}{\bar{c}_a^n} \right)^2 \text{Var}[V_a^{n+1}] + \frac{2\beta(t_a^0)^2}{\bar{c}_a^n} \{E[V_a^{n+2}] - E[V_a^{n+1}]E[V_a]\} \right\} \\ &= \sum_{a \in A} \left\{ (t_a^0)^2 \cdot \text{VMR} \cdot v_a + \left(\frac{\beta t_a^0}{\bar{c}_a^n} \right)^2 \left\{ v_a^{2n} y_a^{4n^2+6n+2} - (v_a^{n+1} y_a^{n^2+n})^2 \right\} + \frac{2\beta(t_a^0)^2}{\bar{c}_a^n} v_a^{n+2} y_a^{n^2+n} (y_a^{2n+2} - 1) \right\} \end{aligned} \quad (58)$$

Differentiating Eq. (58) with respect to the mean link flow yields

$$\begin{aligned} \frac{\partial \text{Var}[TT]}{\partial v_a} &= (t_a^0)^2 \cdot \text{VMR} + \left(\frac{\beta t_a^0}{\bar{c}_a^n} \right)^2 \left\{ \begin{aligned} &\left\{ v_a^{2n} y_a^{4n^2+6n} [(2n+2)v_a - (2n^2+n-1) \cdot \text{VMR}] \right\} \\ &\left\{ -v_a^{2n} y_a^{2n^2+2n-2} [(2n+2)v_a - (n^2-n-2) \cdot \text{VMR}] \right\} \end{aligned} \right\} \\ &+ \frac{2\beta(t_a^0)^2}{\bar{c}_a^n} \left\{ \begin{aligned} &\left\{ v_a^n y_a^{n^2+3n} \left[(n+2)v_a - \frac{(n^2+n-2)}{2} \cdot \text{VMR} \right] \right\} \\ &\left\{ -v_a^n y_a^{n^2+n-2} \left[(n+2)v_a - \frac{(n^2-n-4)}{2} \cdot \text{VMR} \right] \right\} \end{aligned} \right\} \end{aligned} \quad (59)$$

With Eq. (26) we have

$$E[(V_a)^2 T_a] = t_a^0 E[V_a]^2 + \beta t_a^0 E[V_a^{n+2}] E\left[\frac{1}{C_a^n}\right] = t_a^0 v_a^2 y_a^2 + \frac{\beta t_a^0}{\bar{c}_a^n} (v_a^{n+2} y_a^{n^2+3n+2}) \quad (60)$$

Differentiating Eq. (60) with respect to the mean link flow v_a and performing some simple algebraic operations, we have

$$\begin{aligned} \frac{\partial E[(V_a)^2 T_a]}{\partial v_a} &= 2 \cdot t_a^0 v_a y_a^2 - t_a^0 \cdot \text{VMR} \\ &+ \frac{\beta t_a^0}{\bar{c}_a^n} \left[(n+2)v_a^{n+1} y_a^{n^2+3n+2} - \frac{n^2+3n+2}{2} \cdot \text{VMR} \cdot v_a^n y_a^{n^2+3n} \right] \end{aligned} \quad (61)$$

Thus the value of PRSN-MCP in case of DS-SD can be determined by substituting Eqs. (28), (29), (57), (59), and (61) into Eq. (47).

5.2.4 Case D: Deterministic supply, deterministic demand (DS-DD)

Case D degenerates into the MCP model in a deterministic traffic network, in which neither the stochastic link capacity nor stochastic travel demand is considered in travelers' route choice decision making. In this case, the variance of both $\text{Var}[TT]$ and $\text{Var}[T]$ is equal to zero, and $E[T_a] = t_a^0 + \beta t_a^0 v_a^n / C_a^n$. We only need to recalculate $\partial E[V_a T_a] / \partial v_a$, and $\partial E[V_a^2 T_a] / \partial v_a$, respectively.

The expected total travel time can be simplified to

$$\begin{aligned}
 E[TT] &= E\left[\sum_{a \in A} V_a T_a\right] = \sum_{a \in A} \left\{ t_a^0 E[V_a] + \beta t_a^0 E[V_a^{n+1}] E\left[\frac{1}{C_a^n}\right] \right\} \\
 &= \sum_{a \in A} \left\{ t_a^0 v_a + \frac{\beta t_a^0 v_a^{n+1}}{\bar{c}_a^n} \right\}
 \end{aligned} \tag{62}$$

Then we have

$$\frac{\partial E[TT]}{\partial v_a} - E[T_a] = \left[t_a^0 + (n+1) \frac{\beta t_a^0}{\bar{c}_a^n} v_a^n \right] - \left[t_a^0 + \frac{\beta t_a^0}{\bar{c}_a^n} v_a^n \right] = \frac{n \beta t_a^0}{\bar{c}_a^n} v_a^n \tag{63}$$

From Eq. (26) we can obtain

$$E[(V_a)^2 T_a] = t_a^0 E[V_a]^2 + \beta t_a^0 E[V_a^{n+2}] E\left[\frac{1}{C_a^n}\right] = t_a^0 v_a^2 + \frac{\beta t_a^0 v_a^{n+2}}{\bar{c}_a^n} \tag{64}$$

Consequently, we have, upon simplifying

$$\frac{\partial E[(V_a)^2 T_a]}{\partial v_a} - E[T_a] = (2v_a - 1) \cdot t_a^0 - [(n+2)v_a - 1] \frac{\beta t_a^0 v_a^n}{\bar{c}_a^n} \tag{65}$$

By substituting Eqs. (63) and (65) into Eq. (47), the value of PRSN-MCP in case of DS-DD can be expressed as follows:

$$\text{PRSN} - \text{MCP} = (1 + \chi) \left(\frac{n \beta t_a^0}{\bar{c}_a^n} v_a^n \right) + \text{VoR} \cdot \varpi^2 \left\{ (2v_a - 1) \cdot t_a^0 - [(n+2)v_a - 1] \frac{\beta t_a^0 v_a^n}{\bar{c}_a^n} \right\} \tag{66}$$

6. Numerical examples

The purpose of the numerical examples is to illustrate: (1) the effect of the VMR on the performance of the SN-MCP model; (2) the effect of both the demand and supply uncertainties on the performance of the PRSN-MCP model; (3) the importance of incorporating the travelers' perception error in the RSN-MCP model; and (4) the application of the proposed PRSN-MCP model in a medium-scale traffic network. The proposed models in this chapter can be solved by the method of successive averages (MSA).

6.1 Effect of the VMR on the performance of SN-MCP toll scheme

Figure 2 shows a network consisting of 14 nodes and 21 directed links. There are two OD pairs, one is from node 1 to 12, and the other one is from node 1 to 14. The link travel time function is assumed to be the Bureau of Public Roads (BPR) function with the following parameters: $\beta = 0.15$, $n = 4$, which is, $T_a = t_a^0 (1 + 0.15 (V_a/C_a)^4)$, $\forall a \in A$. The free-flow travel time, design capacity, and degradation parameter for each link are given in **Table 1**. In order to test the effects of different demand levels, the potential mean total demand for OD pair 1 and 2 is set as $\bar{q}^1 = 3800z$ and $\bar{q}^2 = 4200z$, respectively. In $0 \leq z \leq 1$, z is the OD demand multiplier.

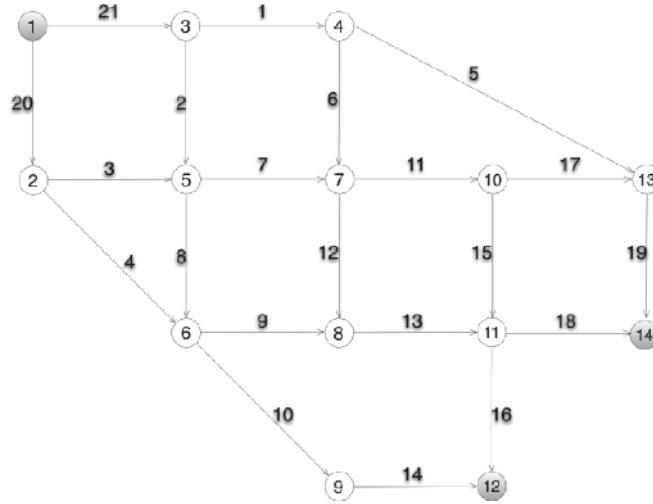


Figure 2.
Traffic network.

Link	Free-flow travel time	Design capacity	Degradation parameter θ_a	Link	Free-flow travel time	Design capacity	Degradation parameter θ_a
1	3	2000	0.95	12	3	1000	0.95
2	3	2000	0.95	13	3	2600	0.95
3	3	2000	0.95	14	3	2000	0.95
4	4.5	1800	0.95	15	3	1400	0.95
5	7.5	1200	0.95	16	3	2000	0.95
6	3	1000	0.95	17	3	800	0.95
7	3	2000	0.95	18	3	2000	0.95
8	3	1800	0.95	19	3	2000	0.95
9	3	1800	0.95	20	3	4000	0.95
10	4.5	1800	0.95	21	3	4000	0.95
11	3	2000	0.95				

Table 1.
Link parameters.

For the first example, we examine the effect of *VMR* on the performance of the SN-MCP model proposed in Section 3. All travelers are assumed to be risk-neutral (i.e., $VoR = 0$). In addition, travelers' perception errors are not considered in the first example. The relationship between the expected total perceived travel time, OD demand level, and *VMR* level under the toll free case and the SN-MCP toll scheme are shown in **Figure 3**. It can be observed that the difference of the expected total perceived travel time (i.e., $U[TT_{\text{toll free}}] - U[TT_{\text{SN-MCP}}]$) between these two scenarios decreases with the OD demand and *VMR* levels. For example, if the demand multiplier z is 0.8 and *VMR* level is 10, $U[TT_{\text{toll free}}] - U[TT_{\text{SN-MCP}}]$ is more than 2900. However, when the demand multiplier z increases to 1 and *VMR* level increases to 50, $U[TT_{\text{toll free}}] - U[TT_{\text{SN-MCP}}]$ is less than 1633. Remember that VMR_w is the variance-to-mean ratio (*VMR*) of random travel demand. This indicates that along with the increase of travel demand variance and congestion level, the performance of the SN-MCP toll scheme decreases.

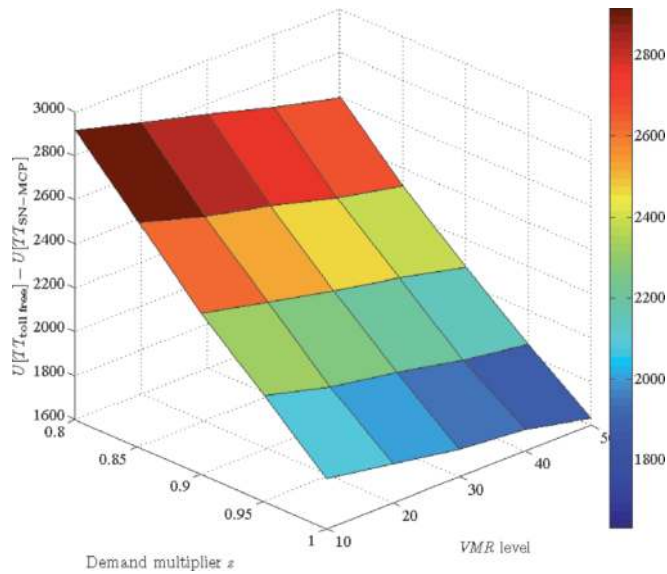


Figure 3. Difference of the expected total perceived travel time between toll free case and SN-MCP under different OD demand multiplier z and VMR levels.

6.2 Importance of incorporating supply and demand uncertainty

6.2.1 Effect of congestion on the performance of different PRSN-MCP toll schemes.

We also use the traffic network shown in **Figure 2** in the following test, in which both supply and travel demand uncertainty and travelers' perception errors will be simulated. To demonstrate the effects of neglecting certain aspects of the stochasticity of the network, we compare the expected total perceived travel time under the four PRSN-MCP scenarios discussed in Section 5.2. These four scenarios are analyzed under different congestion levels (the OD demand multiplier z increases from 0.8 to 1 by interval 0.05). As a reminder, all the four scenarios consider the travelers' perception error, with the following differences: Case A is the most complete and realistic representation of the actual traffic flow as both stochastic fluctuations in supply (or link capacity) and demand are incorporated. In comparison, Case B and C are "incomplete cases," because they neglect certain aspects of the stochastic network. Case D is the classical MCP model in a deterministic traffic network.

In this example, we study the effect of congestion levels on the performance of different toll schemes with fixed VoR (i.e., $VoR = 0.0165$) and VMR_w (i.e., $VMR_w = 1.5$). Furthermore, we assume the perception error distribution of unit travel time follows $N(0.1, 0.2)$. **Table 2** displays the expected total perceived travel time at different congestion levels under the toll free, SS-SD, SS-DD, DS-DD, and DS-SD of the PRSN-MCP toll schemes. The results show that the expected total perceived travel time of the toll free and = other toll schemes increases as the demand multiplier z increases.

Figure 4 demonstrates the percentage improvements in the expected total perceived travel time related to **Table 2**. The "Improvement" in **Figure 4** is, in this case, the percentage of improvement in the expected total perceived travel time from the toll free case compared to the SS-SD tolls case, that is,

$$\text{Improvement} = (U[\tilde{T}\tilde{T}_{\text{toll-free}}] - U[\tilde{T}\tilde{T}_{\text{case}}]) / (U[\tilde{T}\tilde{T}_{\text{toll-free}}] - U[\tilde{T}\tilde{T}_{\text{SS-SD}}]) \times 100\% \quad (67)$$

Demand multiplier (z)	$U[\bar{T}\bar{T}]$				
	Toll free	SS-SD	SS-DD	DS-DD	DS-SD
0.8	132,261	129,158	129,171	129,184	129,300
0.85	142,651	139,878	139,908	139,928	140,084
0.9	153,870	151,438	151,474	151,502	151,695
0.95	166,113	163,979	164,033	164,072	164,283
1	179,550	177,688	177,757	177,801	177,996

Table 2. Comparison of system performance under different modeling scenarios and OD demand multipliers.

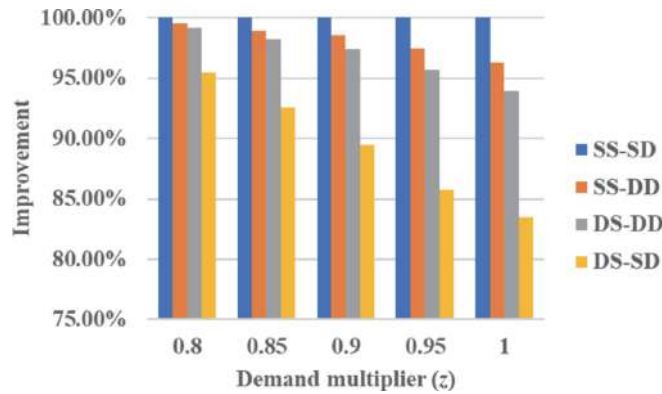


Figure 4. Improvement in system performance under different modeling scenarios and OD demand multipliers.

From the figure above, the improvement in the expected total perceived travel time obtained by the SS-DD, DS-DD, and DS-SD tolls is lower than that obtained by the SS-SD tolls. Besides, the gap between the expected total perceived travel time under the SS-SD tolls and other toll schemes increases as z increases. When traffic is light, all toll schemes achieve similar system performances, revealing that other toll schemes do not lose too much accuracy by ignoring the stochasticity of the traffic network. However, when traffic is heavy, the differences between them become pronounced. Furthermore, for the DS-SD tolls, neglecting the stochastic link capacity makes the system performance decrease rapidly. This indicates that the toll scheme is more sensitive to the stochasticity of link capacity.

6.2.2 Effect of the VoR on the performance of different PRSN-MCP toll schemes

By assuming the levels of congestion and VMR_w are fixed (i.e., $z = 1$, $VMR_w = 1.5$), the effect of the VoR on the expected total perceived travel time under different toll schemes is examined in this section. In **Table 3**, the expected total perceived travel time at different VoR levels under the toll free, SS-SD, SS-DD, DS-DD, and DS-SD of the PRSN-MCP toll schemes are compared. The expected total perceived travel time increases with an increase in the level of the VoR. This is logical: when VoR increases, travelers need to budget a large buffer time to improve their travel time reliability.

Based on Eq. (67) and **Table 3**, we can obtain the percentage improvements in the expected total perceived travel time, as shown in **Figure 5**. It can be seen that the discrepancies between the performance of the SS-SD toll and that of other toll schemes become conspicuously larger as the VoR increases. This implies that the higher the travel time reliability that travelers are concerned with, the worse the

VoR	$U[\tilde{T}\tilde{T}]$				
	Toll free	SS-SD	SS-DD	DS-DD	DS-SD
0.0068	179,233	177,335	177,384	177,413	177,510
0.0085	179,291	177,394	177,445	177,491	177,620
0.0104	179,344	177,468	177,525	177,568	177,716
0.0129	179,432	177,560	177,623	177,668	177,844
0.0165	179,550	177,688	177,757	177,801	177,996

Table 3.
 Comparison of system performance under different modeling scenarios and VoR levels.

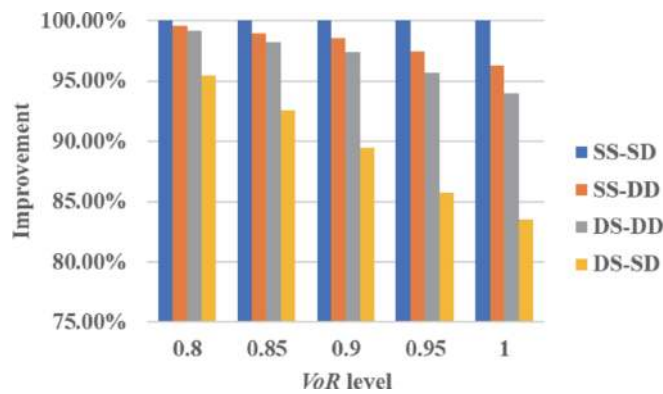


Figure 5.
 Improvement in system performance under different modeling scenarios and VoR levels.

actual effect of other toll schemes, which ignore the effect of stochastic travel demand and link capacity.

From the above analysis, it can be concluded that the discrepancies of these simplifications depend on both the congestion and VoR levels. Capturing the effect of stochastic capacity degradation and stochastic travel demand is critically important.

6.3 Analysis of the essentiality of incorporating the travelers' perception error

The traffic network shown in **Figure 2** is again adopted in examining the PRSN-MCP model. By comparing the difference of the expected total perceived travel time achieved by the RSN-MCP tolls (expressed by $U[TT_{SS-SD}]$) and the PRSN-MCP tolls (denoted by $U[\tilde{T}\tilde{T}_{SS-SD}]$), we examine the effect of incorporating the traveler's perception error into the RSN-MCP tolls. In this example, both stochastic fluctuations in supply (or link capacity) and demand are considered for both toll schemes. **Figure 6** illustrates the influence of various combinations of travel demand level and VoR level on the difference of the expected total perceived travel time achieved by the RSN-MCP tolls and the PRSN-MCP tolls. Based on the survey results of [24], it is reasonable to assume that all the travelers are risk-averse under an uncertain environment. Therefore, we test the VoR level from 0.0068 to 0.0165, and the OD demand multiplier z from 0.8 to 1 with an interval of 0.05. From **Figure 6**, it is clear that $U[TT_{SS-SD}] - U[\tilde{T}\tilde{T}_{SS-SD}]$ increases as the demand level z increases. This implies that the consideration of travelers' perception error in the RSN-MCP tolls may have a more significant impact on system performance

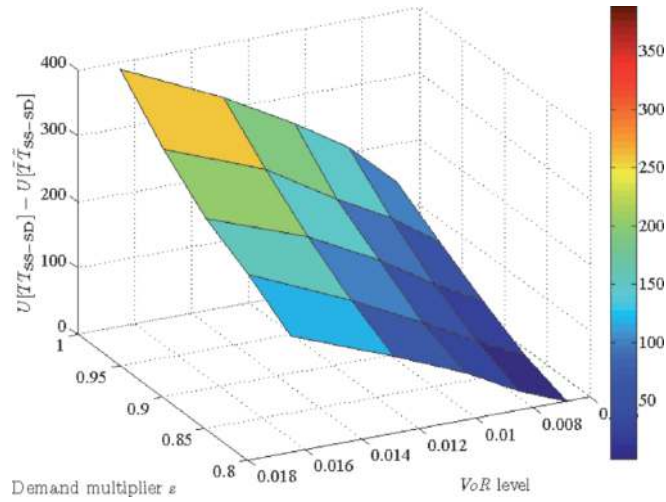


Figure 6. Difference of the expected total perceived travel time between PRSN-MCP and RSN-MCP under different OD demand multiplier z and VoR levels.

under heavier congestion levels. On the other hand, we can see that $U[TT_{SS-SD}] - U[\tilde{T}\tilde{T}_{SS-SD}]$ is increasing while the VoR level increases. This is to be expected, since a higher travel time reliability requires a larger time buffer. Therefore, ignoring the travelers' perception error may significantly reduce the performance of the RSN-MCP tolls, especially when the network congestion level is heavy and travelers require a higher travel time reliability level.

6.4 Application to the Sioux Falls network in the PRSN-MCP (SS-SD) case

The final example illustrates the calculation of the PRSN-MCP (SS-SD) toll in a larger network. This example network is the well-known medium-scale Sioux Falls

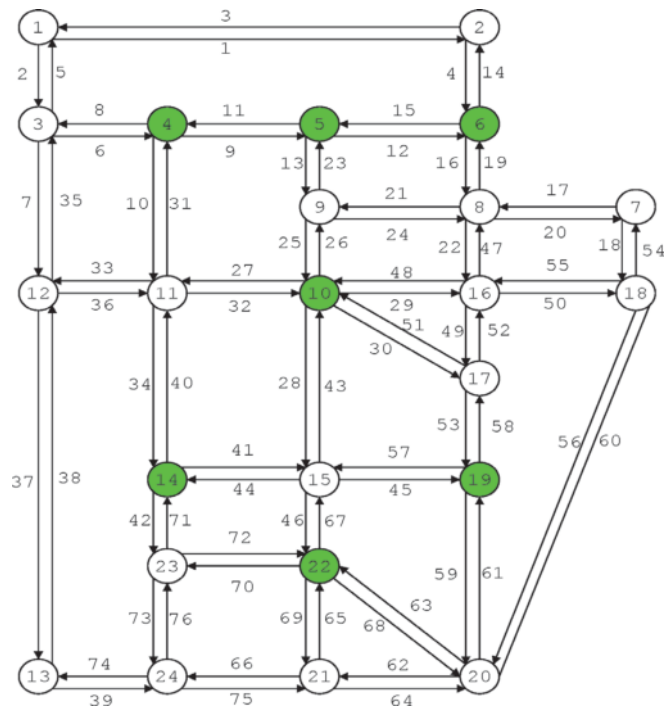


Figure 7. Sioux Falls network.

network (see **Figure 7**), which consists of 24 nodes and 76 links. The link design capacity and link free-flow travel time can be found in [25]. Degradation parameter θ_a for each link is 0.95. In this example, we also assume $VMR_w = 1.5$, and the perception error distribution of a unit travel time follows $N(0.1, 0.2)$. Forty-two OD pairs are considered in the Sioux Falls network and the mean of the lognormal demand for all OD pairs is given in **Table 4**. The stopping tolerance criterion is set at 0.001. Convergence is achieved in 48 iterations as depicted in **Figure 8**.

In this example, we compare two scenarios. One is the toll free case, and the other one is the PRSN-MCP toll scheme. **Table 5** presents the link toll under the PRSN-MCP scenario. By levying these tolls on each link, the network becomes smooth and efficient. At the equilibrium state, the expected total perceived travel time achieved by the toll free case and PRSN-MCP toll scheme is 345,749 and 324,636, respectively. Therefore, the proposed PRSN-MCP model is an efficient method in reducing traffic congestion.

O/D	4	5	6	10	14	19	22
4		800	800	800	800	800	800
5	800		800	800	800	800	800
6	800	800		800	800	800	800
10	800	800	800		800	800	800
14	800	800	800	800		800	800
19	800	800	800	800	800		800
22	800	800	800	800	800	800	

Table 4.
 Means of the stochastic demand for all OD pairs in the Sioux Falls network.

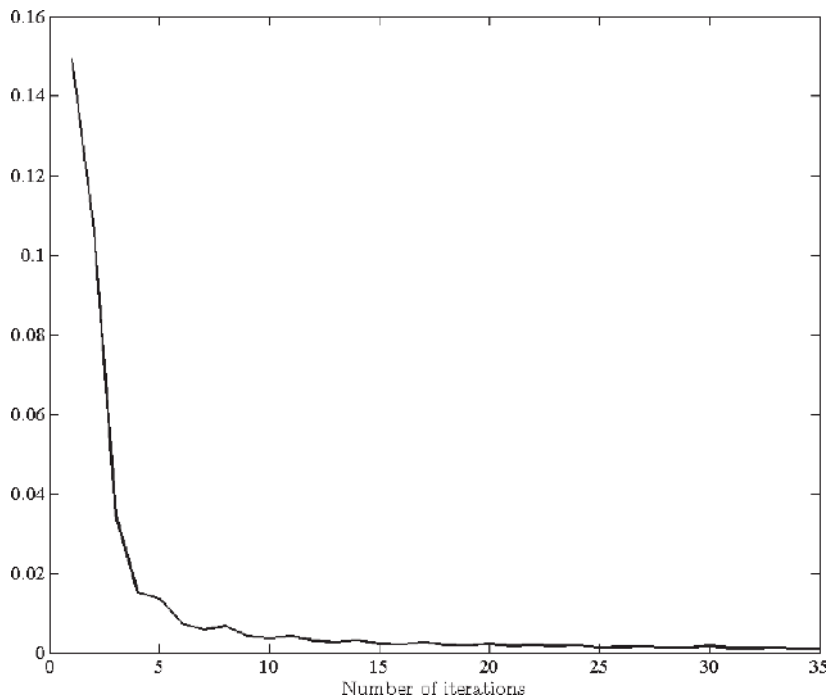


Figure 8.
 Convergence of the MSA for the Sioux Falls network.

Link	Link toll	Link	Link toll	Link	Link toll	Link	Link toll
1	19.37	20	23.16	39	33.54	58	45.96
2	12.81	21	17.97	40	68.29	59	20.13
3	19.37	22	37.02	41	38.43	60	35.53
4	16.08	23	95.53	42	22.83	61	20.13
5	12.81	24	17.97	43	59.94	62	23.86
6	34.19	25	37.12	44	38.43	63	32.55
7	78.43	26	37.12	45	22.83	64	23.86
8	34.19	27	33.15	46	29.97	65	7.68
9	39.22	28	59.94	47	37.02	66	15.86
10	125.82	29	17.53	48	17.53	67	29.97
11	39.22	30	30.77	49	14.00	68	32.55
12	46.12	31	125.82	50	3.07	69	7.68
13	95.53	32	33.15	51	30.77	70	18.04
14	16.08	33	18.94	52	14.00	71	22.83
15	46.12	34	68.29	53	45.96	72	18.04
16	73.25	35	78.43	54	15.33	73	5.24
17	23.16	36	18.94	55	3.07	74	33.54
18	15.33	37	24.96	56	35.53	75	15.86
19	73.25	38	24.96	57	22.83	76	5.24

Table 5.
PRSN-MCP tolls for each link at equilibrium state.

7. Conclusions

To make pricing more efficient and effective, this chapter developed a reliability-based marginal cost pricing model. The new model explicitly accounts for both stochastic link degradation and stochastic demand of road network and perception errors within the travelers' route choice decision process. We consider that the stochastic demand follows a lognormal distribution and the capacity degradation follows a uniform distribution, and VMR across all OD pairs. Based on moment analysis, we derive the mean and variance of the expected total perceived travel time. After performing some derivations, we derived four analytical functions of PRSN-MCP under different simplifications of network uncertainty.

This chapter investigated possible defects associated with ignoring certain aspects of the stochastic behaviors of the network. Through numerical examples, we find that both link capacity degradation and stochastic demand play essential roles in the PRSN-MCP model, especially under high travelers' confidence level and network congestion. We further examined the effect of incorporating the travelers' perception error into the RSN-MCP tolls. The numerical example illustrates that travelers' perception errors have a significant impact on the performance of the PRSN-MCP tolls and, therefore, should not be neglected.

A. Appendix: computation of the MGF of $\tilde{T}\tilde{T}$

The MGF of $\tilde{T}\tilde{T}$ can be represented as follows:

$$\begin{aligned}
 M_{\tilde{T}\tilde{T}}(s) &= \sum_{a \in A} E[\exp(sV_a\tilde{T}_a)] \\
 &= \sum_{a \in A} E\{\exp[sV_a(T_a + \varepsilon_a)]\} \\
 &= \sum_{a \in A} E_{T_a}\{\exp(sV_aT_a)\exp(sV_a\varepsilon_a)\} \\
 &= \sum_{a \in A} E_{T_a}\left\{\exp(sV_aT_a)E_{\varepsilon_a|T_a}[\exp(sV_a\varepsilon_a|T_a)]\right\} \\
 &= \sum_{a \in A} E_{T_a}\left\{\exp(sV_aT_a)M_{\varepsilon_a|T_a}(sV_a)\right\} \\
 &= \sum_{a \in A} E_{T_a}\left\{\exp(sV_aT_a)\exp[sV_aT_a(\chi + \varpi^2sV_a/2)]\right\}
 \end{aligned} \tag{68}$$

The first-order moment is, from the first derivative evaluated at $s = 0$,

$$E[\tilde{T}\tilde{T}] = \sum_{a \in A} (1 + \chi)E[V_aT_a] \tag{69}$$

Similarly, the second-order moment of $\tilde{T}\tilde{T}$ can be derived from the second derivative evaluated at $s = 0$,

$$E[(\tilde{T}\tilde{T})^2] = \sum_{a \in A} \left\{ (1 + \chi)^2 E[(V_aT_a)^2] + \varpi^2 E[V_a^2T_a] \right\} \tag{70}$$

Then we can obtain the variance of $\tilde{T}\tilde{T}$ as follows:

$$\begin{aligned}
 Var[\tilde{T}\tilde{T}] &= E[(\tilde{T}\tilde{T})^2] - E[\tilde{T}\tilde{T}]^2 \\
 &= \sum_{a \in A} \left\{ (1 + \chi)^2 \left\{ E[(V_aT_a)^2] - E[V_aT_a]^2 \right\} + \varpi^2 E[V_a^2T_a] \right\} \\
 &= \sum_{a \in A} \left\{ (1 + \chi)^2 Var[TT] + \varpi^2 E[V_a^2T_a] \right\}
 \end{aligned} \tag{71}$$

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