Nonlinear Channel Equalization Approach for Microwave Communication Systems

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Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/65969

Abstract

The theoretical principles of intersymbol interference (ISI) and channel equalization in wireless communication systems are addressed. Several conventional and well-known equalization techniques are discussed and compared such as zero forcing (ZF) and maximum likelihood (ML). The main section in this chapter is devoted to an abstract concept of equalization approach, namely, dual channel equalization (DCE). The proposed approach is flexible and can be employed and integrated with other linear and nonlinear equalization approaches. Closed expressions for the achieved signal-to-noise ratio (SNR) and bit error rate (BER) in the case of ZF-DCE and ML-DCE are derived. According to the obtained outcomes, the DCE demonstrates promising improvements in the equalization performance (BER reduction) in comparison with the conventional techniques.

Keywords: channel equalization, intersymbol interference (ISI), zero forcing (ZF) equalization, maximum likelihood (ML) equalization, bit error rate (BER), dual channel equalization (DCE)

1. Introduction

All types of microwave wireless communication systems under different digital modulation schemes and antennas configuration suffer from the channel effects and related problems such as attenuation, signal amplitude and phase distortions, time-varying fading (Doppler shift), multipath fading, and intersymbol interference (ISI). A common and well-known wireless channel modeling method is based on the representation of the channel as a band-limited digital filter, i.e., linear time-invariant (LTI) filter with specific transfer function (impulse response). Thus, to alleviate and reduce the channel effects, especially for multipath fading and ISI, it is possible to design a digital filter with transfer function that is inverse to the transfer function of the associated wireless channel. This digital filter is called the equalizer

[1, Chapter 10]. Additionally, the employment of multiple antennas can also help to mitigate the multipath fading consequences (transmit/receive diversity) and increase the data rate (spatial multiplexing).

In practice, the wireless transmission system sends a sequence of messages (one-shot transmission) where these successive transmissions should not interfere even if they are closely spaced to increase the data rate. This interference between the successive transmissions is referred as intersymbol interference (ISI) that is able to complicate and reduce the detection performance [2, Chapter 4]. The simple symbol-by-symbol (SBS) detector (optimal in the case of additive white Gaussian noise AWGN channel) cannot be the maximum likelihood estimator for a sequence of message under ISI problem. A receiver for succession messages detection is shown in **Figure 1**, where the matched filter outputs are processed by the receiver and SBS detector to generate the estimate $\hat{X}_k$ of the input symbol X_k at time k.

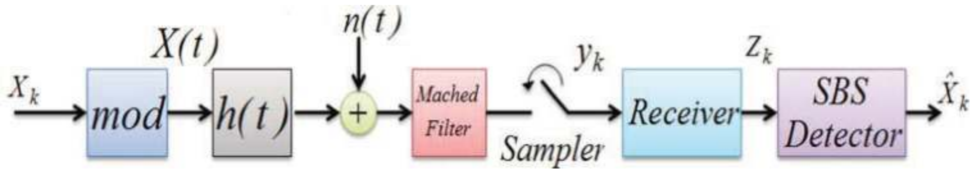

Figure 1. The band-limited channel with receiver and SBS detector.

An equalizer or equalization method can be essentially embedded in the contents of the receiver block presented in **Figure 1**. Different equalization techniques lead to different receiver structures that are not always optimal for detection, but rather are widely implemented as suboptimal cost-effective solutions that alleviate the ISI. Any equalization approach converts the band-limited channel with ISI into memory less appearing channel (AWGN-like) at the receiver output. The wireless channel equalization forms a major challenge in current and future communication networks.

In Section 2 of this chapter, the ISI between successive transmissions is modeled to prove that the distortion caused by this overlapping is unacceptable and some corrective actions should be applied. Some targeted and desired wireless channel responses that exhibit no ISI are discussed in Section 3 with the corresponding Nyquist criterion for the ISI-free channels. In fact, the signal-to-noise ratio (SNR) parameter used to quantify the receiver performance can be consistently considered for equalization techniques evaluation as well. Several conventional and well-known equalization approaches are presented in Section 4 such as zero-forcing equalizer (ZFE), minimum mean square error (MMSE) equalizer, and decision feedback equalizer (DFE). In the last section (Section 5), the proposed equalization approach and its performance are discussed and compared to other equalization techniques under the same initial conditions.

2. Successive message transmission and ISI

The frequency or wireless channel reuse is a common technique to transmit several succession messages separated by the symbol period (T units in time where $1/T$ is the symbol rate). For sending one of M possible messages every T time units, the data rate of the communication system is defined as:

$$R = \frac{log_2(M)}{T}. \tag{1}$$

Increasing the data rate R can be achieved by decreasing T which reduces or narrows the time between message transmissions and as a consequence increases ISI on the band-limited channel. The transmitted signal of K successive transmissions (conveying one of M^K possible messages) is given by

$$x(t) = \sum_{k=0}^{K-1} x_k(t-kT). \tag{2}$$

The detection of M^K messages approach has a complexity that grows exponentially with the block message length K (especially when $K \to \infty$). The SBS detection can be considered as an alternative suboptimal solution that detects independently each of the successive K messages. The ISI problem is the main limitation of the SBS detection approach when the performance degradation increases as T decreases (or R increases). The ISI can be analyzed by rewriting Eq. (2) using the following form:

$$x(t) = \sum_{k=0}^{K-1} \sum_{n=1}^{N} x_{kn}\varphi_n(t-kT), \tag{3}$$

where the original transmissions $x_k(t)$ are decomposed using a standard orthogonal basis set $\{\varphi_n(t)\}$. For instance, in the case of quadrature amplitude modulation (QAM), the baseband basis:

$$\varphi(t) = \frac{1}{\sqrt{T}} sinc\left(\frac{t}{T}\right) \tag{4}$$

produces orthogonal functions for all integer-multiple of T time translations and the successive transmissions sampled at time instants kT are ISI free. Owing to the channel filtering alterations, the filtered basis functions at the channel output are no longer orthogonal and the ISI is introduced.

The band-limited noise-free channel output used for successive transmission of data symbol can be presented as

$$x_p(t) = \sum_{k=0}^{K-1} \sum_{n=1}^{N} x_{kn}\varphi_n(t-kT) * h(t) = \sum_{k=0}^{K-1} \sum_{n=1}^{N} x_{kn}p_n(t-kT), \tag{5}$$

where $p_n(t) = \varphi_n(t)*h(t)$ and when $h(t)\neq\delta(t)$, the functions $p_n(t-kT)$ do not form orthogonal basis (see **Figure 2**). The equalization techniques try to convert the functions $p_n(t-kT)$ to an orthogonal set of functions, thus the SBS detection approach can be applied at the equalized channel output.

The Nyquist criterion [3, Chapter 1] can specify the conditions for ISI-free channel on which SBS detector is optimal. This criterion helps to construct band-limited functions to reduce the

ISI negative effects. Another way to explain the ISI problem is to relate it with the channel frequency response. When consecutive symbols are transmitted using linear modulation over a wireless channel, the frequency response (impulse response) of the channel makes a transmitted symbol to be spread in the time domain. Thus, the ISI is generated because the previously transmitted and currently received symbols are overlapped. The Nyquist theorem could relate the time domain conditions to its equivalent frequency domain ones. Simply, considering the channel impulse response $h(t)$ and the symbol period $h(t)$, the condition of ISI-free response can be expressed as:

$$h(nT) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases} \text{for all integers } n. \tag{6}$$

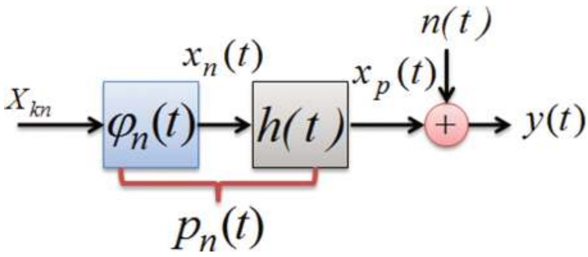

Figure 2. The band-limited channel equivalent impulse response representation.

The last condition can be represented in another form as follows (the Nyquist ISI criterion):

$$\frac{1}{T}\sum_{k=-\infty}^{+\infty} H\left(f - \frac{k}{T}\right) = 1; \ \forall f, \tag{7}$$

where $H(f)$ is the channel frequency response (Fourier transform of $h(t)$). Hence, the $\sin c$ pulse shape allows eliminating the ISI at sampling instants and any filter with excess bandwidth and odd-symmetry around Nyquist frequency can satisfy and meet the ISI-free requirements such as raised cosine filter (RCF).The raised cosine filter with bandwidth equal to $(\frac{1}{2T})(1 + \alpha)$ can be presented using the following form [3]:

$$P_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} \frac{\cos\left(\frac{\alpha \pi t}{T}\right)}{1 - \left(\frac{2\alpha t}{T}\right)^2}; \ 0 \leq \alpha \leq 1, \tag{8}$$

where α is the roll-off factor. The RCF impulse and frequency responses are shown in **Figure 3** for different roll-off factor α values.

The eye diagram is a widely used convenient method to observe the effects of ISI and noise introduced by the channel where the quality of the received signal (the ability to correctly recover the symbols and timing) can be illustrated (oscilloscope presentation). The interpretation of the eye diagram gives important information such as:

http://dx.doi.org/10.5772/65969

- sensitivity to timing error or jitter (smaller is better);
- wasted power;
- amount of distortion at sampling instants;
- amount of noise tolerance (larger is better);
- best time for sampling;
- the matching degree between the transmitter and receiver filters;
- the presence of ISI; and
- measurement of eye opening is performed to estimate the achievable BER.

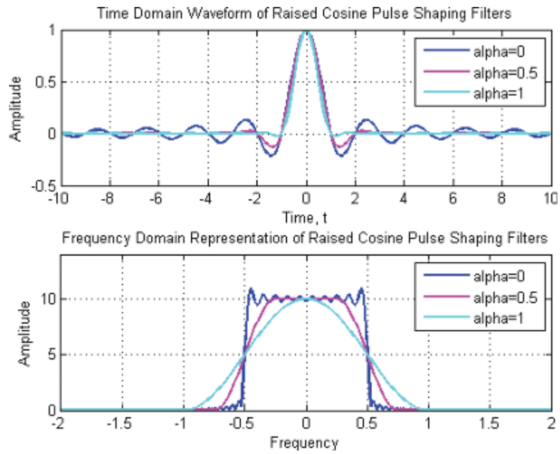

Figure 3. RCF impulse and frequency responses.

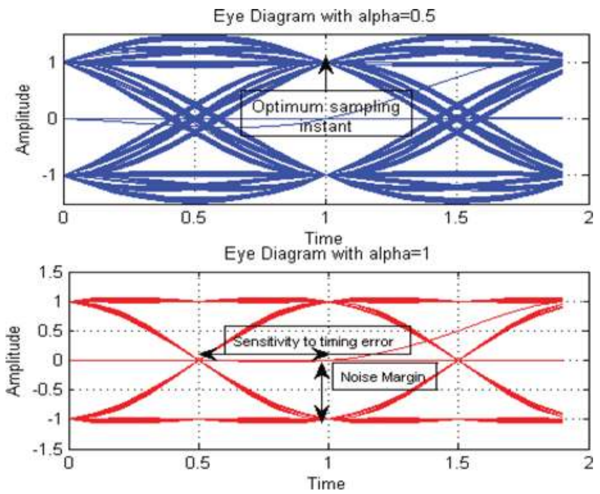

Figure 4. Raised cosine eye diagram for BPSK-modulated symbols.

The eye diagram using raised cosine filtering is presented in **Figure 4** for binary phase shift keying (BPSK)-modulated symbols at two roll-off factor values $\alpha = 0.5; 1$. In general, the ideal sampling instant is at the point where the vertical eye opening is maximum. Under the presence of ISI, the vertical eye opening reduces which leads to higher probability of error (BER). More sensitivity to timing error is presented by smaller horizontal eye opening. From **Figure 4**, it can be observed that the horizontal eye opening is smaller at roll-off factor $\alpha = 0.5$ in comparison with the case when $\alpha = 1$ owing to that the tails of the RCF are stripped down faster. It is important to mention here that higher values of roll-off factor require more transmission bandwidth. Smaller values of roll-off factor α lead to larger errors if the best sampling time (the center of the eye) is not achieved.

3. The equalization main concept

As mentioned before, the equalization technique is the design of a digital filter with inverse or counter transfer function in accordance with the transfer function of the associated wireless channel. This concept is shown in **Figure 5** where the frequency responses of the wireless channel and the equalizer are compared.

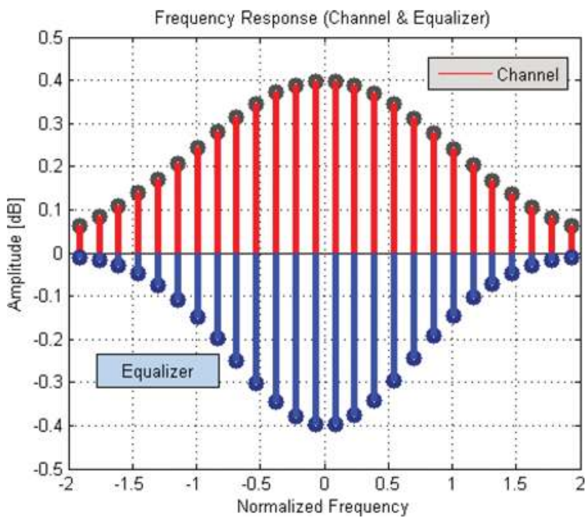

Figure 5. The wireless channel and equalizer frequency responses.

The equalizer design concept is presented in **Figure 6** (a simple block diagram for the channel effect and the equalizer transfer function). The discrete time model that should be considered under the equalization technique designing process is presented using the following form:

$$\hat{C}_k = \sum_{i=0}^{L-1} E_i u_k, \tag{9}$$

where $\hat{C}_k$ is the data sequence at the equalizer output, L is the number of symbols, E_i represents the coefficients of the causal impulse response of the equalizer $H_{EQ}(f)$, and u_k is the input of the equalizer given by

$$u_k = \sum_{j=0}^{L-1} h_j C_k + n_k, \tag{10}$$

where C_k is the transmitted data sequence, the coefficients h_j represents the equivalent causal impulse response (transfer function) of the wireless channel, and n_k is the discrete time additive noise. The channel transfer function $H_{ch}(f)$ can be presented as:

$$H_{ch}(f) = |H_{ch}(f)| \exp [j\theta_{ch}(f)], \tag{11}$$

where $|H_{ch}(f)|$ is the amplitude response, and $\theta_{ch}(f)$ is the phase response. The bit error rate (BER) reduction is achieved when the value of the error given by

$$e_k = C_k – \hat{C}_k \tag{12}$$

is reduced using equalization.

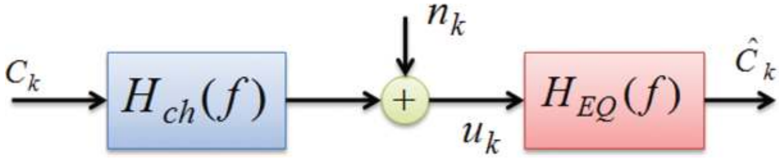

Figure 6. The equalizer main design model.

It is useful at this point to demonstrate the effects of the equalization on the detection performance or quality at the receiver side by presenting the difference between the received symbols with and without applying equalization. This comparison is given in **Figure 7** for the quadrature phase shift keying (QPSK)-modulated symbols. When the equalization is not applied, the uncertainty on the received signal constellation is very high (caused by the channel effect) and as a consequence the detection quality is low (detection performance degradation). Applying the equalization technique, the uncertainty level is lower and the detection performance is improved.

As a last word in this section, the ISI can produce a bias in the SNR value at the receiver. The error e_k presented by Eq. (12) is identical to the additive noise n_k in the case of unbiased detection rule. However, in the case of biased receiver, the probability density function (PDF) of the error e_k is a scaled and shifted version of the noise n_k PDF (the mean value depends on the actual data symbol C_k). The last condition can be formulated as follows:

$$E[\hat{C}_k|C_k] = C_k, \text{Unbiased Receiver} \tag{13}$$

For a given unbiased receiver [4] for a detection decision rule on a general signal constellation C_k, the maximum unconstrained SNR corresponding to the same receiver under any biased decision rule is given by

$$SNR_b = SNR_u + 1, \tag{14}$$

where SNR_b and SNR_u are the SNR for biased and unbiased receivers, respectively. The previous apparent discrepancy is more understandable recalling that the error rate is a monotone function of the SNR only. Thus, from Eq. (14), the negative effect of ISI on the SNR at the receiver is observed.

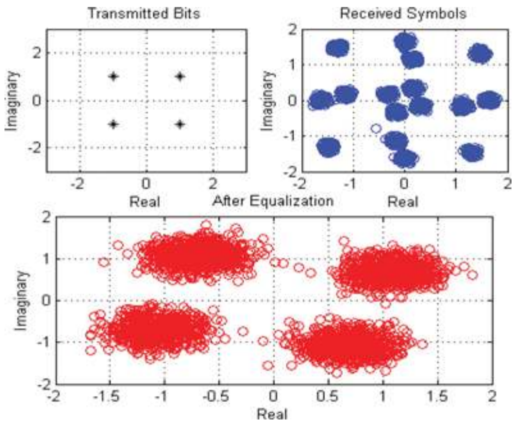

Figure 7. QPSK received symbols with and without equalization.

4. Equalization techniques

In general, the channel equalization techniques are classified to linear and nonlinear algorithms or to blind (without training sequence) and nonblind based on the degree of knowledge. The earliest mentions of digital equalization techniques are made under different design criterions, for example, zero forcing (ZF) equalization [5], minimum mean square error (MMSE) equalization [6], maximum likelihood (ML) equalization [7], decision feedback equalization (DFE) [8], and maximum *a posteriori* (MAP) equalization [9]. In this chapter, a new nonlinear equalization approach is proposed based on the dual channel equalization (DCE) idea with the purpose to improve the equalization performance, namely, the BER reduction, in comparison with the other widely used equalizers under multiple input multiple output (MIMO) wireless channel. The simulation results demonstrate considerable and promising performance improvements applying the suggested equalization approach in comparison with the conventional techniques. In order to help the reader to acquire the basics of equalization concepts, a few well-known and conventional equalization techniques will be reviewed and discussed in the next subsections.

4.1. System model

The complex baseband MIMO wireless channel model is considered with number of transmit antennas equal to N and number of receive antennas equal to M.

The received signal at the input of the receiver (the I/O relation of the MIMO channel) can be presented in the following form:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \tag{15}$$

where $\mathbf{y} \in C^{M1}$ is the vector form of the received signal, $\mathbf{H} \in C^{MN}$ is the matrix form of the Rayleigh fading channel with independent and identically distributed (i.i.d) coefficients obeying the circularly symmetric complex Gaussian distribution at zero mean and variance $\sigma_h^2 = 1$, denoted as $h_{ij} \tilde{} CN(0, 1)$ for $1 \leq i \leq M, 1 \leq j \leq N$, $\mathbf{x} \in C^{N1}$ is the vector form of the transmitted signal, and $\mathbf{z}$ is the circularly symmetric complex white Gaussian noise with zero mean and variance σ_n^2, i.e., $\mathbf{z} \tilde{} CN(0, \sigma_n^2 \mathbf{I})$. It is assumed that the N data substreams have uniform power which means $\mathbf{x} \in C^{N1}$ has a covariance matrix given by

$$E[\mathbf{x}\mathbf{x}^*] = \sigma_x^2 \mathbf{I}_N, \tag{16}$$

where $E[.]$ represents the mathematical expectation, $(.)^*$ is the conjugate transpose, and $\mathbf{I}_N$ is NN identity matrix. The signal-to-noise ratio (SNR) at the receiver input is expressed as:

$$SNR = \frac{\sigma_x^2}{\sigma_n^2}. \tag{17}$$

The channel matrix $\mathbf{H} \in C^{MN}$ is equalized or inverted by the weight matrix (the equalizer matrix) $\mathbf{W} \in C^{MN}$. Thus, the obtained signal $\hat{\mathbf{x}}$ at the equalizer output is defined as follows:

$$\hat{\mathbf{x}} = \mathbf{W}\mathbf{y}. \tag{18}$$

4.2. ZF equalization

ZF is a linear equalization that applies the inverse of the wireless channel frequency response to alleviate the channel effects on the received signal and restore the transmitted symbols. The name is assigned based on the reduction of ISI down to zero in the noise-free channel case (forcing the residual ISI to zero). The ZF equalizer can be designed using finite of infinite impulse response filters (FIR or IIR filters).

Employing ZF equalization technique with matrix $\mathbf{W}_{\mathrm{ZF}}$, the equalizer directly applies the inverse of the channel response to the received signal $\mathbf{y}$. Thus, the ZF equalizer satisfies the following condition:

$$\mathbf{W}_{\mathrm{ZF}}\mathbf{H} = \mathbf{I}_{\mathrm{MN}}. \tag{19}$$

The general form of the ZF equalizer matrix $\mathbf{W}_{\mathrm{ZF}}$ is defined as:

$$\mathbf{W}_{\text{ZF}} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H, \tag{20}$$

where $(.)^H$ represents Hermitian transpose operation. The BER in the case of ZF equalization technique and BPSK modulation can be defined using the following form [10]:

$$BER_{\text{ZF}} = \left[\frac{1}{2}\left(1 - \sqrt{\frac{SNR}{1+SNR}}\right)\right]^{M-N+1} \sum_{m=0}^{M-N+1} \binom{M-N+m}{m} \left(\frac{1+\sqrt{SNR/1+SNR}}{2}\right), \tag{21}$$

where m is the index of the received signal substream. The last equation is derived considering the Neyman-Pearson (NP) detection criteria. Thus, the SNR of the obtained M decoupled substreams is defined as [10]:

$$SNR_{\text{ZF},m} = \frac{SNR}{[(\mathbf{H}^H\mathbf{H})^{-1}]_{mm}};\ 1 \le m \le M, \tag{22}$$

where $[.]_{mm}$ corresponds to the mth diagonal element. The denominator of the achieved partial SNR given in Eq. (22) can be presented in terms of the mth column $\mathbf{h}_m$ of $\mathbf{H}$ as follows:

$$[(\mathbf{H}^H\mathbf{H})^{-1}]_{mm} = \frac{1}{\mathbf{h}_m^H\mathbf{h}_m - \mathbf{h}_m^H\mathbf{H}_m(\mathbf{H}_m^H\mathbf{H}_m)^{-1}\mathbf{H}_m^H\mathbf{h}_m}. \tag{23}$$

The ZF equalizer tries to null and cancel out all the interfering terms that are sometimes accompanied with noise amplification, for this reason, ZF is not optimal under very noisy channels. In the case of FIR filter use (to deal with noncausal components a decision delay is applied), a complete elimination of ISI problem is not possible owing to the finite filter length. Alternative criterion called peak-distortion criterion can be applied to minimize the maximum possible signal distortion due to ISI at the equalizer output.

4.3. MMSE equalization

The main objective of MMSE equalizer is to minimize the variance of the error signal e_k in Eq. (12). The MMSE equalizer ensures the trade-off between residual ISI and noise enhancement (reduce the total noise power). Under conditions more close to practice, MMSE equalizer can achieve lower BER compared to ZF equalizer at low-to-moderate SNRs. Thus, the MMSE equalization is applied to minimize the value of the mean given by

$$E[(\mathbf{W}_{\text{MMSE}}\mathbf{y} - \mathbf{x})(\mathbf{W}_{\text{MMSE}}\mathbf{y} - \mathbf{x})^H], \tag{24}$$

where $\mathbf{W}_{\text{MMSE}}$ is the MMSE equalizer matrix that is presented by the following expression [11]:

$$\mathbf{W}_{\text{MMSE}} = \left[\mathbf{H}^H\mathbf{H} + \frac{1}{SNR}\mathbf{I}\right]^{-1}\mathbf{H}^H. \tag{25}$$

The MMSE matrix $\mathbf{W}_{\text{MMSE}}$ is multiplied by the received signal vector $\mathbf{y}$ to obtain M decoupled substreams with SNR equal to [11]:

$$SNR_{\text{MMSE},m} = \frac{SNR}{\left[\left(\mathbf{H}^H\mathbf{H} + \frac{1}{SNR}\mathbf{I}\right)^{-1}\right]_{mm}} - 1; 1 \le m \le M, \tag{26}$$

The BER for this equalizer considering the NP criterion and BPSK modulation can be approximated using the following form [11]:

$$BER_{\text{MMSE}} = E[Q\sqrt{2SNR_{\text{MMSE},m}}], \tag{27}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$ is the standard Gaussian Q-function. The output SNRs of the M decoupled substreams using ZF and MMSE equalizers are related as follows:

$$SNR_{\text{MMSE},m} = SNR_{\text{ZF},m} + \delta_{\text{SNR}.m}; 1 \le m \le M, \tag{28}$$

where $\delta_{\text{SNR},m}$ is nondecreasing function of SNR ($SNR_{\text{ZF},m}$ and $\delta_{\text{SNR},m}$ are statistically independent). Moreover, the ratio of the output SNR gains for these two equalizers for any full rank channel realization goes to unity (in dB) [11]:

$$10\log_{10}\left(\frac{SNR_{\text{MMSE},m}}{SNR_{\text{ZF},m}}\right) = 10\log_{10}\left(1 + \frac{\delta_{\text{SNR},m}}{SNR_{\text{ZF},m}}\right) \to 0;\ as\ \text{SNR} \to \infty \tag{29}$$

Again the MMSE equalizer can be implemented with FIR and IIR filters and in both cases the error signal e_k in Eq. (12) depends on the estimated symbols $\hat{C}_k$. According to the orthogonality principle of MMSE optimization, the error e_k and the input of the MMSE equalizer must be orthogonal. The achieved SNR can be defined based on the error variance σ_e^2. For instance, in the case of MMSE-IIR equalizer, the SNR is given by

$$SNR_{\text{MMSE, IIR}} = \frac{1 - \sigma_e^2}{\sigma_e^2} \tag{30}$$

which is more general form of SNR in comparison with the form in Eq. (26).

4.4. ML equalization

The ML equalization technique tests all the possible data symbols and chooses the one that has the maximum probability of correctness at the output (optimal in the sense of minimizing the probability of error $P_e = P(\mathbf{x} \neq \hat{\mathbf{x}})$, where $\hat{\mathbf{x}}$ is the estimated or chosen signal at the equalizer output). The Euclidian distance between the received signal vector and the products of all possible transmitted signal vectors is calculated, and the signal with minimum distance is considered. The ML estimation of $\mathbf{x}$ takes the following form [12]:

$$\hat{\mathbf{x}}_{\text{ML}} = \arg\min_{\hat{\mathbf{x}} \in \mathbf{x}} \underbrace{\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\|^2}_{\mathbf{J}} \tag{31}$$

As follows, the ML-based equalizer selects the data sequence $\hat{\mathbf{x}}$ that yields the smallest distance $\mathbf{J}_{\text{min}}$ between the received signal vector $\mathbf{y}$ and the estimated or hypothesized message $\mathbf{H}\hat{\mathbf{x}}$. For NP-based receiver, the obtained SNR at ML equalizer output is related to $\mathbf{J}_{\text{min}}$ and given by

$$SNR_{\mathrm{ML}} = \frac{\mathbf{J}_{\mathrm{min}}}{4\sigma_n^2}. \tag{32}$$

The achieved BER of ML equalizer is based on $\mathbf{J}_{\mathrm{min}}$ as well as defined as [12]:

$$BER_{\mathrm{ML}} = Q\left(\sqrt{\frac{\mathbf{J}_{\mathrm{min}}}{4\sigma_n^2}}\right). \tag{33}$$

In **Figure 8**, a comparison between the ZF, MMSE, and ML equalizer performances in terms of BER as a function of energy per bit E_b to the noise power spectral density N_0 ratio $\left(\frac{E_b}{N_0}\right)$ is presented in the case of BPSK modulation and for MIMO antenna configuration with $N = M = 2$ and Rayleigh fading channel. As shown in **Figure 8**, the ML equalizer has the best performance and ZF equalizer has the worst one.

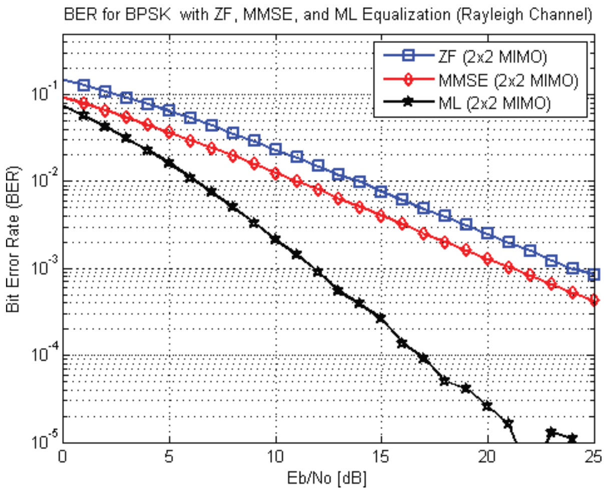

Figure 8. ZF, MMSE, and ML equalizer performances comparison.

4.5. Other equalization techniques

The main linear equalization drawback is that the equalizer filter enhances the noise at the output (increases the noise variance), and additionally, the noise is colored (especially for severely distorted channels). Employing noise prediction technique helps to avoid the described problem. The last short discussion leads to the basic idea about decision feedback equalization (DFE). The DFE structure consists of two filters: a feed forward-filter whose input is the channel output signal and a feedback-filter that feeds back the previous decisions for noise prediction process. This can be achieved by combining linear equalization technique like ZF or MMSE with noise variance prediction (ZF-DFE and MMSE-DFE). A simple block diagram for DFE is presented in **Figure 9**.

The maximum *a posteriori* (MAP) equalizer [13] estimates the transmitted symbol $x[n]$ at discrete-time index n that maximize the *a posteriori* probability $P(x[n] = x|\mathbf{y})$ as follows:

$$\hat{x}[n] = \text{arg max } P(x[n] = x|\mathbf{y}). \tag{34}$$

The MAP equalizer can be used as SBS detector with maximum-likelihood sequence estimation (MLSE) for transmission over rapidly time-varying (TV) wireless channel as shown in Ref. [13].

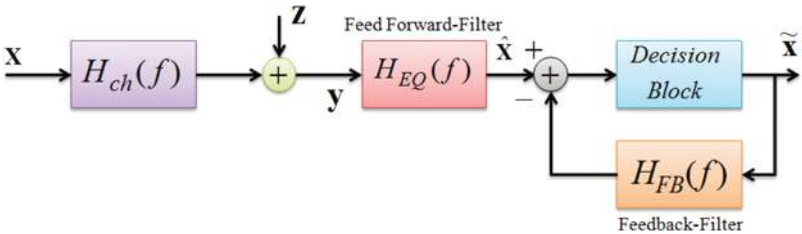

Figure 9. Basic DFE structure.

When some signal properties are used for the determination of the instantaneous error which updates the adaptive filter coefficients or weights, the fractionally spaced equalization (FSE) is an effective approach under the absence of the training sequences (blind equalization) [14]. The FSE receives number of input samples equal to N_{FSE} before it produces one output sample. The adaptive filter weights are updated employing a special algorithm such as constant modulus algorithm (CMA) which uses the constant modularity as the desired signal property. Thus, if the output rate is $1/T$, the input sample rate is N_{FSE}/T. Thus, the tap spacing of the FSE is a fraction of the baud spacing or the transmitted period. The FSE can be modeled as a parallel combination of several baud spaced equalizers known as multichannel model of FSE where the oversampling factor defines the tap spacing as follows:

$$\text{Tap Spacing} = \frac{T}{\text{Oversampling Factor}}. \tag{35}$$

In Ref. [15], a brief discussion about the recent equalization requirements and approaches is presented along with some important related references.

In optical communication field, a novel and efficient multiplier-less finite impulse response filter (FIR) based on chromatic dispersion equalization (CDE) is proposed for coherent receivers [16]. An iterative receiver is designed [17] for joint phase noise estimation, equalization, and decoding in a coded communication system with combined belief propagation, mean field, and expectation propagation (BP-MF-EP). In the frequency domain-based equalization, many important contributions are made recently, for example, in Ref. [18] and for single carrier frequency domain equalization (SC-FDE) in broadband wireless communication systems, a robust design under imperfect channel knowledge is considered based on a statistical channel estimation model where the equalization coefficients are defined under mean square error minimization criterion.

Another work deals with frequency domain equalization for faster-than-Nyquist (FTN) signaling is presented in Ref. [19] for doubly selective channels (DSCs) based on low complexity receivers with variational methods implemented in order to handle the interference of frequency domain symbols instead of using MMSE equalizer that involves high complexity in DSCs.

The frequency domain equalization is also proposed to be employed for broadband power line communications (PLC) as in Ref. [20]. PLC performance can benefit from frequency domain equalization techniques in the context of a cyclic-prefix single carrier modulation schemes. The study in Ref. [20] presents an equalization algorithm based on the properties of complementary sequences (CSs) to reduce the complexity by performing all the operations in the frequency domain without the necessity of noise variance estimator.

5. The new nonlinear equalization approach

5.1. Channel equalization with DCE

The proposed new nonlinear equalization approach applying dual channel equalization (DCE) is presented in this section. The DCE idea can be implemented and coupled with any standard channel equalizers such as ZF or ML equalizers. **Figure 10** shows a simple flow chart for the DCE idea presented as a coupling process between two digital filters. These filters can have the same or different transfer functions and for simplicity of analysis, the similarity case is considered. The topology in **Figure 10** is flexible and able to work with various equalization techniques. The squaring device block $(.)^2$ and the transfer function of the equalization filter $H_{\mathrm{EQ}}(f)$ are the sources of the nonlinearity in this approach. The band-stop filter $H_{\mathrm{EQ}}(f)$ main function is to filter out the received signal in order to obtain the reference colored noise ψ at the output of the second equalization filter. In fact, the squaring device and the equalization filter form a familiar preliminary design for square-law demodulation or square-law envelop detector.

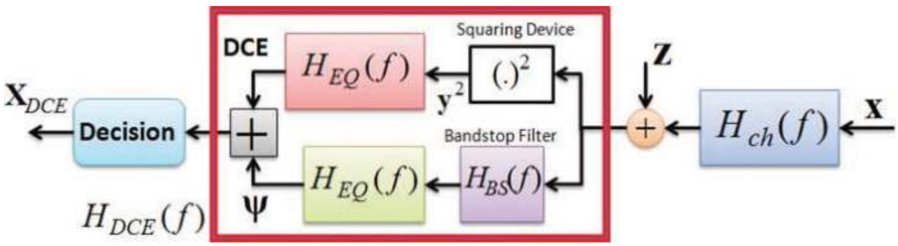

Figure 10. The DCE flow chart.

5.2. ZF equalization with DCE

The system model presented in Section 4.1 is valid in the analysis of ZF-DCE. The signal matrix $\mathbf{X}_{\mathrm{DCE}}$ at the output can be presented using the following form:

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$$\mathbf{X}_{\text{DCE}} = \mathbf{W}_{\text{DCE–ZF}}\mathbf{y}^2 + \psi = \hat{\mathbf{X}}^2 + \psi, \tag{36}$$

where $\mathbf{W}_{\text{DCE–ZF}}$ is the DCE matrix that takes the following form:

$$\mathbf{W}_{\text{DCE–ZF}} = (\mathbf{H}^H\mathbf{H})^{-2}(\mathbf{H}^H)^2. \tag{37}$$

Taking into consideration Eq. (37), the given form in Eq. (36) can be rewritten as:

$$\mathbf{X}_{\text{DCE}} = \hat{\mathbf{X}}^2 + \psi = \hat{\mathbf{X}}^2 + (\mathbf{H}^H\mathbf{H})^{-2}(\mathbf{H}^H)^2\mathbf{Z}_{\text{BS}}, \tag{38}$$

where $\mathbf{Z}_{\text{BS}}$ is the matrix form of the noise at the stopband filter. To derive the closed expressions for the achieved SNR and BER under the use of ZF-DCE, a complete description for the noise component ψ forming at the output is required (the error performance is strongly related to the power of ψ). Using the standard complex matrix decomposition (singular value decomposition SVD), the complex channel matrix ψ can be decomposed as follows:

$$\begin{cases} \mathbf{H} = \mathbf{UTV}^H \\ \mathbf{H}^H\mathbf{H} = \mathbf{UTT}^H\mathbf{U}^H = \mathbf{U}\Lambda\mathbf{U}^H, \end{cases} \tag{39}$$

where $\mathbf{U}$ is MM complex unitary matrix that contains the $\mathbf{HH}^H$ eigenvectors ($\mathbf{U}^H\mathbf{U} = \mathbf{I}_M$, $\mathbf{U}^H = \mathbf{U}^{-1}$), $\mathbf{V}$ is NN complex unitary matrix that contains the $\mathbf{H}^H\mathbf{H}$ eigenvectors, $\mathbf{T} = diag(\lambda_1, ..., \lambda_N)$ is MN real diagonal matrix of the positive square roots of the corresponding eigenvalues, and Λ is MM diagonal matrix. The diagonal elements of $\mathbf{T}$ and Λ are defined by the following form:

$$\lambda_i = \begin{cases} \mathrm{d}_\mathrm{i}^2, \mathrm{i} = 1, 2, ..., \min(N, M) \\ 0, \mathrm{i} = \min(N, M) + 1,..., \mathrm{M} \end{cases} \tag{40}$$

where the squared singular values $\{d_i^2\}$ are the channel matrix $\mathbf{H}$ eigenvalues. The squared Euclidian norm $\| . \|$ of the channel matrix $\mathbf{H}$ is given by

$$\|\mathbf{H}\|^2 = tr(\Lambda) = \sum_{i=1}^{\min(N,M)} \lambda_i = \sum_{i=1}^{\min(N,M)} d_i^2, \tag{41}$$

where $tr(.)$ is the matrix trace. Based on Eqs. (39) and (41), the power of the noise ψ at the ZF-DCE output can be presented using the expression:

$$\begin{aligned} \|\psi\|^2 &= \|(\mathbf{H}^H\mathbf{H})^{-2}(\mathbf{H}^H)^2\mathbf{Z}_{\text{BS}}\|^2 = \|(\mathbf{VT}^2\mathbf{V}^H)^{-2}(\mathbf{VTU}^H)^2\mathbf{Z}_{\text{BS}}\|^2 \\ &= \|\mathbf{V}^{-2}\mathbf{T}^{-4}(\mathbf{V}^H)^{-2}\mathbf{V}^2\mathbf{T}^2(\mathbf{U}^H)^2\mathbf{Z}_{\text{BS}}\|^2 = \|\mathbf{V}^{-2}\mathbf{T}^{-2}(\mathbf{U}^H)^2\mathbf{Z}_{\text{BS}}\|^2 \\ &= \|(\mathbf{V}^{-1}\mathbf{T}^{-1}\mathbf{U}^H)^2\mathbf{Z}_{\text{BS}}\|^2. \end{aligned} \tag{42}$$

The mathematical expectation of the noise power given by Eq. (42) is defined as follows:

$$E\{\|\psi\|^2\} = E\{\|(\mathbf{V}^{-1}\mathbf{T}^{-1}\mathbf{U}^H)^2\mathbf{Z}_{\text{BS}}\|^2\} = E\{\|(\mathbf{VTU})^{-2}\mathbf{Z}_{\text{BS}}\|^2\} \tag{43}$$

The unitary matrix lemma states that the multiplication with a unitary matrix will not change the vector norm. Thus, owing to the fact that is $\mathbf{V}$ a unitary matrix, the presented form in Eq. (43) can be simplified:

$$\begin{aligned} E\{\|\psi\|^2\} &= E\{\|(\mathbf{VTU})^{-2}\mathbf{Z}_{\rm BS}\|^2\} = E\{\|\mathbf{T}^{-2}\mathbf{U}^{-2}\mathbf{Z}_{\rm BS}\|^2\} \\ &= E\{tr[(\mathbf{T}^{-2}\mathbf{U}^{-2})\mathbf{Z}_{\rm BS}\mathbf{Z}_{\rm BS}^H(\mathbf{U}^2\mathbf{T}^{-2})]\} = tr[(\mathbf{T}^{-2}\mathbf{U}^{-2})E\{\mathbf{Z}_{\rm BS}\mathbf{Z}_{\rm BS}^H\}(\mathbf{U}^2\mathbf{T}^{-2})] \\ &= tr[\sigma_z^2(\mathbf{T}^{-2}\mathbf{U}^{-2})(\mathbf{U}^2\mathbf{T}^{-2})] = \sigma_z^2 tr[(\mathbf{T}^{-2}\mathbf{U}^{-2})(\mathbf{U}^2\mathbf{T}^{-2})] = \sigma_z^2 tr[\mathbf{T}^{-4}] \\ &= \sigma_n^2 \sum_{i=1}^{N} \lambda_i^{-2}. \end{aligned} \tag{44}$$

where σ_z^2 is the variance of the noise $\mathbf{Z}_{\rm BS}$ at the bandstop filter output. The last form is obtained considering that $\sigma_z^2 = \sigma_n^2$ (in the frequency band of interest, the noise power spectral density is constant). The SNR at the ZF-DCE output can be defined based on Eq. (44) as follows:

$$SNR_{\rm ZF\text{-}DCE} = \frac{\sigma_x^2}{\sigma_n^2 \sum_{i=1}^{N} \lambda_i^{-2}}. \tag{45}$$

The general form used to determine the BER in the case of ZF equalization technique under BPSK modulation can be defined as [10]:

$$BER_{\rm ZF} = \int_0^{\infty} Q(\sqrt{2SNR_{\rm ZF}x})\frac{1}{(M-N)!}x^{M-N}e^{-x}dx \tag{46}$$

The BER form in Eq. (21) [10] is the solution of the last integral in Eq. (46) and together with Eq. (45) can present the final BER closed expression of ZF-DCE as follows:

$$BER_{\rm ZF} = \left[\frac{1}{2}\left(1-\sqrt{\frac{\sigma_x^2}{\sigma_x^2+\sigma_n^2\sum_{i=1}^{N}\lambda_i^{-2}}}\right)\right]^{M-N+1} \sum_{m=0}^{M-N+1}\binom{M-N+m}{m}\left(\frac{1}{2}+\frac{1}{2}\sqrt{\frac{\sigma_x^2}{\sigma_x^2+\sigma_n^2\sum_{i=1}^{N}\lambda_i^{-2}}}\right). \tag{47}$$

As noticed in Eqs. (45) and (47), the derived forms are based on λ_i that also considered as the ith eigenvalue of the sample covariance matrix $\mathbf{R_y}$ given by

$$\mathbf{R_y} = \frac{1}{N_s}\mathbf{y}\mathbf{y}^H. \tag{48}$$

where N_s is the number of samples received by each antenna. The direct relation between λ_i and $\mathbf{y}$ can be illustrated using the following form:

$$\frac{1}{2\sigma_n^2}\sum_{k=0}^{N_s-1} y(k)^H y(k) = \frac{N_s}{2\sigma_n^2}\sum_{i=1}^{M}\lambda_i \tag{49}$$

5.3. ML equalization with DCE

For this case, the DCE flow chart presented in **Figure 10** can be considered for ML-DCE design excluding the squaring device. The corresponding equalizer or channel matrix takes the following format:

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$$\mathbf{W}_{\text{ML–DCE}} = \mathbf{H} = (\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y} \tag{50}$$

Based on Eq. (31), the ML-DCE estimation is defined as:

$$\hat{\mathbf{x}}_{\text{ML–DCE}} = \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}} \| \mathbf{y}–\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y} \|^2 \tag{51}$$

The presented form in Eq. (51) can be simplified by the following operations:

$$\begin{aligned}\hat{\mathbf{x}}_{\text{ML–DCE}} &= \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}} [\mathbf{y}–\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]^H[\mathbf{y}–\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]\\ &= \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}}[\mathbf{y}^H–\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H][\mathbf{y}–\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]\\ &= \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}}[\mathbf{y}^H\mathbf{y}–\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}–\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y} + \mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]\\ &= \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}}[\mathbf{y}^H\mathbf{y}–2\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y} + \mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]\\ &= \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}} \underbrace{[\mathbf{y}^H\mathbf{y}–\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]}_{\text{J}_{\text{DCE}}}.\end{aligned} \tag{52}$$

It is obvious that to minimize the form in Eq. (52), the term $\left(\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}\right)$ should be maximized as follows:

$$\begin{aligned}\mathbf{J}_{\text{ML}} &= \arg\max_{\hat{\mathbf{x}}\in\mathbf{x}} \left(\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}\right) = \arg\max_{\hat{\mathbf{x}}\in\mathbf{x}}[\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}]\\ &= \arg\max_{\hat{\mathbf{x}}\in\mathbf{x}}[\left(\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}\right)^H\left(\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}\right)] = \arg\max_{\hat{\mathbf{x}}\in\mathbf{x}} \|\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}\|^2.\end{aligned} \tag{53}$$

The related SNR at the ML-DCE output is determined using $\mathbf{J}_{\text{min,DCE}}$ as in the presented form Eq. (32) and the covariance matrix of the reference noise $\mathbf{Z}_{\text{BS}}$ forming at the bandstop filter given by

$$C_{Z_{\text{BS}}}=E[Z_{\text{BS}}^H Z_{\text{BS}}]=\sigma_z^2 I=\sigma_n^2 I \tag{54}$$

Defining the achieved SNR for ML-DCE, it is possible to calculate the BER using the same form in Eq. (**33**). Taking into account the sample covariance matrix $\mathbf{R}_y$ in Eq. (**48**), Eq. (**52**) can be represented as:

$$\hat{\mathbf{x}}_{ML–DCE} = \arg\min_{\hat{\mathbf{x}}\in\mathbf{x}}[N_s\mathbf{R}_\mathbf{y}–\mathbf{y}^H\hat{\mathbf{x}}(\hat{\mathbf{x}}^H\hat{\mathbf{x}})^{-1}\hat{\mathbf{x}}^H\mathbf{y}] \tag{55}$$

The use of two equalization filters and the bandstop filter (the main DCE idea) helps us to construct new equalization matrices and convert linear equalizers such as ZF equalizer to nonlinear one. Additionally, the reference noise forming at the second equalizer output contributes in the definitions of achieved BER and SNR and can be employed to estimate the noise variance (power) at the equalizer input. In this section, two equalizers are introduced, namely, ZF-DCE and ML-DCE. The DCE concept can be implemented with other equalization techniques like MMSE and DFE as well. The proposed DCE performance is evaluated in the following subsection.

5.4. Simulation process and results

In this section, *NM* MIMO channel is considered where the number of transmit antennas N= 2 and the number of receive antennas M= 2 under Rayleigh fading channel and for BPSK modulation. The overall simulation process for this system employing the conventional and DCE types of equalization techniques are presented in **Figure 11**. The evolution and comparison criterion is based on the BER (probability of error) as a function of energy per bit E_b to the noise power spectral density N_0 ratio ($\frac{E_b}{N_0}$). The relation between $\frac{E_b}{N_0}$ and the SNR can be simply presented as:

$$SNR = \frac{E_b}{N_0}\frac{D_R}{BW}, \tag{56}$$

where D_R is the data rate and BW is the bandwidth.

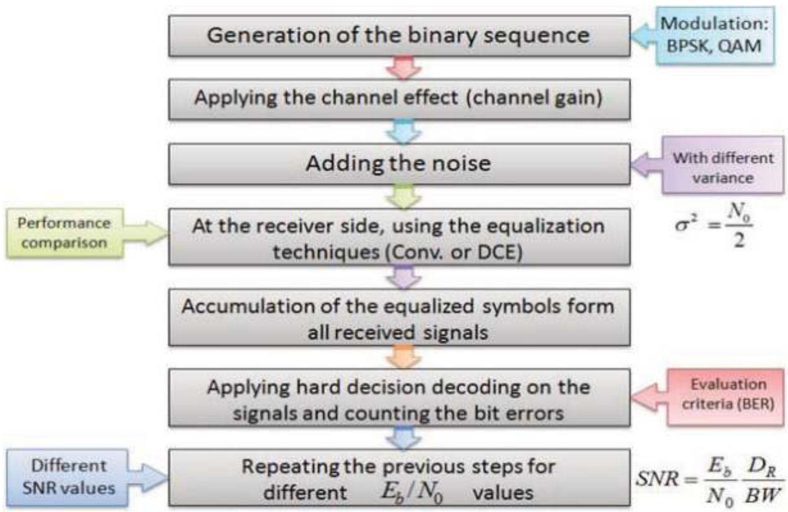

Figure 11. The simulation and evaluation processes.

In **Figure 12**, a comparison between the conventional ZF equalization technique and the modified ZF using dual channel equalization (DCE) is presented. The ZF-DCE approach demonstrates better performance (lower BER) comparing with the conventional ZF at the same $\frac{E_b}{N_0}$ (or SNR) range. The two equalizers have the same performance under relatively lower SNR ($\frac{E_b}{N_0}$≤5[dB]).

The ML-DCE and the conventional ML equalizer performances are compared in **Figure 13** under the same initial conditions but for different SNR ($\frac{E_b}{N_0}$) range with the purpose of demonstrating the

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effective and efficient SNR values for DCE employment. Once again the ML-DCE outperforms the conventional ML in performance and for lower SNR compared with the case of ZF equalization where the two equalizers present asymptotic performance for SNR less than 0 dB or $\frac{E_b}{N_0} \leq 0[dB]$ (roughly speaking, the ML-DCE performance is slightly better at $\frac{E_b}{N_0} \leq 0[dB]$).

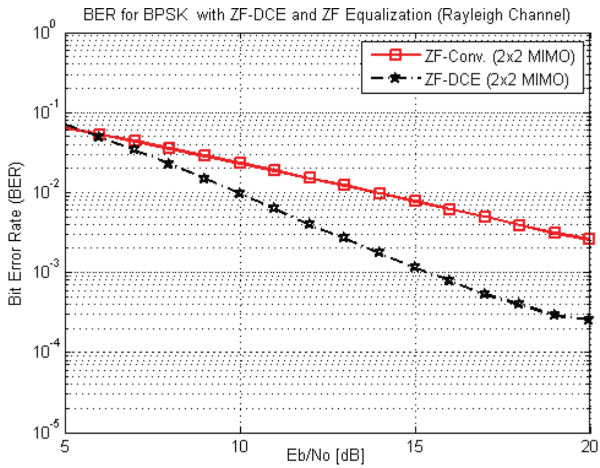

Figure 12. The comparison between conventional ZF and the ZF-DCE.

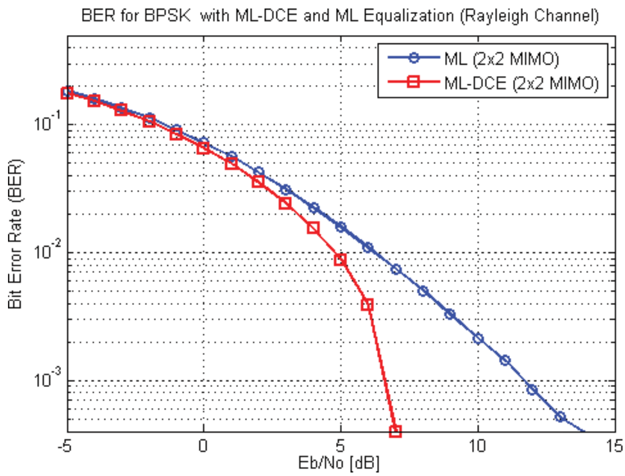

Figure 13. The comparison between conventional ML and the ML-DCE.

6. Conclusion remarks

The discussion of this chapter can be ended by making several remarks. The new equalization approach (DCE) shows promising outcomes in terms of improving the equalization performance by reducing the BER (sequence error probability).The obtained results can be simply generalized for other equalization techniques and for frequency domain equalization as well. Additionally, it is easy to prove that the DCE symbol error performance can be better than that of the conventional equalizers. The type of filters used to design the DCE equalizer is not mentioned or discussed but both FIR and IIR filters are possible and reasonable candidates.

Although the design and implementation complexity issues (the computational cost to design the equalizer and to equalize the channel, respectively) for the proposed above equalization structure are not discussed in this chapter, an ostensible comments can be made. The DCE design relies on other equalization approaches and does not exhibit significant overhead complexity. Thus, the complexity increases employing DCE but not with overwhelming degree. Finally, a complete analysis based on practical conditions and for time-invariant (TIV) and time-varying (TV) channel models under specific scenarios is required before addressing the feasibility of the presented DCE approach.

Acknowledgements

This work is performed by Radio Physics Research Group (RPRG) at Polytechnic University of San Luis Potosi (UPSLP), San Luis Potosi, Mexico.

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