Some sufficient conditions for graphs to be (g, f, n)-critical graphs

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Abstract. Let G be a graph of order p , and let a and b and n be nonnegative integers with $1 < a < b$, and let g and f be two integer-valued functions defined on $V(G)$ such that $a \leq g(x) \leq f(x) \leq b$ for all $x \in V(G)$. A (g, f) -factor of graph G is defined as a spanning sub graph F of G such that $g(x) \leq d_F(x) \leq f(x)$ for each $x \in V(G)$. Then a graph G is called a (g, f, n) -critical graph if after deleting any n vertices of G the remaining graph of G has a (g, f) -factor. In this paper, we prove that every graph G is a (g, f, n) -critical graph if its minimum degree is greater than $p + a + b - 2\sqrt{(a+1)p - bn} + 1$. Furthermore, it is showed that the result in this paper is best possible in some sense.

Keywords: graph, minimum degree, (g, f) -factor, (g, f, n) -critical graph.

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1. INTRODUCTION

In this paper, we consider a finite graph G with vertex set $V(G)$ and edge set $E(G)$, which has neither loops nor multiple edges. For any vertex x of G , we denote by $d_G(x)$ the degree of x in G . We denote by $\delta(G)$ the minimum vertex degree of G . For any $S \subseteq V(G)$, the subgraph of G induced by S is denoted by $G[S]$ and $G - S = G[V(G) \setminus S]$.

Let g and f be two nonnegative integer-valued functions defined on $V(G)$ such that $g(x) \leq f(x)$ for each $x \in V(G)$. A (g, f) -factor of graph G is defined as a spanning subgraph F of G such that $g(x) \leq d_F(x) \leq f(x)$ for each $x \in V(G)$ (Where of course d_F -denotes the

degree in F). If $g(x)=f(x)$ for each $x \in V(G)$, then a (g, f) -factor is called an f -factor. If $g(x)=a$ and $f(x)=b$ for all $x \in V(G)$, then a (g, f) -factor is called an $[a, b]$ -factor. If $g(x)=f(x)=k$ for all $x \in V(G)$, then a (g, f) -factor is called a k -factor. A graph G is called a (g, f, n) -critical graph if after deleting any n vertices of G the remaining graph of G has a (g, f) -factor. If G is a (g, f, n) -critical graph, then we also say that G is (g, f, n) -critical. If $g(x)=f(x)$ for each $x \in V(G)$, then a (g, f, n) -critical graph is an (f, n) -critical graph. If $g(x)=a$ and $f(x)=b$ for all $x \in V(G)$, then a (g, f, n) -critical graph is an (a, b, n) -critical graph. If $a=b=k$, then an (a, b, n) -critical graph is simply called a (k, n) -critical graph. In particular, a $(1, n)$ -critical graph is simply called an n -critical graph. The other notations and definitions not given in this paper can be found in [1].

Q. Yu [2] gave the characterization of n -critical graphs. O. Favaron [3] studied the properties of n -critical graphs. G. Liu and Q. Yu [4] studied the characterization of (k, n) -critical graphs. The characterization of (a, b, n) -critical graphs with $a < b$ was given by G. Liu and J. Wang [5]. S. Zhou [6,7,8,13] gave some sufficient conditions for graphs to be (a, b, n) -critical graphs. J. Li [9] showed two sufficient conditions for graphs to be (a, b, n) -critical graphs. S. Zhou [10] obtained a sufficient condition for graphs to be (g, f, n) -critical graphs. The characterization of (g, f, n) -critical graphs was given by J. Li and H. Matsuda [11]. In this paper, we obtain two new sufficient conditions for graphs to be (g, f, n) -critical graphs. The main results will be given in the following section. The following results on k -factors and (a, b, k) -critical graphs and (g, f, n) -critical graphs are known.

In [12], Y. Egawa and H. Enomoto proved the following result for the existence of k -factors.

Theorem 1.^[12] Let $k \geq 2$ be an integer, and let G be a graph of order n , kn is even. If

$$\delta(G) > n + 2k - 2\sqrt{kn+1},$$

then G has a k -factor.

In [8], S. Zhou and M. Zong gave a sufficient condition for graphs to be (a, b, k) -critical graphs.

Theorem 2.^[8] Let a, b and k be nonnegative integers such that $1 \leq a < b$ and G be a graph with order $n \geq \frac{(a-1)(a+1)(a+b)(a+b-1)}{a(b-1)} - \frac{(a+b)(ab+b-1)}{ab(b-1)} + k$. If $\delta(G) \geq a+k$, and

$$\max\{d_G(x), d_G(y)\} \geq \frac{an+bk}{a+b}$$

for any vertices x and y of $V(G)$ with $d(x, y) = 2$. Then G is an (a, b, k) -critical graph.

In [10], S. Zhou showed a sufficient condition for graphs to be (g, f, n) -critical graphs.

Theorem 3.^[10] Let G be a graph, and let g and f be two nonnegative integer-valued functions defined on $V(G)$ such that $g(x) < f(x)$ for each $x \in V(G)$. If $g(x) \leq d_G(x)$ and $f(x)(d_G(y) - n) \geq d_G(x)g(y)$ for each $x, y \in V(G)$, then G is a (g, f, n) -critical graph. Here n is a nonnegative integer.

2. THE PROOF OF MAIN THEOREMS

In this paper, we obtain a new sufficient condition for graphs to be (g, f, n) -critical graphs. Our result is the extension of Theorem 1.

Theorem 4. Let G be a graph of order p , and let a, b and n be nonnegative integers such that $1 \leq a < b$, and let g and f be two integer-valued functions defined on $V(G)$ such that $a \leq g(x) < f(x) \leq b$ for each $x \in V(G)$. If

$$\delta(G) > p + a + b - 2\sqrt{(a+1)p - bn + 1}, \quad (1)$$

then G is a (g, f, n) -critical graph.

In Theorem 4, if $n=0$, then we obtain the following corollary.

Corollary 1. Let G be a graph of order p , and let a and b be integers such that $1 \leq a < b$, and let g and f be two integer-valued functions defined on $V(G)$ such that $a \leq g(x) < f(x) \leq b$ for each $x \in V(G)$. If

$$\delta(G) > p + a + b - 2\sqrt{(a+1)p + 1},$$

then G has a (g, f) -factor.

According to Corollary 1 and the definition of (g, f, n) -critical graph, we obtain easily the following result.

Theorem 5. Let G be a graph of order p , and let a, b and n be nonnegative integers such that $1 \leq a < b$, and let g and f be two integer-valued functions defined on $V(G)$ such that $a \leq g(x) < f(x) \leq b$ for each $x \in V(G)$. If

$$\delta(G) > p + a + b - 2\sqrt{(a+1)p + 1} + n,$$

then G is a (g, f, n) -critical graph.

Let S and T be disjoint subsets of $V(G)$. We write $e_G(S, T) = |\{xy \in E(G) : x \in S, y \in T\}|$, $f(S) = \sum_{x \in S} f(x)$, $d_{G-S}(T) = \sum_{x \in T} d_{G-S}(x)$, and $g(T) = \sum_{x \in T} g(x)$. Our proof of Theorem 4 relies heavily on the following theorem.

Theorem 6.^[11] Let G be a graph, $n \geq 0$ an integer, and let g and f be two integer-valued functions defined on $V(G)$ such that $g(x) < f(x)$ for each $x \in V(G)$. Then G is a (g, f, n) -

critical graph if and only if

$$\begin{aligned}\delta_G(S, T) &= f(S) + d_{G-S}(T) - g(T) \\ &\geq \max\{f(N) : N \subseteq S, |N| = n\}\end{aligned}$$

for all disjoint subsets S and T of $V(G)$ with $|S| \geq n$.

Proof of Theorem 4. Suppose a graph G satisfies the condition of the theorem, but it is not a (g, f, n) -critical graph. Then, by Theorem 6, there exist disjoint subsets S and T of $V(G)$ with $|S| \geq n$ such that

$$\delta_G(S, T) = f(S) + d_{G-S}(T) - g(T) \leq \max\{f(N) : N \subseteq S, |N| = n\} - 1. \quad (2)$$

We choose subsets S and T such that $|T|$ is minimum and S and T satisfy (2).

Claim 1. $d_{G-S}(x) \leq g(x) - 1 \leq b - 2$ for each $x \in T$.

Proof. Suppose that there exists a vertex $x \in T$ such that $d_{G-S}(x) \geq g(x)$. Then the subsets S and $T - \{x\}$ satisfy (2), which contradicts the choice of T . Completing the proof of Claim 1.

If $T = \emptyset$, then by (2), $f(S) - 1 \geq \max\{f(N) : N \subseteq S, |N| = n\} - 1 \geq \delta_G(S, T) = f(S)$, a contradiction. Hence, $T \neq \emptyset$. Let

$$h = \min\{d_{G-S}(x) : x \in T\}.$$

According to Claim 1, we have

$$0 \leq h \leq b - 2,$$

and

$$\delta(G) \leq h + |S|. \quad (3)$$

According to (2) and $|S| + |T| \leq p$, we get that

$$\begin{aligned}bn - 1 &\geq \max\{f(N) : N \subseteq S, |N| = n\} - 1 \\ &\geq \delta_G(S, T) = f(S) + d_{G-S}(T) - g(T) \\ &\geq (a+1)|S| + d_{G-S}(T) - (b-1)|T| \\ &\geq (a+1)|S| + h|T| - (b-1)|T| \\ &= (a+1)|S| - (b-h-1)|T| \\ &\geq (a+1)|S| - (b-h-1)(p-|S|) \\ &= (a+b-h)|S| - (b-h-1)p.\end{aligned}$$

Thus, we obtain

$$|S| \leq \frac{(b-h-1)p + bn - 1}{a+b-h}. \quad (4)$$

In view of (3) and (4), we have

$$\delta(G) \leq h + |S| \leq h + \frac{(b-h-1)p + bn - 1}{a+b-h}. \quad (5)$$

Let $f(h) = h + \frac{(b-h-1)p + bn - 1}{a+b-h}$. In the range of $h \leq a+b$, the function $f(h)$ attains its maximum value at $h = a+b - \sqrt{(a+1)p - bn + 1}$. Since $0 \leq h \leq b-2$, then we have

$$\begin{aligned} f(h) &\leq f(a+b - \sqrt{(a+1)p - bn + 1}) \\ &= p + a + b - 2\sqrt{(a+1)p - bn + 1}, \end{aligned}$$

that is,

$$\delta(G) \leq p + a + b - 2\sqrt{(a+1)p - bn + 1},$$

this contradicts (1).

From the argument above, we deduce the contradiction. Hence, G is a (g, f, n) -critical graph.

Completing the proof of Theorem 4.

The binding number $\text{bind}(G)$ of G is the minimum value of $\frac{|N_G(X)|}{|X|}$ taken over all non-empty subsets X of $V(G)$ such that $N_G(X) \neq V(G)$. The binding number condition on (a, b, k) -critical graphs was given by Sizhong Zhou and Jiashang Jiang [7].

Theorem 7.^[7] Let G be a graph of order n , and let a, b and k be nonnegative integers such that $1 \leq a < b$. If the binding number $\text{bind}(G) > \frac{(a+b-1)(n-1)}{bn - (a+b) - bk + 2}$ and $n \geq \frac{(a+b-1)(a+b-2)}{b} + \frac{bk}{b-1}$, then G is an (a, b, k) -critical graph.

In Theorem 7, if $k=0$, then we get the following corollary.

Corollary 2. Let G be a graph of order n , and let a and b be two integers such that $1 \leq a < b$. If the binding number $\text{bind}(G) > \frac{(a+b-1)(n-1)}{bn - (a+b) + 2}$ and $n \geq \frac{(a+b-1)(a+b-2)}{b}$, then G has an $[a, b]$ -factor.

Let a, b and k be nonnegative integers such that $1 \leq a < b$. The proof of Theorem 7 relies heavily on the following theorem.

Theorem 8.^[5] Let G be a graph of order $n \geq a + k + 1$. Then G is (a, b, k) -critical if and only if for any $S \subseteq V(G)$ and $|S| \geq k$

$$\sum_{j=0}^{a-1} (a-j)p_j(G-S) \leq b|S| - bk, \text{ or}$$

$$\delta_G(S, T) = b|S| + d_{G-S}(T) - a|T| \geq bk,$$

where $T = \{x : x \in V(G) \setminus S, d_{G-S}(x) \leq a-1\}$.

Proof of Theorem 7. Suppose a graph G satisfies the condition of the theorem, but it is not an (a, b, k) -critical graph. Then, by Theorem 8, there exists a subset S of $V(G)$ with $|S| \geq k$ such that

$$\delta_G(S, T) = b|S| + d_{G-S}(T) - a|T| \leq bk - 1, \quad (6)$$

where $T = \{x : x \in V(G) \setminus S, d_{G-S}(x) \leq a-1\}$. We choose subsets S and T such that $|T|$ is minimum and S and T satisfy (6).

If $T = \emptyset$, then by (6), $bk - 1 \geq \delta_G(S, T) = b|S| \geq bk$, a contradiction. Hence, $T \neq \emptyset$.

Let

$$h = \min\{d_{G-S}(x) : x \in T\}.$$

According to the definition of T , we have

$$0 \leq h \leq a - 1.$$

We shall consider various cases according to the value of h and derive contradictions.

Case 1. $h = 0$.

At first, we prove the following claim.

Claim 2. $\frac{bn - (a+b) - bk + 2}{n-1} > 1$.

Proof. Since $n \geq \frac{(a+b-1)(a+b-2)}{b} + \frac{bk}{b-1}$, then we have

$$\begin{aligned}
bn - (a+b) - bk + 2 - (n-1) &= (b-1)n - (a+b) - bk + 3 \\
&\geq (b-1)\left(\frac{(a+b-1)(a+b-2)}{b} + \frac{bk}{b-1}\right) - (a+b) - bk + 3 \\
&= \frac{(b-1)(a+b-1)(a+b-2)}{b} - (a+b) + 3 \\
&\geq (a+b-2) - (a+b) + 3 > 0
\end{aligned}$$

Thus, we have

$$\frac{bn - (a+b) - bk + 2}{n-1} > 1.$$

Completing the proof of Claim 2.

Let $m = |\{x : x \in T, d_{G-S}(x) = 0\}|$, and let $Y = V(G) \setminus S$. Then $N_G(Y) \neq V(G)$ since $h=0$. In view of the definition of the binding number $bind(G)$, we get that

$$|N_G(Y)| \geq bind(G) |Y|.$$

Thus, we obtain

$$n - m \geq |N_G(Y)| \geq bind(G) |Y| = bind(G)(n - |S|),$$

that is,

$$|S| \geq n - \frac{n-m}{bind(G)}. \quad (7)$$

Using $|S| + |T| \leq n$ and (6) and (7) and Claim 2, we have

$$\begin{aligned}
bk - 1 &\geq \delta_G(S, T) = b|S| + d_{G-S}(T) - a|T| \\
&\geq b|S| - (a-1)|T| - m \\
&\geq b|S| - (a-1)(n - |S|) - m \\
&= (a+b-1)|S| - (a-1)n - m \\
&\geq (a+b-1)\left(n - \frac{n-m}{bind(G)}\right) - (a-1)n - m \\
&= bn - (a+b-1)\frac{n-m}{bind(G)} - m \\
&> bn - (a+b-1)\frac{(n-m)(bn - (a+b) - bk + 2)}{(a+b-1)(n-1)} - m \\
&= bn - \frac{(n-m)(bn - (a+b) - bk + 2)}{n-1} - m
\end{aligned}$$

$$\begin{aligned}
&\geq bn - \frac{(n-1)(bn - (a+b) - bk + 2)}{n-1} - 1 \\
&= bk + (a+b) - 3 \\
&\geq bk,
\end{aligned}$$

which is a contradiction.

Case 2. $1 \leq h \leq a-1$.

Let x_1 be a vertex in T such that $d_{G-S}(x_1) = h$, and let $Y = (V(G) \setminus S) \setminus N_{G-S}(x_1)$. Then $x_1 \in Y \setminus N_G(Y)$, so $Y \neq \emptyset$ and $N_G(Y) \neq V(G)$. In view of the definition of the binding number $bind(G)$, we obtain

$$\frac{|N_G(Y)|}{|Y|} \geq bind(G).$$

Thus, we get that

$$n-1 \geq |N_G(Y)| \geq bind(G) |Y| = bind(G)(n-h-|S|),$$

that is,

$$|S| \geq n-h - \frac{n-1}{bind(G)}. \quad (8)$$

By $|S| + |T| \leq n$ and (6) and (8), we obtain

$$\begin{aligned}
bk-1 &\geq \delta_G(S, T) = b|S| + d_{G-S}(T) - a|T| \\
&\geq b|S| - (a-h)|T| \\
&\geq b|S| - (a-h)(n-|S|) \\
&= (a+b-h)|S| - (a-h)n \\
&\geq (a+b-h)\left(n-h - \frac{n-1}{bind(G)}\right) - (a-h)n \\
&> (a+b-h)\left(n-h - \frac{bn-(a+b)-bk+2}{a+b-1}\right) - (a-h)n,
\end{aligned}$$

that is,

$$bk-1 > (a+b-h)\left(n-h - \frac{bn-(a+b)-bk+2}{a+b-1}\right) - (a-h)n. \quad (9)$$

Let $f(h) = (a+b-h)\left(n-h - \frac{bn-(a+b)-bk+2}{a+b-1}\right) - (a-h)n$. In fact, the function $f(h)$ attains its minimum value at $h=1$ since $1 \leq h \leq a-1$ is an integer. Then, we have

$$f(h) \geq f(1).$$

Combining this with (9), we obtain

$$\begin{aligned} bk - 1 &> f(1) = (a + b - 1)(n - 1 - \frac{bn - (a + b) - bk + 2}{a + b - 1}) - (a - 1)n \\ &= (a + b - 1)(n - 1) - (bn - (a + b) - bk + 2) - (a - 1)n \\ &= bk - 1, \end{aligned}$$

that is a contradiction.

From the argument above, we deduce the contradictions, so the hypothesis cannot hold. Hence, G is (a, b, k) -critical.

Completing the proof of Theorem 7.

Remark 1. Let us show that the condition $\delta(G) > p + a + b - 2\sqrt{(a+1)p - bn + 1}$ in Theorem 4 cannot be replaced by $\delta(G) \geq p + a + b - 2\sqrt{(a+1)p - bn + 1}$. Let $a = 2$, $b = 3$, and $n \geq 0$ an integer. Let $H = K_{n+1} \vee (K_a \cup K_a)$. Then $p = 2a + n + 1$ and $\delta(H) = a + n$. Thus, we obtain easily $p + a + b - 2\sqrt{(a+1)p - bn + 1} = a + n$, that is, $\delta(H) = p + a + b - 2\sqrt{(a+1)p - bn + 1}$. Let $S = V(K_{n+1}) \subseteq V(H)$, $T = V(K_a \cup K_a) \subseteq V(H)$. Since $a \leq g(x) < f(x) \leq b$ and $b = a + 1$, then we have $g(x) = a$ and $f(x) = b = a + 1$. Thus, we get

$$\begin{aligned} \delta_H(S, T) &= f(S) + d_{H-S}(T) - g(T) \\ &= b|S| + (a - 1)|T| - a|T| \\ &= b|S| - |T| \\ &= b(n + 1) - 2a \\ &= bn + b - 2a \\ &= bn - 1 \quad (\text{Since } a=2 \text{ and } b=3) \\ &< bn = \max\{f(N) : N \subseteq S, |N| = n\}. \end{aligned}$$

By Theorem 6, H is not a (g, f, n) -critical graph. In the above sense, the result of Theorem 4 is best possible.

Remark 2. We may adopt the similar way to argue the condition $\delta(G) > p + a + b - 2\sqrt{(a+1)p + 1 + n}$ in Theorem 5, and the condition $\delta(G) > p + a + b - 2\sqrt{(a+1)p + 1 + n}$ in Theorem 5 is the best possible in some sense.

Remark 3. Let us show that the condition $\text{bind}(G) > \frac{(a+b-1)(n-1)}{bn-(a+b)-bk+2}$ in Theorem 7 cannot be replaced by $\text{bind}(G) \geq \frac{(a+b-1)(n-1)}{bn-(a+b)-bk+2}$. Let $b > a \geq 2, k \geq 0$ be three integers such that $a + b + k$

is odd, and let $n = \frac{(a+b-1)(a+b-2)+(a+b-2)+(a+2b-1)k}{b}$ is an integer, and let $l = \frac{a+b+k-1}{2}$ and $m = n - 2l = n - (a+b+k-1) = \frac{(a+b-1)(a-2)+(a+b-2)+(a+b-1)k}{b}$. Clearly, m is an integer. Let $H = K_m \vee K_2$. Let $X = V(K_2)$, for any $x \in X$, then $|N_H(X \setminus x)| = n - 1$. By the definition of $\text{bind}(H)$, $\text{bind}(H) = \frac{|N_H(X \setminus x)|}{|X \setminus x|} = \frac{n-1}{2l-1} = \frac{n-1}{a+b+k-2} = \frac{(a+b-1)(n-1)}{bn-(a+b)-bk+2}$. Let $S = V(K_m) \subseteq V(H)$, $T = V(K_2) \subseteq V(H)$, then $|S| = m \geq k$, $|T| = 2l$. Thus, we get

$$\begin{aligned} \delta_H(S, T) &= b|S| - a|T| + d_{H-S}(T) \\ &= b|S| - a|T| + |T| = b|S| - (a-1)|T| \\ &= b \frac{(a+b-1)(a-2) + (a+b-2) + (a+b-1)k}{b} - (a-1)(a+b+k-1) \\ &= bk - 1 < bk. \end{aligned}$$

By Theorem 8, H is not an (a, b, k) -critical graph. In the above sense, the result in Theorem 7 is best possible.

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