Extended Source Size Correction Factor in Antenna Gain Measurements

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1. Introduction

In this chapter we consider the extended source size correction factor that is widely utilized in antenna gain measurements when extraterrestrial extended radio sources are in use. Extended radio sources having an angular size that is comparable or even larger than the antenna's far-field power pattern Half Power Beam Width (HPBW) are often used to determine various antenna parameters including gain of electrically large antenna apertures (Baars, 1973). The use of such sources often becomes almost inevitable since the far-field distance of electrically large antennas can reach tens or even hundreds kilometers, which makes it impractical if not impossible to employ the conventional far-field antenna transmitter-receiver test range technique.

To illustrate the importance of the problem let's consider 30m radio astronomy antenna at 10GHz and 96GHz frequency bands. Its far-field zone distances are 60km and 580km with the HPBW equal to 4' and 0.42' respectively. For the comparison, among the strongest cosmic radio sources, Cassiopeia A has 4' and Sygnus A has 0.7' of their disk angular sizes (Guidici & Castelli, 1971). Even for the 7m communication antenna working at 20 GHz, the far-field distance is 6.5km and the far-field patterns HPBW is 1.4°, while the angular size of the Sun or the Moon disks are about of 0.5° (Guidici & Castelli, 1971).

When the radio source angular size is comparable with the antenna HPBW, the antenna radiation pattern is averaged within the solid spatial angle subtended to the source. Therefore, the measured antenna gain value appears to be less than what would be expected for the antenna's effective collecting area and the aperture illumination and the resulting gain measurements must be corrected by the extended source size correction factor to account for the convolution of the extended radio source angular size, angular source brightness distribution and the shape of the antenna's far-field radiation pattern. In this chapter two kinds of extended radio sources, having either uniform or Gaussian brightness distributions over the source disk (Baars, 1973; Kraus, 1986), along with three kinds of the most usable "Polynomial-on-Pedestal," Gaussian, and Taylor antenna aperture illuminations are examined for circular and rectangular antenna apertures.

As a result of the above considerations and based on the literature survey, the complete set of simple analytical expressions that accurately approximate the value of the extended source size correction factor have been derived and/or developed for circular and

rectangular antenna apertures and for all the above combinations of extended radio source brightness distributions and antenna aperture illuminations. Those expressions eliminate the need to perform complicated and often impractical numerical integrations in order to evaluate the extended source size correction factor value for the case of particular measurement. The approximate analytical expressions for the extended source size correction factor for rectangular antenna apertures along with their tolerances for circular and rectangular apertures are obtained for the first time in literature.

Because the extended source size correction factor most conveniently can be expressed through the ratio between the extended source angular size (or its HPBW) and the antenna's far-field radiation pattern HPBW, the approximations of the antenna's HPBW for all three types of antenna aperture illuminations and for circular and rectangular antenna apertures are considered as a supplementary problem. As a result, numerous simple and accurate analytical expressions for the antenna's far-field pattern HPBW for circular and rectangular antenna apertures and for all three types of antenna aperture illuminations have been developed. This also eliminates the need to perform complicated and often impractical numerical integrations in order to evaluate the antenna's far-field pattern HPBW value for the particular antenna size(s) and aperture illumination(s). In addition, for circular and rectangular antenna apertures these expressions are shown in the form of plots.

While in this chapter, we consider the extended source size correction factor from the prospective of the antenna gain measurement, it should be noted that the same factor can also be utilized for the solution of the inverse problem: the measurement of the unknown temperature and/or flux density of a randomly polarized extended radio source using the electrically large antenna with a known antenna far-field power pattern (Ko, 1961).

2. Extended Cosmic Radio Sources and Extended Source Size Correction Factor in Antenna Gain Measurements

The IEEE Standard Std 149-1979 for the antenna test procedures (Kummer et al., 1979) defines the extended source size correction factor K by the following expression:

$$K = \frac{\iint_{\Omega_s} B_s(\Omega) d\Omega}{\iint_{\Omega_s} B_s(\Omega) F_n(\Omega) d\Omega} \quad (1)$$

where $B_s(\Omega)$ is the angular brightness distribution of the extended radio source, $F_n(\Omega)$ is the normalized pencil beam antenna far-field power pattern, such that at the antenna boresight (direction of the peak of the main beam) $F_n(0) = 1$, and Ω_s is the solid angle subtended to the extended radio source. To obtain the correct antenna gain, the gain value measured using the extended radio source should be multiplied by K .

In order to understand the meaning and the applicability of definition (1) let's first establish the relations between the power pattern $F_n(\Omega)$ and the antenna far-field radiation power pattern $F(\Omega)$ that is normalized the way that the total power transmitted or received by the antenna within the entire 4π steradian solid angle equals to one power unit:

$$\iint_{4\pi} F(\Omega) d\Omega = \iint_{4\pi} A \times F_n(\Omega) d\Omega = 1 \quad (2)$$

where A is a constant multiplier that needs to be found. In case of the omnidirectional antenna when $F_n(\Omega) = 1$ at any direction of the solid angle Ω , condition (2) gives the following value of that multiplier A_{omni} :

$$\iint_{4\pi} A_{\text{omni}} d\Omega = 1 \Rightarrow A_{\text{omni}} = \frac{1}{4\pi} \quad (3)$$

By definition (Balanis, 2005), the maximum antenna gain G_{max} is the ratio of the maximum antenna radiation intensity, i. e. A , to the radiation intensity averaged over all directions, i. e. A_{omni} :

$$G_{\text{max}} = \frac{A}{A_{\text{omni}}} = 4\pi A \Rightarrow A = \frac{G_{\text{max}}}{4\pi} \quad (4)$$

Based on (2) and (4) one can notice that the denominator in (1) is proportional to the power $P_{\text{extended source}}$ received by the antenna whose maximum gain value is G_{max} in the case when its far-field pattern HPBW is comparable or greater than the size of the extended radio source spatial angle:

$$P_{\text{extended source}} = \iint_{\Omega_s} B_s(\Omega) \frac{G_{\text{max}}}{4\pi} F_n(\Omega) d\Omega \quad (5)$$

The numerator in (1) is proportional to the power $P_{\text{point source}}$ received by the same antenna in the case when its far-field antenna pattern HPBW is much bigger than the angular size of extended radio source and thus, the value $F_n(\Omega)$ in (5) can be substituted by 1 within the source spatial angle:

$$P_{\text{point source}} = \iint_{\Omega_s} B_s(\Omega) \frac{G_{\text{max}}}{4\pi} d\Omega \quad (6)$$

The relations (5) and (6) can be used by both ways: to find the unknown maximum antenna gain and the radiation pattern based on the measured power received by the antenna and the known source brightness distribution or, conversely, to find the unknown source brightness distribution based on the measured power received by the antenna and the known maximum antenna gain and the radiation pattern. Either way, the ratio between the received power (6) and the received power (5) is equal to the extended source size correction factor defined by (1) and represents the correction that should be made if the extended radio source, but not the point source, is used in the measurement procedure.

In order to compute the extended source size correction factor using its definition (1), both the extended source brightness $B_s(\Omega)$ and the normalized antenna power pattern $F_n(\Omega)$ as a function of the solid angle subtended to the extended source should be known. In following sections we review expressions for the extended source correction factor already existed in literature, discuss and specify the considered set of functions $B_s(\Omega)$ and $F_n(\Omega)$, its relations with actual extended radio sources and antenna configurations and disclose numerous

novel simple analytical expressions that accurately approximate the value of extended source size correction factor for those combinations of $B_s(\Omega)$ and $F_n(\Omega)$.

3. Existing Approximate Analytical Formulae for Extended Source Size Correction Factor

Based on definition (1) of the extended source size correction factor and making various simplifying assumptions about the source brightness distribution $B_s(\Omega)$ and the normalized antenna far-field power pattern $F_n(\Omega)$, several approximate analytical expressions for the extended source correction factor have been developed in literature.

For the case of a Gaussian source brightness distribution and a Gaussian antenna far-field power pattern (Guidici & Castelli, 1971) and (Baars, 1973) stated the following expression for the extended source correction factor K :

$$K = 1 + s^2 \quad (7)$$

where s is the ratio of the extended source HPBW to the antenna HPBW. For the case of a uniform source brightness distribution and a Gaussian antenna far-field power pattern (Ko, 1961) expresses value of K as:

$$K = \frac{(s\sqrt{\ln 2})^2}{1 - \exp[-(s\sqrt{\ln 2})^2]} \quad (8)$$

where s is the ratio of the disk source angular diameter to the antenna HPBW. For the case of a uniform source brightness distribution and an antenna far-field power pattern that corresponds with a uniform antenna aperture illumination (Ko, 1961) expresses the value of K as:

$$K = \frac{(1.616s)^2}{4[1 - J_1^2(1.616s) - J_0^2(1.616s)]} \quad (9)$$

where s is the ratio of the source disk diameter to the antenna HPBW.

It should be noted that expressions (7) – (9) were derived under the assumption of a small value of variable s , i. e., when the extended source angular size is noticeably less than the antenna pattern HPBW and only for the circular antenna aperture.

In Figure 1, the approximate values of the extended source size correction factor as a function of the ratio of the source diameter or the source HPBW to the antenna HPBW and given by expressions (7) – (9) are shown in comparison. As is seen from Figure 1, the values given by expressions (8) and (9) are very close to each other, while being significantly different from values given by the expression (7). Expressions (7) – (9) were selected among few others because they represent the best approximations of the extended source size correction factor available in the literature for both uniform and Gaussian extended source brightness distributions that are used the most in antenna measurements and/or calibrations.

However, there are also several problems that are associated with these approximations. First, approximations (7) and (8) are based on the shape of the antenna far-field power

pattern rather than based on the shape of the antenna aperture illumination and its edge illumination taper, which are actually known in practice. Secondly, in order to use expressions (7) – (9), the antenna HPBW as a function of the type of the aperture illumination and its edge illumination taper has to be known. Third, not all combinations of aperture illuminations and extended source brightness distributions used in practice are covered by (7) – (9). Fourth, all expressions (7) – (9) were derived for the small value of the source size to antenna HPBW ratio and its accuracy for the case when this ratio isn't small is unknown. Fifth, these expressions were derived only for circular antenna apertures and do not cover rectangular apertures that become increasingly popular for the modern solid state antennas. Sixth, expressions (7) – (9) do not explicitly depend on the antenna aperture edge illumination taper, which is manifestly wrong since the normalized antenna power pattern $F_n(\Omega)$ and therefore, the extended source size correction factor do depend on it. Further in this chapter we will amend and expand expressions (7) – (9) the way that eliminates all its deficiencies that were mentioned above.

Fig. 1. Approximate expressions for extended source size correction factor in comparison.
 Plot legend: solid red – expression (9); long-dashed blue – expression (7); and short-dashed green – expression (8);

4. Brightness Distributions of Extended Cosmic Radio Sources Used in Antenna Gain Measurements

The detailed description of most cosmic extended radio sources that are used in the electrically large antenna measurements and calibrations along with the discussion of various aspects of such measurements is given, for example, in (Baars, 1973), (Guidici & Castelli, 1971) and (Kraus, 1986).

From the extended source correction factor evaluation standpoint it's enough to notice that most of these extended radio sources have circular disk shape with either axially symmetrical Gaussian or uniform brightness distributions over the solid angle that is subtended to the extended radio source disk (Baars, 1973), (Kraus, 1986). Those sources are almost entirely incoherent and that is why in (1) the antenna far-field power pattern, as oppose to the antenna far-field radiation field pattern, is in use. For example, the Cassiopeia A has a uniform brightness distribution, while the Orion A has an axially symmetrical Gaussian brightness distribution over a source disk (Baars, 1973).

Thus, for the purpose of this chapter, we will consider only these two, the uniform and the Gaussian extended radio source brightness distributions, which can be written in following forms:

$$B_{S\text{ uni}}(\theta, \varphi) = \begin{cases} 1, & \theta \leq \theta_s / 2 \\ 0, & \theta > \theta_s / 2 \end{cases} \quad (10)$$

$$B_{S\text{ Gauss}}(\theta, \varphi) = \exp \left[-\ln 2 \left(\frac{2\theta}{\theta_s} \right)^2 \right] \quad (11)$$

where θ_s is the angular size of the extended radio source. In the case of the uniform source brightness distribution θ_s is the physical angular size of the source disk. In the case of the Gaussian source brightness distribution, the source angular size θ_s is defined as a source HPBW, i. e., the angular size at which the brightness of the source is half of its maximum at the center of the source disk.

It should be stressed that the assumed properties of the extended radio source namely, the radiation incoherence and the types of source brightness distribution (10) and (11), are essential for the correct simulation of the extended source size correction factor.

5. Illumination Functions and Antenna Patterns for Circular and Rectangular Antenna Apertures

5.1 Circular Antenna Aperture Case

In order to calculate the value of the extended source size correction factor (1) except of the extended source brightness distribution function $B_s(\mathcal{Q})$ that was described in previous section, the antenna far-field power pattern $F_n(\mathcal{Q})$ needs to be known. Unlike the extended source brightness distribution $B_s(\mathcal{Q})$ the antenna far-field power pattern $F_n(\mathcal{Q})$ for almost all practical cases is not an analytically defined function and is rather defined through the particular illumination of the antenna aperture.

For the purpose of this chapter we chose the three most usable circularly symmetrical aperture illumination functions: "Polynomial-on-Pedestal," Gaussian, and Taylor. For convenience, we use following forms of these aperture illumination functions for the circular aperture:

$$f_{ApPoly} = B + (1 - B) \left[1 - \left(\frac{2r}{d} \right)^2 \right]^2 \quad (12)$$

$$f_{ApGauss} = \exp \left[- \left(\sqrt{-\ln B} \frac{2r}{d} \right)^2 \right] \quad (13)$$

$$f_{ApTaylor} = \frac{J_0 \left[j\pi\beta \sqrt{1 - \left(\frac{2r}{d} \right)^2} \right]}{J_0[j\pi\beta]} \quad (14)$$

where r is the value of the radius vector from the center of the circular aperture, d is the diameter of the antenna aperture, B is the aperture edge illumination taper ($0 \leq B \leq 1$), and the constant β in the expression (14) can be found from the equation:

$$J_0[j\pi\beta] = \frac{1}{B} \quad (15)$$

The aperture edge illumination taper is usually expressed in dB, thus for convenience, we introduce the constant c that is associated with constant B by:

$$c = -20 \log_{10} B \quad (16)$$

Here's another useful constant that is associated with constant B and will be extensively used throughout the chapter:

$$b = 1 - B \quad (17)$$

The example of all three circular aperture illumination functions (12) - (14) are shown in Figure 2 for comparison. As is seen from plots in Figure 2, expression (12) - (14) are normalized so that all aperture illuminations (12) - (14) have the same maximum value of 1 at the center of the aperture and the same minimum value at the edge of the aperture that is equal to the value of the aperture edge illumination taper B .

Knowing the antenna aperture illuminations (12) - (14) and using the aperture approach for the antenna pattern calculation, the normalized antenna power pattern for the circular antenna aperture can be expressed, according to (Johnson at all, 1993) as follows:

Fig. 2. Comparison between “Polynomial-on-Pedestal” (solid red), Gaussian (long-dashed blue) and Taylor (short-dashed green) aperture illuminations for 10dB (upper) and 30dB (lower) circular aperture edge illumination taper.

$$F_{in}(\theta) = \left[\frac{\int_0^{0.5d} f_i(r, B) J_0(kr \sin \theta) dr}{\int_0^{0.5d} f_i(r, B) dr} \right]^2 \quad (18)$$

where $k = 2\pi/\lambda$ is the wavenumber, θ is the antenna pattern off-boresight angle, and the index i stands for any of aperture illumination functions (12) – (14). For instance, $i = 1$ corresponds with the expression (12), $i = 2$ corresponds with the expression (13) and $i = 3$ corresponds with the expression (14).

It should be noted that in spite of well known deficiencies of the aperture approach for the antenna pattern calculation that approach is quite adequate for this particular application since in (1) only values of function (18) in the vicinity of the antenna main lobe are actually used. Plots in Figure 3 illustrate the difference between antenna patterns computed using (18) for all three aperture illuminations (12) – (14).

As is seen from these plots the difference between antenna patterns for all three aperture illuminations becomes noticeable well outside of the 3dB beamwidth off-boresight direction even for heavily tapered aperture illuminations. It should be also noticed that when the aperture edge illumination taper approaches 0dB, which means that the constant B

approaches 1, all three aperture illumination functions (12) – (14) approach the uniform illumination and thus, their antenna patterns are also converged to each other.

Fig. 3. Comparison between antenna patterns for “Polynomial-on-Pedestal” (solid red), Gaussian (long-dashed blue), and Taylor (short-dashed green) aperture illumination functions with 10dB (upper) and 30dB (bottom) circular aperture edge illumination taper.

5.2 Rectangular Antenna Aperture Case

It was assumed that for the case of the rectangular aperture the same aperture illumination functions (12) – (14) are applied in combination along each of the Cartesian coordinate x and y in a form of its direct product:

$$f_{ij}(x, B_x, y, B_y) = f_i(x, B_x) \times f_j(y, B_y) \quad (19)$$

where i and j stand for any of aperture illuminations (12) – (14). For example, the rectangular aperture illumination that is the “Polynomial-on-Pedestal” along the X-axis and is the Gaussian one along the Y-axis is described by the following expression:

$$f_{12}(x, B_x, y, B_y) = f_1(x, B_x) \times f_2(y, B_y) = \left\{ B_x + (1 - B_x) \left[1 - \left(\frac{2x}{d_x} \right)^2 \right]^2 \right\} \times \exp \left[- \left(\sqrt{-\ln B_y} \frac{2y}{d_y} \right)^2 \right] \quad (20)$$

where d_x and d_y are the rectangular aperture width and height, x and y are coordinates in the aperture plane with the origin in the center of the aperture, and B_x and B_y are aperture edge illumination tapers that, in general, have different values along X and Y axes.

Based on the rectangular aperture illumination function (19) and using the same aperture approach that was used for the case of the circular aperture, the normalized antenna power pattern for the rectangular aperture can be written, according to (Johnson, R. C. at all, 1993) as follows:

$$F_{ijn}(\theta, \varphi) = \left[\frac{\int_0^{0.5d_x} \int_0^{0.5d_y} f_{ij}(x, B_x, y, B_y) \exp[k \sin \theta (x \cos \varphi + y \sin \varphi)] dx dy}{\int_0^{0.5d_x} \int_0^{0.5d_y} f_{ij}(x, B_x, y, B_y) dx dy} \right]^2 \quad (21)$$

where the integration can be done just across the quarter of the aperture because all integrand functions are even in respect to variables x and y . Because all integrand functions are also separable in respect to variables x and y , the expression (21) can be simplified even further and present the antenna pattern of the rectangular aperture as a product of two terms one of which contains integrals only along the x coordinate axis and the other contains integrals only along the y coordinate axis:

$$F_{ijn}(\theta, \varphi) = \left[\frac{\int_0^{0.5d_x} f_i(x, B_x) \exp[kx \sin \theta \cos \varphi] dx}{\int_0^{0.5d_x} f_i(x, B_x) dx} \times \frac{\int_0^{0.5d_y} f_j(y, B_y) \exp[ky \sin \theta \sin \varphi] dy}{\int_0^{0.5d_y} f_j(y, B_y) dy} \right]^2 \quad (22)$$

The expression (22) is valid for the rectangular aperture antenna pattern with separable aperture illuminations along x and y axes and means, for instance, that the antenna pattern in principal plane at $\varphi = 0^\circ$ depends only on the illumination function $f_i(x, B_x)$, while in the other principal plane at $\varphi = 90^\circ$, it depends only on the illumination function $f_j(y, B_y)$. Plots in Figure 4 illustrate the difference between antenna patterns in principal planes computed using (22) for all three aperture illuminations (12) – (14). These plots were calculated for the rectangular aperture with the same area as the area of the circular aperture that was used to calculate plots in Figure 3. As is seen from these plots, similarly to the circular aperture case differences in antenna patterns for all three aperture illuminations (12) – (14) become

noticeable well outside of the 3dB beamwidth off-boresight angles even for heavily tapered aperture illuminations. The comparison between antenna patterns for circular (Fig. 3) and rectangular (Fig. 4) antennas shows just some quantitative but not qualitative differences between them.

Fig. 4. Comparison between antenna patterns for “Polynomial-on-Pedestal” (solid red), Gaussian (long-dashed blue), and Taylor (short-dashed green) aperture illuminations with 10dB (upper) and 30dB (bottom) rectangular aperture edge illumination taper.

6. Simple and Accurate Approximation of Antenna HPBW for Circular and Rectangular Apertures

6.1 Circular Antenna Aperture Case

Based on the general definition of the extended source correction factor (1) and using expressions (10), (11) for the source brightness distribution and expressions (18), (22) for the circular and rectangular aperture antenna power pattern, it is possible now to calculate the exact value for the extended source size correction factor. However, in order to compare the exact value of the extended source size correction factor with its approximations given by expressions (7) – (9), the value of the antenna pattern HPBW should be known with a high

degree of accuracy for each aperture illuminations (12) – (14) for circular and rectangular apertures. This is ultimately needed because the argument s of the extended source size correction factor approximate expressions (7) – (9) is defined as:

$$s = \frac{\text{source diameter or source HPBW}}{\text{antenna HPBW}} \quad (23)$$

According, for example, to (Johnson at all, 1993), the circular antenna HPBW can be estimated through the simple formula:

$$\text{HPBW} = \alpha \frac{\lambda}{d} \quad (24)$$

where λ is a wavelength, d is the antenna diameter and the α is the beamwidth multiplier in degrees that depends on the type of aperture illumination and edge illumination taper. The rough estimation of α as a function of the aperture edge illumination taper, without taking into account the type of the aperture illumination, was given in (Johnson at all, 1993):

$$\alpha = 55.9486 + 1.05238 c \quad (25)$$

where c is define in (16) as an absolute value of the edge illumination taper in dB. The number of significant digits in (25) is misleading because the six digits computational accuracy implied by the expression (25) can not be achieved based solely on the value of the edge illumination taper, regardless of the type of the aperture illumination.

The numerical simulations of the circular antenna beamwidth using the expression (18) for the circular antenna radiation pattern and for all three type of aperture illuminations (12) – (14) give the values of the beamwidth multiplier α that are summarized in Table 1 and illustrated in Figure 5.

6.2 Rectangular Antenna Aperture Case

As it was mentioned in section 5.2 for the rectangular antenna aperture illuminated by the separable aperture illumination function (19), antenna pattern cuts in principal planes (along the X or Y axes) are independent of each other and, as it follows from (22), depend exclusively on its own aperture illumination function regardless of illumination function that is applied along the opposite axis in the aperture plane. Thus, to calculate the rectangular antenna HPBW in principal planes, one can still employ the expression (24) substituting the circular aperture diameter d by the width d_x or the height d_y of the rectangular aperture, respectively.

The numerical simulations of the rectangular antenna beamwidth using the expression (22) for the rectangular antenna radiation pattern and for all three type of aperture illuminations (12) – (14) give the values of the beamwidth multiplier α that are summarized in Table 2 and illustrated in Figure 6.

Fig. 5. Approximate 3dB beamwidth multiplier a used in (24) as a function of aperture edge illumination taper for circular aperture and "Polynomial-on-Pedestal" (solid red), Gaussian (long-dashed blue) and Taylor (short-dashed green) aperture illuminations.

Antenna Aperture Illumination	Approximate Beamwidth Multiplier a in degrees Circular Aperture, $c \leq 40$ dB	Max Error, %
Polynomial	$58.862 + 0.53523c + 0.039795c^2 - 0.001575c^3 + 0.00001562c^4$	0.13
Gaussian	$58.862 + 0.5865c + 0.01089c^2 - 0.000094c^3$	0.055
Taylor	$58.862 + 0.6247c + 0.0048c^2 - 0.000086c^3$	0.095

Table 1. Approximate formulae for antenna pattern beamwidth multiplier a used in (24) for circular aperture.

Fig. 6. Approximate 3dB beamwidth multiplier a used in (24) as a function of edge illumination taper for rectangular aperture and “Polynomial-on-Pedestal” (solid red), Gaussian (long-dashed blue) and Taylor (short-dashed green) aperture illuminations.

Antenna Aperture Illumination	Approximate Beamwidth Multiplier a in degrees Rectangular Aperture, $c \leq 40$ dB	Max Error, %
Polynomial	$50.677 + 0.9449c + 0.2964c^2 - 0.001694c^3 + 0.00001971c^4$	0.26
Gaussian	$50.677 + 0.8236c + 0.011283c^2 - 0.0001402c^3$	0.037
Taylor	$50.677 + 0.863c + 0.0006c^2 - 0.000056c^3$	0.15

Table 2. Approximate formulae for antenna pattern beamwidth multiplier a used in (24) for rectangular aperture.

The comparison between antenna beamwidth multipliers a for circular and rectangular apertures shows that they are close to each other and have very similar behavior as a function of the aperture illumination and its edge illumination taper. As it appears from Figures 5 and 6 and Tables 1 and 2, the antenna beamwidth multiplier for rectangular aperture is slightly less than for the circular one. However, if one would compare the circular aperture with diameter d_{circle} and the square apertures with the size d_{square} that have the same area of aperture (26), which results the smaller linear size of the square aperture than the diameter of the corresponded circular aperture, the antenna beamwidth multipliers a and therefore the antenna HPBW for circular and rectangular apertures become virtually the same.

$$\frac{\pi d_{circle}^2}{4} = d_{square}^2 \quad (26)$$

Using (24) and the appropriate value of the beamwidth multiplier a derived from the Figures 5 and 6 or Tables 1 and 2 it's easy to calculate the antenna HPBW for the respective aperture geometry and illumination with a high degree of accuracy shown in Tables 1 and 2 without the use of complicated and sometime impractical integration. It should be noted that this possibility is very instrumental by itself, regardless of the calculation of the extended source size correction factor.

7. Approximation of Exact Value of Extended Source Size Correction Factor

7.1 Circular Antenna Aperture Case

Based on some earlier studies (Solovey & Mittra, 2008) and on an extensive numerical simulations of the extended source size correction factor using its definition (1) with all combinations of the extended source brightness distributions (10) and (11) and aperture illuminations (12) – (14) for circular aperture, it was found that the most convenient way to approximate the exact value of the extended source size correction factor is to present it in the following form:

$$10 \log_{10} K = 10 \log_{10} K_{approx} - \text{Corrective Term} \quad (27)$$

where K_{approx} is given by one of the expressions (7) or (9) and the value of the *Corrective Term* should be calculated using the formulae in Tables 3 – 6 or picked up from plots in Figures 7 – 10. In those Tables and Figures the *Corrective Term* is expressed in dB and presented as a function of the ratio of the extended source size or HPBW to the antenna HPBW, the type of the aperture illumination and the illumination taper.

(a) *Uniform Source Brightness Distribution*. It was found that for the extended radio source with the uniform source brightness distribution (10) and for the circular antenna aperture, the best approximation of the extended source size correction factor is achieved when expression (9) for the K_{approx} is used in (27):

$$10 \log_{10} K = 10 \log_{10} \left[\frac{(1.616s)^2}{4[1 - J_1^2(1.616s) - J_0^2(1.616s)]} \right] - \text{Corrective Term} \quad (28)$$

where the *Corrective Term* can be found either from plots in Figures 7 and 8 or from formulae in Tables 3 and 4 and the value of s is restricted by $s < 3$, while the value of c is restricted by $c < 30$. As a reminder, the variable s in (28) and Tables 3 and 4 is defined by (23) and variables c and b in Tables 3 and 4 are defined by (16) and (17). In addition, the calculation errors of the extended source correction factor with and without the *Corrective Term* are also shown in Tables 3 and 4. As is seen from these Tables, the application of the *Corrective Term* decreases the calculation error of the extended source correction factor for the extended radio sources with the uniform source brightness distribution from the 0.038dB – 0.32dB to the 0.0051dB – 0.050dB.

By and large, the complexity of the *Corrective Term* polynomial can be increased to achieve even a smaller calculation error of the extended source correction factor. However, here and further in this chapter, the complexity of the polynomial was chosen on an ad hoc basis as a compromise between the tolerance delivered by the *Corrective Term* and the complexity of its polynomial. Another reason why one should not try to increase the complexity of the *Corrective Term* polynomial out of proportion in order to achieve increasingly lower error of the extended source correction factor approximation (27), especially for the values of s that are essentially more than 1 is this. While for the small values of s that are well within the antenna pattern main lobe, the expressions (18) and (22) for the circular and rectangular antenna patterns describe the shape of antenna pattern of real antennas fairly accurately, for the $s > 1$ the accuracy of the theoretical antenna pattern representation becomes progressively lower. The antenna pattern of real antennas outside of the main lobe essentially depends on many fine details of the particular antenna system including but not limited to the aperture blockage by the feeder(s) and struts, the fine differences between the real aperture illumination and its approximate representation by the formulae (12) – (14), etc. While those fine details have very limited influence on the shape of the main lobe of antenna pattern, they greatly affect the shape of even closest antenna sidelobes.

That is why the extended source size correction factor approximation (27) was done just within the $s < 3$ ratio and that is why it makes little sense to increase the complexity of the *Corrective Term* polynomial far more than it is necessarily to achieve a few hundredth of dB tolerance for $s < 1.5$ values and a few tenth of dB tolerance for $s < 3$ values.

(b) *Gaussian Source Brightness Distribution*. It was found that for the extended radio source with the Gaussian source brightness distribution (11) and the circular antenna aperture the best approximation of the extended source size correction factor is achieved when the expression (7) for the K_{approx} is used in (27):

$$10\log_{10} K = 10\log_{10} \left[1 + x^2 \right] - \text{Corrective Term} \quad (29)$$

where the value of the *Corrective Term* can be found either from plots in Figures 9 and 10 or from formulae in Tables 5 and 6 and the value of s is restricted by $s < 3$, while the value of c is restricted by $c < 30$. In addition, the errors in calculation of the extended source correction factor with and without the *Corrective Term* are also shown in Tables 5 and 6. As is seen from these Tables, the application of the *Corrective Term* decreases the calculation error of the extended source size correction factor for the extended radio sources with the Gaussian source brightness distribution from 0.19dB – 0.29dB to 0.025dB – 0.045dB.

Fig. 7. Corrective Term for circular aperture and uniform source brightness distribution.

Plots legend for aperture illuminations: "Polynomial-on-Pedestal" (upper), Gaussian (mid) and Taylor (bottom) aperture illuminations;

Curves legend for aperture illumination tapers: 0dB (solid red), 4dB (long-dashed dark blue), 8dB (short-dashed green), 12dB (long-dashes-dotted light blue), 15dB (long-dashes-double-dotted yellow) and 30dB (long-dashes-triple-dotted purple);

Fig. 8. Corrective Term for circular aperture and uniform source brightness distribution.

Plots legend for aperture illuminations: "Polynomial-on-Pedestal" (upper), Gaussian (mid) and Taylor (bottom) aperture illuminations;

Curves legend for aperture illumination tapers: 0dB (solid red), 4dB (long-dashed dark blue), 8dB (short-dashed green), 12dB (long-dashes-dotted light blue), 15dB (long-dashes-double-dotted yellow) and 30dB (long-dashes-triple-dotted purple);

Fig. 9. *Corrective Term* for circular aperture and Gaussian source brightness distribution.

Plots legend for aperture illuminations: "Polynomial-on-Pedestal" (upper), Gaussian (mid) and Taylor (bottom) aperture illuminations;

Curves legend for aperture illumination tapers: 0dB (solid red), 4dB (long-dashed dark blue), 8dB (short-dashed green), 12dB (long-dashes-dotted light blue), 15dB (long-dashes-double-dotted yellow) and 30dB (long-dashes-triple-dotted purple);

Fig. 10. Corrective Term for circular aperture and Gaussian source brightness distribution.

Plots legend for aperture illuminations: "Polynomial-on-Pedestal" (upper), Gaussian (mid) and Taylor (bottom) aperture illuminations;

Curves legend for aperture illumination tapers: 0dB (solid red), 4dB (long-dashed dark blue), 8dB (short-dashed green), 12dB (long-dashes-dotted light blue), 15dB (long-dashes-double-dotted yellow) and 30dB (long-dashes-triple-dotted purple);

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Circular Aperture, $s \leq 1.5$, $c \leq 30$ dB Maximum Error without <i>Corrective Term</i> 0.038dB	rms max
Polynomial	$(0.00249s - 0.00516s^2 + 0.00197s^3) + b(-0.00559s - 0.01423s^2 + 0.01541s^3) + b^2(0.01411s - 0.07224s^2 + 0.04739s^3)$	0.0004 0.0017
Gaussian	$(-0.0115s + 0.0153s^2 - 0.005s^3) + b(0.0518s - 0.0504s^2 + 0.0064s^3) + b^2(-0.0318s - 0.0672s^2 + 0.0735s^3)$	0.0010 0.0051
Taylor	$(-0.0061s + 0.0079s^2 - 0.0027s^3) + b(0.0292s - 0.0401s^2 + 0.01294s^3) + b^2(-0.0155s - 0.0483s^2 + 0.0481s^3)$	0.00054 0.0027

Table 3. *Corrective Term* for Extended Radio Source with Uniform Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Circular Aperture, $1.5 \leq s \leq 3.0$, $c \leq 30$ dB Maximum Error without <i>Corrective Term</i> 0.32dB	rms max
Polynomial	$(-0.03761s + 0.03033s^2 - 0.00622s^3) + b(-0.0663s + 0.29041s^2 - 0.0705s^3) + b^2(-0.23992s + 0.20878s^2 - 0.03424s^3)$	0.0075 0.021
Gaussian	$(0.002s - 0.0007s^2 + 0.0018s^3) + b(-0.2597s + 0.0739s^2 - 0.0348s^3) + b^2(-0.5513s + 0.532s^2 - 0.0915s^3)$	0.017 0.050
Taylor	$(-0.0089s + 0.0082s^2 - 0.0009s^3) + b(-0.1152s + 0.1237s^2 - 0.0378s^3) + b^2(-0.3863s + 0.3728s^2 - 0.0675s^3)$	0.0093 0.028

Table 4. *Corrective Term* for Extended Radio Source with Uniform Brightness Distribution.

7.2 Rectangular Antenna Aperture Case

Based on the extensive numerical simulations of the extended source size correction factor using its definition (1) with all combinations of the extended source brightness distributions (10) and (11) and aperture illuminations (12) – (14) along X and Y axes for the rectangular antenna aperture, it was found that the most convenient way to approximate the exact value of the extended source size correction factor is to present it by the same formula (27) that was used in the circular aperture case. Indeed, the particular expressions for the K_{approx} and the *Corrective Term* for rectangular and circular apertures are different. Too many combinations of aperture illumination functions and its tapers along each X and Y axes for the rectangular aperture make it impractical to present the *Corrective Term* in the form of plots. Instead, for rectangular apertures, the *Corrective Term* is presented by formulae only.

(a) *Uniform Source Brightness Distribution*. It was found that for the extended radio source with the uniform source brightness distribution (10) and the rectangular antenna aperture the best approximation of the extended source size correction factor is achieved when the expressions (8) for the K_{approx} are used in (27) along X and Y axes of rectangular aperture:

$$10 \log_{10} K = 10 \log_{10} \left[\frac{(s_1 \sqrt{\ln 2})^2}{1 - \exp[-(s_1 \sqrt{\ln 2})^2]} \times \frac{(s_2 \sqrt{\ln 2})^2}{1 - \exp[-(s_2 \sqrt{\ln 2})^2]} \right] - \text{Corrective Term} \quad (30)$$

where the *Corrective Term* can be found from formulae in Tables 7 and 8 and values of s_1 and s_2 are restricted by $s_1, s_2 < 3$, while values of c_1 and c_2 are restricted by $c_1, c_2 < 30$. Indexes 1

and 2 correspond with X and Y axes in plane of the rectangular aperture. In addition, the calculation errors of the extended source correction factor with and without the *Corrective Term* are also shown in Tables 7 and 8.

As is seen from these Tables, the application of the *Corrective Term* decreases the calculation error of the extended source correction factor for the extended radio sources with the uniform source brightness distribution from 0.076dB – 0.34dB to 0.011dB – 0.072dB.

(b) *Gaussian Source Brightness Distribution*. It was found that for extended radio sources with the Gaussian source brightness distribution (11) and the rectangular antenna aperture the best approximation of the extended source size correction factor is achieved when the expressions (7) for the K_{approx} are used in (27) along X and Y axes of rectangular aperture:

$$10\log_{10} K = 10\log_{10} \left[\sqrt{(1 + s_1^2) \times (1 + s_2^2)} \right] - \text{Corrective Term} \quad (31)$$

where the value of the *Corrective Term* can be found from formulae in Tables 9 – 11 and values of s_1 and s_2 are restricted by $s_1, s_2 < 3$, while values of c_1 and c_2 are restricted by $c_1, c_2 < 30$. In addition, calculation errors of the extended source correction factor with and without the *Corrective Term* are also shown in Tables 9 – 11. As is seen from these Tables, the application of the *Corrective Term* decreases the calculation error of the extended source correction factor for the extended radio sources with the Gaussian source brightness distribution from 0.085dB – 2.18dB to 0.011dB – 0.40dB.

7.3 Extended Source Size Correction Factor Calculation Scheme

Here is a short, step by step summary of how to calculate the extended source size correction factor using the approach discussed in this chapter:

- Based on the given antenna aperture illumination taper(s) and using (16), find the value of variable c for circular or values of variables c_1 and c_2 for rectangular antennas.
- Based on the given antenna aperture illumination taper(s) and using (17), find the value of variable b for circular or values of variables b_1 and b_2 for rectangular antennas.
- Based on the given antenna aperture illumination and its taper(s), find the antenna HPBW multiplier a using the appropriate formula from Table 1 or plot from Fig. 5 for circular antenna aperture. For the rectangular antenna use Table 2 or plots from Fig. 6.
- Using (24) find the antenna HPBW for circular or two values of the antenna HPBW in the principal planes for rectangular antennas.
- Based on the given source brightness distribution and its angular size, and using (23), find the value of variable s for circular or values of variables s_1 and s_2 for rectangular antennas.
- Based on the given source brightness distribution and antenna aperture illumination, find the approximate value of the extended source size correction factor K using either formulae (28) or (29) for circular or formulae (30) or (31) for rectangular antennas. The *Corrective Term* in those formulae can be found from either plots in Figures 7 – 10 or Tables 3 – 7 for the circular aperture. For the rectangular aperture the *Corrective Term* can be found from Tables 7 – 11.

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Circular Aperture, $s \leq 1.5$, $c \leq 30$ dB Maximum Error without Corrective Term 0.19dB	rms max
Polynomial	$(0.1388s - 0.2485s^2 + 0.0433s^3) + b(0.0973s - 0.1767s^2 + 0.081s^3) + b^2(-0.0171s + 0.3195s^2 - 0.1144s^3)$	0.0051 0.016
Gaussian	$(0.0575s - 0.0822s^2 - 0.0297s^3) + b(0.4417s - 0.8687s^2 + 0.3551s^3) + b^2(-0.4637s + 0.9003s^2 - 0.3299s^3)$	0.0067 0.025
Taylor	$(0.0824s - 0.1285s^2 - 0.0117s^3) + b(0.3336s - 0.6783s^2 + 0.298s^3) + b^2(-0.353s + 0.7089s^2 - 0.283s^3)$	0.0059 0.020

Table 5. Corrective Term for Extended Radio Source with Gaussian Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Circular Aperture, $1.5 \leq s \leq 3.0$, $c \leq 30$ dB Maximum Error without Corrective Term 0.29dB	rms max
Polynomial	$(-0.1533s + 0.0001s^2 + 0.0097s^3) + b(0.1888s - 0.1559s^2 + 0.0186s^3) + b^2(-0.0528s + 0.1031s^2 - 0.0163s^3)$	0.0051 0.014
Gaussian	$(-0.1442s + 0.0038s^2 + 0.0086s^3) + b(0.0864s - 0.168s^2 + 0.0261s^3) + b^2(0.052s + 0.1223s^2 - 0.0256s^3)$	0.018 0.045
Taylor	$(-0.1528s + 0.0063s^2 + 0.008s^3) + b(-0.011s - 0.0322s^2 - 0.002s^3) + b^2(0.1657s - 0.0478s^2 + 0.0104s^3)$	0.0089 0.026

Table 6. Corrective Term for Extended Radio Source with Gaussian Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Rectangular Aperture, $s \leq 1.5$, $c \leq 30$ dB Maximum Error without Corrective Term 0.076dB	rms max
Polynomial	$0.0193(s_1 + s_2) + 0.0228(s_1^2 + s_2^2) - 0.0083(s_1 s_2) - 0.0271(s_1 s_2)^2$	0.0014
Polynomial	$-0.0254(s_1 b_1 + s_2 b_2) + 0.0143(s_1^2 b_1 + s_2^2 b_2) + 0.00013 s_1 s_2 (b_1 + b_2)$	0.0076
Polynomial	$0.0215(s_1 + s_2) + 0.0215(s_1^2 + s_2^2) - 0.0087(s_1 s_2) - 0.0269(s_1 s_2)^2$	0.0023
Gaussian	$-0.0297(s_1 b_1 + s_2 b_2) + 0.0167(s_1^2 b_1 + s_2^2 b_2)$	0.011
Polynomial	$0.02(s_1 + s_2) + 0.0223(s_1^2 + s_2^2) - 0.0075(s_1 s_2) - 0.0274(s_1 s_2)^2$	0.0016
Taylor	$-0.0253(s_1 b_1 + s_2 b_2) + 0.0143(s_1^2 b_1 + s_2^2 b_2)$	0.0077
Gaussian	$0.0239(s_1 + s_2) + 0.0202(s_1^2 + s_2^2) - 0.0092(s_1 s_2) - 0.0267(s_1 s_2)^2$	0.0027
Gaussian	$-0.034(s_1 b_1 + s_2 b_2) + 0.0192(s_1^2 b_1 + s_2^2 b_2)$	0.011
Gaussian	$0.0223(s_1 + s_2) + 0.021(s_1^2 + s_2^2) - 0.008(s_1 s_2) - 0.0271(s_1 s_2)^2$	0.0024
Taylor	$-0.0296(s_1 b_1 + s_2 b_2) + 0.0167(s_1^2 b_1 + s_2^2 b_2)$	0.012
Taylor	$0.02075(s_1 + s_2) + 0.02176(s_1^2 + s_2^2) - 0.00686(s_1 s_2) - 0.02762(s_1 s_2)^2$	0.0017
Taylor	$-0.02528(s_1 b_1 + s_2 b_2) + 0.0143(s_1^2 b_1 + s_2^2 b_2)$	0.0077

Table 7. Corrective Term for Extended Radio Source with Uniform Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB Rectangular Aperture, $s \leq 3.0$, $c \leq 30$ dB Maximum Error without <i>Corrective Term</i> 0.34dB	Error, dB rms max
Polynomial Polynomial	$-0.0284(s_1+s_2) + 0.0325(s_1^2+s_2^2) - 0.1447(s_1s_2) + 0.0097(s_1^3+s_2^3) - 0.0994(s_1s_2^2 + s_2s_1^2) - 0.0029(s_1^4+s_2^4) + 0.0183(s_1s_2^3 + s_2s_1^3) + 0.0041(s_1s_2)^2 + 0.0256(s_1b_1+s_2b_2) - 0.0059(s_1b_2+s_2b_1)$	0.011 0.049
Polynomial Gaussian	$-0.0429(s_1+s_2) - 0.07451(s_1^2+s_2^2) + 0.15911(s_1s_2) + 0.14348(s_1^3+s_2^3) - 0.10923(s_1s_2^2 + s_2s_1^2) - 0.03264(s_1^4 + s_2^4) + 0.02107(s_1s_2^3 + s_2s_1^3) + 0.00388(s_1s_2)^2 + 0.02566(s_1b_1+s_2b_2) - 0.00593(s_1b_2+s_2b_1)$	0.013 0.047
Polynomial Taylor	$-0.0141(s_1+s_2) + 0.0065(s_1^2+s_2^2) + 0.1502(s_1s_2) + 0.0224(s_1^3+s_2^3) - 0.1007(s_1s_2^2 + s_2s_1^2) - 0.0049(s_1^4 + s_2^4) + 0.0182(s_1s_2^3 + s_2s_1^3) + 0.0046(s_1s_2)^2 + 0.0256(s_1b_1+s_2b_2) - 0.0059(s_1b_2+s_2b_1)$	0.012 0.053
Gaussian Gaussian	$-0.026(s_1+s_2) + 0.027(s_1^2+s_2^2) + 0.143(s_1s_2) + 0.014(s_1^3+s_2^3) - 0.099(s_1s_2^2 + s_2s_1^2) - 0.004(s_1^4 + s_2^4) + 0.018(s_1s_2^3 + s_2s_1^3) + 0.004(s_1s_2)^2 - 0.026(s_1b_1+s_2b_2) + 0.006(s_1b_2+s_2b_1) - 0.014(s_1b_1^2+s_2b_2^2) + 0.06(s_1b_2^2+s_2b_1^2)$	0.016 0.072
Gaussian Taylor	$-0.02926(s_1+s_2) + 0.039(s_1^2+s_2^2) + 0.14537(s_1s_2) + 0.00598(s_1^3+s_2^3) - 0.1002(s_1s_2^2 + s_2s_1^2) - 0.0023(s_1^4 + s_2^4) + 0.01853(s_1s_2^3 + s_2s_1^3) + 0.00414(s_1s_2)^2 - 0.01783(s_1b_1+s_2b_2) + 0.0042(s_1b_2+s_2b_1) - 0.0107(s_1b_1^2+s_2b_2^2) + 0.04706(s_1b_2^2+s_2b_1^2) - 0.00019(s_1+s_2)b_1b_2$	0.015 0.064
Taylor Taylor	$-0.02138(s_1+s_2) + 0.02502(s_1^2+s_2^2) + 0.14732(s_1s_2) + 0.01115(s_1^3+s_2^3) - 0.10045(s_1s_2^2 + s_2s_1^2) - 0.00296(s_1^4 + s_2^4) + 0.01849(s_1s_2^3 + s_2s_1^3) + 0.00424(s_1s_2)^2 + 0.02554(s_1b_1+s_2b_2) - 0.00592(s_1b_2+s_2b_1)$	0.013 0.060

Table 8. *Corrective Term* for Extended Radio Source with Uniform Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB Rectangular Aperture, $s \leq 1.0$, $c \leq 30$ dB Maximum Error without <i>Corrective Term</i> 0.085dB	Error, dB rms max
Polynomial Polynomial	$0.0601(s_1+s_2) - 0.0371(s_1^2+s_2^2) - 0.0033(s_1s_2) + 0.0378(s_1s_2)^2 - 0.031(s_1b_1+s_2b_2) + 0.0315(s_1b_1^2+s_2b_2^2)$	0.0032 0.010
Polynomial Gaussian	$0.0597(s_1+s_2) - 0.037(s_1^2+s_2^2) + 0.003(s_1s_2) + 0.0356(s_1s_2)^2 - 0.0346(s_1b_1+s_2b_2) + 0.0369(s_1b_1^2+s_2b_2^2) - 0.0042s_1s_2(b_1+b_2)$	0.0034 0.010
Polynomial Taylor	$0.0433(s_1+s_2) - 0.0164(s_1^2+s_2^2) - 0.0028(s_1s_2) + 0.037(s_1s_2)^2 - 0.0016(s_1b_1+s_2b_2) - 0.0044(s_1b_1^2+s_2b_2^2)$	0.0036 0.011
Gaussian Gaussian	$0.066(s_1+s_2) - 0.0437(s_1^2+s_2^2) - 0.009(s_1s_2) + 0.0392(s_1s_2)^2 - 0.0334(s_1b_1+s_2b_2) + 0.044(s_1^2b_1+s_2^2b_2) + 0.0033s_1s_2(b_1+b_2) - 0.0115(s_1b_1^2+s_2b_2^2) - 0.0009(s_1 + s_2)b_1b_2$	0.0033 0.012
Gaussian Taylor	$0.0643(s_1+s_2) - 0.0417(s_1^2+s_2^2) - 0.0081(s_1s_2) + 0.0379(s_1s_2)^2 - 0.0291(s_1b_1+s_2b_2) + 0.0382(s_1^2b_1+s_2^2b_2) + 0.0032s_1s_2(b_1+b_2) - 0.0094(s_1b_1^2+s_2b_2^2) + 0.0001(s_1 + s_2)b_1b_2$	0.0034 0.011
Taylor Taylor	$0.044(s_1+s_2) - 0.0176(s_1^2+s_2^2) - 0.0024(s_1s_2) + 0.0363(s_1s_2)^2 + 0.0007(s_1b_1+s_2b_2) - 0.0065(s_1b_1^2+s_2b_2^2)$	0.0037 0.010

Table 9. *Corrective Term* for Extended Radio Source with Gaussian Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Rectangular Aperture, $s \leq 1.5$, $c \leq 30$ dB Maximum Error without Corrective Term 0.15dB	rms max
Polynomial Polynomial	$-0.0216(s_1+s_2) - 0.0578(s_1^2+s_2^2) - 0.0747(s_1s_2) + 0.148(s_1^3+s_2^3)$ $- 0.067(s_1^4+s_2^4) + 0.0306(s_1s_2^3 + s_2s_1^3) + 0.0502(s_1s_2)^2$ $- 0.0033(s_1b_1+s_2b_2) - 0.0016(s_1b_2+s_2b_1)$	0.0033 0.018
Polynomial Gaussian	$0.0187(s_1+s_2) + 0.0553(s_1^2+s_2^2) - 0.0558(s_1s_2) + 0.0226(s_1^3+s_2^3)$ $- 0.0144(s_1s_2^2 + s_2s_1^2) - 0.0701(s_1^4 + s_2^4) + 0.0336(s_1s_2^3 + s_2s_1^3)$ $+ 0.0551(s_1s_2)^2 - 0.0021(s_1b_1+s_2b_2) - 0.0024(s_1b_2+s_2b_1)$	0.0038 0.019
Polynomial Taylor	$0.0217(s_1+s_2) + 0.0631(s_1^2+s_2^2) - 0.0782(s_1s_2) + 0.0071(s_1^3+s_2^3)$ $+ 0.0016(s_1s_2^2 + s_2s_1^2) - 0.0642(s_1^4 + s_2^4) + 0.0309(s_1s_2^3 + s_2s_1^3)$ $+ 0.0492(s_1s_2)^2 - 0.0036(s_1b_1+s_2b_2) - 0.0015(s_1b_2+s_2b_1)$	0.0034 0.017
Gaussian Gaussian	$0.017(s_1+s_2) + 0.063(s_1^2+s_2^2) - 0.07(s_1s_2) + 0.012(s_1^3+s_2^3)$ $- 0.003(s_1s_2^2 + s_2s_1^2) - 0.066(s_1^4 + s_2^4) + 0.031(s_1s_2^3 + s_2s_1^3)$ $+ 0.051(s_1s_2)^2 - 0.001(s_1b_1+s_2b_2) - 0.003(s_1b_2+s_2b_1)$	0.0042 0.018
Gaussian Taylor	$0.0188(s_1+s_2) + 0.0499(s_1^2+s_2^2) - 0.037(s_1s_2) + 0.0311(s_1^3+s_2^3)$ $- 0.0302(s_1s_2^2 + s_2s_1^2) - 0.0735(s_1^4 + s_2^4) + 0.0379(s_1s_2^3 + s_2s_1^3)$ $+ 0.06(s_1s_2)^2 - 0.0024(s_1b_1+s_2b_2) - 0.0023(s_1b_2+s_2b_1)$	0.0038 0.021
Taylor Taylor	$0.0179(s_1+s_2) + 0.0691(s_1^2+s_2^2) - 0.0535(s_1s_2) + 0.0055(s_1^3+s_2^3)$ $- 0.0198(s_1s_2^2 + s_2s_1^2) - 0.0647(s_1^4 + s_2^4) + 0.037(s_1s_2^3 + s_2s_1^3)$ $+ 0.0553(s_1s_2)^2 - 0.0038(s_1b_1+s_2b_2) - 0.0013(s_1b_2+s_2b_1)$	0.0034 0.019

Table 10. Corrective Term for Extended Radio Source with Gaussian Brightness Distribution.

Antenna Aperture Illumination	Corrective Term, dB	Error, dB
	Rectangular Aperture, $s \leq 3.0$, $c \leq 30$ dB Maximum Error without Corrective Term 2.18dB	rms max
Polynomial Polynomial	$-0.0714(s_1+s_2) + 0.4084(s_1^2+s_2^2) - 0.4646(s_1s_2) - 0.4509(s_1^3+s_2^3)$ $+ 0.375(s_1s_2^2 + s_2s_1^2) + 0.1035(s_1^4 + s_2^4) - 0.088(s_1s_2^3 + s_2s_1^3)$ $- 0.03091(s_1s_2)^2 + 0.0369(s_1b_1+s_2b_2)$	0.044 0.27
Polynomial Gaussian	$-0.073(s_1+s_2) + 0.412(s_1^2+s_2^2) - 0.452(s_1s_2) - 0.45(s_1^3+s_2^3) + 0.369(s_1s_2^2 + s_2s_1^2)$ $+ 0.103(s_1^4 + s_2^4) - 0.087(s_1s_2^3 + s_2s_1^3) - 0.038(s_1s_2)^2 - 0.022(s_1b_1+s_2b_2)$ $+ 0.009(s_1b_2+s_2b_1) - 0.018(s_1b_1^2+s_2b_2^2) + 0.072(s_1b_2^2+s_2b_1^2) - 0.0008(s_1+s_2)b_1b_2$	0.048 0.36
Polynomial Taylor	$-0.0473(s_1+s_2) + 0.0121(s_1^2+s_2^2) + 0.3153(s_1s_2) - 0.1709(s_1^3+s_2^3)$ $+ 0.0852(s_1s_2^2 + s_2s_1^2) + 0.0527(s_1^4 + s_2^4) - 0.062(s_1s_2^3 + s_2s_1^3) + 0.0181(s_1s_2)^2$ $+ 0.0134(s_1b_1+s_2b_2) - 0.0133(s_1b_2+s_2b_1) + 0.0056(s_1b_1^2+s_2b_2^2) + 0.0295(s_1b_2^2+s_2b_1^2) - 0.0014(s_1+s_2)b_1b_2$	0.043 0.30
Gaussian Gaussian	$-0.0175(s_1+s_2) - 0.0526(s_1^2+s_2^2) + 0.2655(s_1s_2) - 0.1384(s_1^3+s_2^3)$ $+ 0.1023(s_1s_2^2 + s_2s_1^2) + 0.0475(s_1^4 + s_2^4) - 0.0634(s_1s_2^3 + s_2s_1^3)$ $+ 0.0153(s_1s_2)^2 + 0.0546(s_1b_1+s_2b_2)$	0.077 0.40
Gaussian Taylor	$-0.0193(s_1+s_2) - 0.0554(s_1^2+s_2^2) + 0.211(s_1s_2) - 0.1505(s_1^3+s_2^3)$ $+ 0.1306(s_1s_2^2 + s_2s_1^2) + 0.0512(s_1^4 + s_2^4) - 0.0694(s_1s_2^3 + s_2s_1^3)$ $+ 0.0135(s_1s_2)^2 - 0.0427(s_1b_1+s_2b_2) + 0.0142(s_1b_2+s_2b_1)$ $- 0.0223(s_1b_1^2+s_2b_2^2) + 0.093(s_1b_2^2+s_2b_1^2) - 0.0018(s_1+s_2)b_1b_2$	0.071 0.34
Taylor Taylor	$-0.0156(s_1+s_2) + 0.2169(s_1^2+s_2^2) + 0.2902(s_1s_2) - 0.2838(s_1^3+s_2^3)$ $+ 0.0916(s_1s_2^2 + s_2s_1^2) + 0.0716(s_1^4 + s_2^4) - 0.0621(s_1s_2^3 + s_2s_1^3)$ $+ 0.0164(s_1s_2)^2 + 0.037(s_1b_1+s_2b_2)$	0.071 0.25

Table 11. Corrective Term for Extended Radio Source with Gaussian Brightness Distribution.

8. Conclusion

The calculation of the extended source size correction factor that is widely utilized in antenna gain measurements of electrically large antennas using the cosmic radio sources whose size is comparable or even larger than the antenna beamwidth was considered.

The analytical expressions for the extended source size correction factor developed in the literature were examined and their areas of applicability and associated errors were investigated for uniform and Gaussian radio source brightness distributions, "Polynomial-on-Pedestal," Gaussian and Taylor antenna aperture illuminations with up to 30dB aperture edge illumination taper(s) and for circular and rectangular antenna apertures.

Accurate analytical expressions for the extended source size correction factor for all of the above combinations of source brightness distributions, aperture illuminations and antenna shapes along with their tolerances were found for the case when the extended cosmic radio source size or its beamwidth is up to 3 times bigger than the antenna beamwidth. In addition, accurate analytical expressions for the antenna beamwidth for same combinations of source brightness distributions, aperture illuminations and antenna shapes along with their tolerances were also found.

Attained results eliminate the need to perform complicated and often impractical numerical integrations in case of the particular measurement. The approximate analytical expressions of the extended source size correction factor for rectangular antenna aperture and accurate analytical expressions for the antenna beamwidth as well as the assessment of their tolerances are obtained for the first time in literature.

9. References

- Baars, J. W. M. (1973). The measurement of large antennas with cosmic radio sources. *IEEE Transactions on Antennas and Propagation*, Vol. AP-21, No. 4, (July 1973) pp. (461-474), ISSN 0018-926X
- Balanis, C. A. (2005). *Antenna Theory*. John Wiley & Sons, ISBN 0-471-66782-X, Hoboken, New Jersey
- Guidici, D.A. & Castelli, J.P. (1971). The use of extraterrestrial radio sources in the measurement of antenna parameters. *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-7, No. 2, (March 1971) pp. (226-234), ISSN 0018-9251
- Johnson, R. C. at all. (1993). *Antenna Engineering Handbook*. McGraw-Hill, ISBN 0-07-032381-X, New York
- Ko, H. C. (1961). On the determination of the disk temperature and the flux density of a radio source using a high-gain antenna. *IRE Transactions on Antennas and Propagation*, Vol. AP-9, No. 5, (September 1961) pp. (500-501), ISSN 0096-1973
- Kraus, J. D. (1986). *Radio Astronomy*, Cygnus-Quasar Books, ISBN 1-882484-00-2, Powell, Ohio
- Kummer, W. H. at al. (1979). *IEEE Standard Test Procedures for Antennas*, John Wiley & Sons, ISBN 0-471-08032-2, New York.
- Solovey, A. & Mittra, R. (2008). Extended source size correction factor in antenna gain measurements, *Proceedings of the 38th European Microwave Conference*, pp. 983-986, (available online only), Amsterdam, October 2008.

**Advanced Microwave and Millimeter Wave Technologies
Semiconductor Devices Circuits and Systems**

Edited by Moumita Mukherjee

ISBN 978-953-307-031-5

Hard cover, 642 pages

Publisher InTech

Published online 01, March, 2010

Published in print edition March, 2010

This book is planned to publish with an objective to provide a state-of-the-art reference book in the areas of advanced microwave, MM-Wave and THz devices, antennas and system technologies for microwave communication engineers, Scientists and post-graduate students of electrical and electronics engineering, applied physicists. This reference book is a collection of 30 Chapters characterized in 3 parts: Advanced Microwave and MM-wave devices, integrated microwave and MM-wave circuits and Antennas and advanced microwave computer techniques, focusing on simulation, theories and applications. This book provides a comprehensive overview of the components and devices used in microwave and MM-Wave circuits, including microwave transmission lines, resonators, filters, ferrite devices, solid state devices, transistor oscillators and amplifiers, directional couplers, microstripeline components, microwave detectors, mixers, converters and harmonic generators, and microwave solid-state switches, phase shifters and attenuators. Several applications area also discusses here, like consumer, industrial, biomedical, and chemical applications of microwave technology. It also covers microwave instrumentation and measurement, thermodynamics, and applications in navigation and radio communication.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Aleksey Solovey and Raj Mitra (2010). Extended Source Size Correction Factor in Antenna Gain Measurements, Advanced Microwave and Millimeter Wave Technologies Semiconductor Devices Circuits and Systems, Moumita Mukherjee (Ed.), ISBN: 978-953-307-031-5, InTech, Available from:
<http://www.intechopen.com/books/advanced-microwave-and-millimeter-wave-technologies-semiconductor-devices-circuits-and-systems/extended-source-size-correction-factor-in-antenna-gain-measurements>

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