
Enhanced Principles in Design of Adaptive Fault Observers

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Additional information is available at the end of the chapter

<http://dx.doi.org/0.5772/67133>

Abstract

In this chapter, modified techniques for fault estimation in linear dynamic systems are proposed, which give the possibility to simultaneously estimate the system state as well as slowly varying faults. Using the continuous-time adaptive observer form, the considered faults are assumed to be additive, thereby the principles can be applied for a broader class of fault signals. Enhanced algorithms using H_∞ approach are provided to verify stability of the observers, giving algorithms with improved performance of fault estimation. Exploiting the procedure for transforming the model with additive faults into an extended form, the proposed technique allows to obtain fault estimates that can be used for fault compensation in the fault tolerant control scheme. Analyzing the ambit of performances given on the mixed H_2/H_∞ design of the fault tolerant control, the joint design conditions are formulated as a minimization problem subject to convex constraints expressed by a system of linear matrix inequalities. Applied enhanced design conditions increase estimation rapidity also in noise environment and formulate a general framework for fault estimation using augmented or adaptive observer structures and active fault tolerant control in linear dynamic systems.

Keywords: linear dynamic systems, additive fault estimation, fault tolerant control design, enhanced bounded real lemma, linear matrix inequalities, H_∞ norm, H_2/H_∞ control strategy

1. Introduction

A model-based fault tolerant control (FTC) can be realized as control-laws set dependent, exploiting fault detection and isolation decision to reconfigure the control structure or as fault estimation dependent, preferring fault compensation within robust control framework. While integration of FTC with the fault localization decision technique requires a selection of optimal residual thresholds as well as a robust and stable reconfiguration mechanism [1], the fault

estimation-dependent FTC structures eliminate a threshold subjectivism and integrate FTC and estimation problems into one robust optimization task [2]. The realization is conditioned by observers, which performs the state reconstruction from the available signals.

The approach, in which faults estimates are used in a control structure to compensate the effects of acting faults, is adopted in modern FTC techniques [3, 4]. FTC with fault estimation for linear systems subject to bounded actuator or sensor faults, are proposed in [5]. The observer structures are in the Luenberger form [6] or realized as unknown input fault observers [7]. To guarantee the desired time response, a linear matrix inequality (LMI) based regional pole placement design strategy is proposed in [8] but such formulation introduces additive LMIs, which increase conservatism of the solutions. To minimize the set of LMIs of the circle regional pole placement is used; a modified approach in LMI construction is proposed in Ref. [9].

To estimate the actuator faults for the linear time-invariant systems without external disturbance the principles based on adaptive observers are frequently used, which make the estimation of the actuator faults by integrating the system output errors [10]. First introduced in Ref. [11], this principle was applied also for descriptor systems [7], linear systems with time delays [12], system with nonlinear dynamics [13], and a class of nonlinear systems described by Takagi-Sugeno fuzzy models [14, 15]. Some generalizations can be found in [16].

The H_2 -norm is one of the most important characteristics of linear time-invariant control systems and so the problems concerning H_2 , as well as H_∞ , control have been studied by many authors (see, e.g. [17–20] and the references therein). Adding H_2 objective to H_∞ control design, a mixed H_2/H_∞ control problem was formulated in Ref. [21], with the goal to minimize H_2 norm subject to the constraint on H_∞ norm of the system transfer function. Such integrated design strategy corresponds to the optimization of the design parameters to satisfy desired specifications and to optimize the performance of the closed-loop system. Because of the importance of the control systems with these properties, considerable attention was dedicated to mixed H_2/H_∞ closed-loop performance criterion in design [22, 23] as well as to formulate the LMI-based computational technique [24, 25] to solve them or to exploit multiobjective algorithms for nonlinear, nonsmooth optimization in this design task [26, 27].

To guarantee suitable dynamics, new LMI conditions are proposed in the chapter for designing the fault observers as well as FTCs. Comparing with Ref. [5], the extended approach to the \mathcal{D} -stability introduced in Ref. [28] is used to minimize the number of LMIs in mixed H_2/H_∞ formulation of the FTC design and the eigenvalue circle clustering in fault observer design. In addition, different from Ref. [29], PD fault observer terms are comprehended through the enhanced descriptor approach [30], and a new design criterion is constructed in terms of LMIs. Since extended Lyapunov functions are exploited, the proposed approach offers the same degree of conservatism as the standard formulations [2, 31] but the H_∞ conditions are regularized under acting of H_2 constraint. Over and above, the \mathcal{D} -stability approach supports adjusting the fault estimator characteristics according to the fault frequency band.

The content and scope of the chapter are as follows. Placed after the introduction presented in Section 1, the basic preliminaries are given in Section 2. Section 3 reviews the definition and results concerning the adaptive fault observer design for continuous-time linear systems, Section 4 details the observer dynamic analysis and derives new results when using

the \mathcal{D} -stability circle criterion and Section 5 recasts the extended design conditions in the framework of LMIs based on structured matrix parameters. Then, in response to fault compensation principle for such type of fault observers, Section 6 derives the design conditions for the fault tolerant control structures, reflecting the joined H_2/H_∞ control idea. The relevance of the proposed approach is illustrated by a numerical example in Section 7 and Section 8 draws some concluding remarks.

2. Basic preliminaries

In order to analyze whether a linear MIMO system is stable under defined quadratic constraints, the basic properties can be summarized by the following LMI forms.

Considering linear MIMO systems

$$\dot{q}(t) = Aq(t) + Bu(t) + Dd(t) \tag{1}$$

$$y(t) = Cq(t) \tag{2}$$

where $q(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, and $y(t) \in \mathbb{R}^m$ are vectors of the system state, input, and output variables, respectively, $d(t) \in \mathbb{R}^w$ is the unknown disturbance vector, $A \in \mathbb{R}^{n \times n}$ is the system dynamic matrix, $D \in \mathbb{R}^{n \times w}$ is the disturbance input matrix, and $B \in \mathbb{R}^{n \times r}$, $C \in \mathbb{R}^{m \times n}$ are the system input and output matrices, then the system transfer functions matrices are

$$G(s) = C(sI_n - A)^{-1}B, \quad G_d(s) = C(sI_n - A)^{-1}D \tag{3}$$

where $I_n \in \mathbb{R}^{n \times n}$ is an unitary matrix and the complex number s is the transform variable (Laplace variable) of the Laplace transform [32].

To characterize the system properties the following lemmas can be used.

Lemma 1 (Lyapunov inequality) [33] *The matrix A is Hurwitz if there exists a symmetric positive definite matrix $T \in \mathbb{R}^{n \times n}$ such that*

$$T = T^T > 0, \quad A^T T + TA < 0 \tag{4}$$

Lemma 2 [34] *The matrix A is Hurwitz and $\|G(s)\|_2 < \gamma_2$ if there exists a symmetric positive definite matrix $V \in \mathbb{R}^{n \times n}$ and a positive scalar $\gamma_2 \in \mathbb{R}$, such that*

$$V = V^T > 0 \tag{5}$$

$$AV + VA^T + BB^T < 0 \tag{6}$$

$$\text{tr}(CVC^T) < \gamma_2^2 \tag{7}$$

where $\gamma_2 > 0$, $\gamma_2 \in \mathbb{R}$ is H_2 norm of the transfer function matrix $G(s)$.

Lemma 3 (Bounded real lemma) [35] *The matrix A is Hurwitz and $\|G_d(s)\|_\infty < \gamma_\infty$ if there exists a symmetric positive definite matrix $U \in \mathbb{R}^{n \times n}$ and a positive scalar $\gamma_\infty \in \mathbb{R}$ such that*

$$U = U^T > 0 \quad (8)$$

$$\begin{bmatrix} UA + A^T U & * & * \\ \mathbf{D}^T U & -\gamma_\infty \mathbf{I}_w & * \\ \mathbf{C} & \mathbf{0} & -\gamma_\infty \mathbf{I}_m \end{bmatrix} < 0, \quad (9)$$

where $\mathbf{I}_w \in \mathbb{R}^{w \times w}$, $\mathbf{I}_m \in \mathbb{R}^{m \times m}$ are identity matrices and $\gamma_\infty > 0$, $\gamma_\infty \in \mathbb{R}$ is H_∞ norm of the disturbance transfer function matrix $\mathbf{G}_d(s)$.

Hereafter, * denotes the symmetric item in a symmetric matrix.

Lemma 4 [28] *The matrix \mathbf{A} is \mathcal{D} -stable Hurwitz if for given positive scalars $a, \rho \in \mathbb{R}$, $a > \rho$, there exists a symmetric positive definite matrix $\mathbf{T} \in \mathbb{R}^{n \times n}$ such that*

$$\mathbf{T} = \mathbf{T}^T > 0, \quad (10)$$

$$\begin{bmatrix} -\rho \mathbf{T} & * \\ \mathbf{T}\mathbf{A} + a\mathbf{T} & -\rho \mathbf{T} \end{bmatrix} < 0, \quad (11)$$

while the eigenvalues of \mathbf{A} are clustered in the circle with the origin $c_0 = (-a + 0i)$ and radius ρ within the complex plane S .

Lemma 5 (Schur complement) [36] *Let \mathbf{O} be a real matrix, and $\mathbf{N}(\mathbf{M})$ be a positive definite symmetric matrix of appropriate dimension, then the following inequalities are equivalent*

$$\begin{bmatrix} \mathbf{M} & \mathbf{O} \\ \mathbf{O}^T & -\mathbf{N} \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} \mathbf{M} + \mathbf{O}\mathbf{N}^{-1}\mathbf{O}^T & \mathbf{0} \\ \mathbf{0} & -\mathbf{N} \end{bmatrix} < 0 \Leftrightarrow \mathbf{M} + \mathbf{O}\mathbf{N}^{-1}\mathbf{O}^T < 0, \quad \mathbf{N} > 0, \quad (12)$$

$$\begin{bmatrix} -\mathbf{M} & \mathbf{O} \\ \mathbf{O}^T & \mathbf{N} \end{bmatrix} < 0 \Leftrightarrow \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{N} + \mathbf{O}^T\mathbf{M}^{-1}\mathbf{O} \end{bmatrix} < 0 \Leftrightarrow \mathbf{N} + \mathbf{O}^T\mathbf{M}^{-1}\mathbf{O} < 0, \quad \mathbf{M} > 0. \quad (13)$$

Lemma 6 (Krasovskii lemma) [37] *The autonomous system (1) is asymptotically stable if for a given symmetric positive semidefinite matrix $\mathbf{L} \in \mathbb{R}^{n \times n}$ there exists a symmetric positive definite matrix $\mathbf{T} \in \mathbb{R}^{n \times n}$ such that*

$$\mathbf{T} = \mathbf{T}^T > 0, \quad (14)$$

$$\mathbf{A}^T \mathbf{T} + \mathbf{T}\mathbf{A} + \mathbf{L} < 0, \quad (15)$$

where \mathbf{L} is the weight matrix of an integral quadratic constraint interposed on the state vector $\mathbf{q}(t)$.

3. Proportional adaptive fault observers

To characterize the role of constraints in the proposed methodology and ease of understanding the presented approach, the theorems' proofs are restated in a condensed form in this section and also for theorems already being presented by the authors, e.g., in Refs. [38–40].

Despite different definitions, the best description for the formulation of the problem is based on the common state-space description of the linear dynamic multiinput, multioutput (MIMO) systems in the presence of unknown faults of the form

$$\dot{q}(t) = Aq(t) + Bu(t) + Ff(t), \tag{16}$$

$$y(t) = Cq(t), \tag{17}$$

where $q(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, and $y(t) \in \mathbb{R}^m$ are vectors of the system, input, and output variables, respectively, $f(t) \in \mathbb{R}^p$ is the unknown fault vector, $A \in \mathbb{R}^{n \times n}$ is the system dynamics matrix, $F \in \mathbb{R}^{n \times p}$ is the fault input matrix, and $B \in \mathbb{R}^{n \times r}$ and $C \in \mathbb{R}^{m \times n}$ are the system input and output matrices, $m, r, p < n$,

$$\text{rank} \begin{bmatrix} A & F \\ C & 0 \end{bmatrix} = n + p, \tag{18}$$

and the couple (A, C) is observable.

Limiting to the time-invariant system (16) and (17) to estimate the faults and the system states simultaneously, as well as focusing on slowly varying additive faults, the adaptive fault observer is considered in the following form [41]

$$\dot{q}_e(t) = Aq_e(t) + Bu(t) + Ff_e(t) + J(y(t) - y_e(t)), \tag{19}$$

$$y_e(t) = Cq_e(t), \tag{20}$$

where $q_e(t) \in \mathbb{R}^n$, $y_e(t) \in \mathbb{R}^m$, and $f_e(t) \in \mathbb{R}^p$ are estimates of the system states vector, the output variables vector, and the fault vector, respectively, and $J \in \mathbb{R}^{n \times m}$ is the observer gain matrix.

The observer (19) and (20) is combined with the fault estimation updating law of the form [42]

$$\dot{f}_e(t) = GH^T e_y(t), \quad e_y(t) = y(t) - y_e(t) = Ce_q(t), \quad e_q(t) = q(t) - q_e(t), \tag{21}$$

where $H \in \mathbb{R}^{m \times p}$ is the gain matrix and $G = G^T > 0$, $G \in \mathbb{R}^{p \times p}$ is a learning weight matrix that has to be set interactively in the design step.

In order to express unexpectedly changing faults as a function of the system and observer outputs and to apply the adaptive estimation principle, it is considered that the fault vector is piecewise constant, differentiable, and bounded, i.e., $\|f(t)\| \leq f_{\max} < \infty$, the upper bound norm f_{\max} is known, and the value of $f(t)$ is set to zero vector until a fault occurs. This assumption, in general, implies that the time derivative of $e_f(t)$ can be considered as

$$\dot{f}_e(t) \approx 0, \quad \dot{e}_f(t) = -\dot{f}_e(t), \quad e_f(t) = f(t) - f_e(t). \tag{22}$$

These assumptions have to be taking into account by designing the matrix parameters of the observers to ensure asymptotic convergence of the estimation errors, Eqs. (21) and (22). The task is to design the matrix J in such a way that the observer dynamics matrix $A_e = A - JC$ is stable and $f_e(t)$ approximates a slowly varying actuator fault $f(t)$.

3.1. Design conditions

If single faults influence the system through different input vectors (columns of the matrix F), it is possible to avoid designing the estimators with the tuning matrix parameter $G > 0$ and formulate the design task through the set of LMIs and a linear matrix equality.

Theorem 1 *The adaptive fault observer (19) and (20) is stable if there exists a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ and matrices $H \in \mathbb{R}^{n \times p}$, $Y \in \mathbb{R}^{n \times m}$ such that*

$$P = P^T > 0, \quad (23)$$

$$PA + A^T P - YC - C^T Y^T < 0, \quad (24)$$

$$PF = C^T H. \quad (25)$$

When the above conditions hold, the observer gain matrix is given by

$$J = P^{-1} Y \quad (26)$$

and the adaptive fault estimation algorithm is

$$\dot{f}_e(t) = GH^T C e_q(t), \quad (27)$$

where

$$e_q(t) = q(t) - q_e(t) \quad (28)$$

and $G \in \mathbb{R}^{p \times p}$ is a symmetric positive definite matrix which values are set interactive in design.

Proof. From the system models (16) and (17) and the observer models (19) and (20), it can be obtained that

$$\begin{aligned} \dot{e}_q(t) &= Aq(t) + Bu(t) + Ff(t) - Aq_e(t) - Bu(t) - Ff_e(t) - J(y(t) - y_e(t)) = \\ &= (A - JC)e_q(t) + Fe_f(t) = A_e e_q(t) + Fe_f(t), \end{aligned} \quad (29)$$

where the observer system matrix is

$$A_e = A - JC. \quad (30)$$

Since $e_q(t)$ is linear with respect to the system parameters, it is possible to consider the Lyapunov function candidate in the following form

$$v(e_q(t)) = e_q^T(t) P e_q(t) + e_f^T(t) G^{-1} e_f(t) > 0, \quad (31)$$

where P , G are real, symmetric, and positive definite matrices. Then, the time derivative of $v(e_q(t))$ is

$$\dot{v}(e_q(t)) = \dot{v}_0(e_q(t)) + \dot{v}_1(e_q(t)) < 0, \quad (32)$$

where

$$\begin{aligned} \dot{v}_0(e_q(t)) &= \dot{e}_q^T(t) P e_q(t) + e_q^T(t) P \dot{e}_q(t) = \\ &= (A_e e_q(t) + F e_f(t))^T P e_q(t) + e_q^T(t) P (A_e e_q(t) + F e_f(t)) = \\ &= e_q^T(t) (A_e^T P + P A_e) e_q(t) + e_q^T(t) P F e_f(t) + e_f^T(t) F^T P e_q(t), \end{aligned} \quad (33)$$

$$\dot{v}_1(e_q(t)) = \dot{e}_f^T(t) G^{-1} e_f(t) + e_f^T(t) G^{-1} \dot{e}_f(t) = -f_e^T(t) G^{-1} e_f(t) - e_f^T(t) G^{-1} \dot{f}_e(t). \quad (34)$$

Inserting Eq. (21) into Eq. (34) leads to

$$\begin{aligned} \dot{v}_1(e_q(t)) &= -e_q^T(t) C^T H G G^{-1} e_f(t) - e_f^T(t) G^{-1} G H^T C e_q(t) \\ &= -e_q^T(t) C^T H e_f(t) - e_f^T(t) H^T C e_q(t) \end{aligned} \quad (35)$$

and substituting Eq. (35) with Eq. (30) into Eq. (33), the following inequality is obtained

$$\begin{aligned} \dot{v}(e_q(t)) &= e_q^T(t) \left((A - J C)^T P + P (A - J C) \right) e_q(t) \\ &+ e_q^T(t) (P F - C^T H) e_f(t) + e_f^T(t) (F^T P - H^T C) e_q(t) < 0. \end{aligned} \quad (36)$$

It is clear that the requirement

$$e_q^T(t) (P F - C^T H) e_f(t) + e_f^T(t) (F^T P - H^T C) e_q(t) = 0 \quad (37)$$

can be satisfied when Eq. (25) is satisfied.

Using the above given condition (37), the resulting formula for $\dot{v}(e_q(t))$ takes the form

$$\dot{v}(e_q(t)) = e_q^T(t) \left((A - J C)^T P + P (A - J C) \right) e_q(t) < 0, \quad (38)$$

and the LMI, defining the observer stability condition, is presented as

$$P(A - J C) + (A - J C)^T P < 0. \quad (39)$$

Introducing the notation

$$P J = Y \quad (40)$$

it is possible to express Eq. (39) as Eq. (24). This concludes the proof.

3.2. Enhanced design conditions

The observer stability analysis could be carried out generally under the assumption (29), i.e., using the forced differential equation of the form

$$\dot{e}_q(t) = (A-JC)e_q(t) + Fe_f(t), \quad (41)$$

$$e_y(t) = Ce_q(t), \quad (42)$$

while

$$G_f(s) = C(A-JC)^{-1}F. \quad (43)$$

It is evident now that $e_f(t)$ acts on the state error dynamics as an unknown disturbance and, evidently, this differential equation is so not autonomous after a fault occurrence. Reflecting this fact, the enhanced approach is proposed to decouple Lyapunov matrix P from the system matrices A , C by introducing a slack matrix Q in the observer stability condition, as well as to decouple the tuning parameter δ from the matrix G in the learning rate setting and using δ to tune the observer dynamic properties. Since the design principle for unknown input observer cannot be used, the impact of faults on observer dynamics is moreover minimized with respect to the H_∞ norm of the transfer functions matrix of $G_f(s)$, while a reduction in the fault amplitude estimate is easily countervailing using the matrix G . In this sense the enhanced design conditions can be formulated in the following way.

Theorem 2 *The adaptive fault observer (19) and (20) is stable if for a given positive $\delta \in \mathbb{R}$ there exist symmetric positive definite matrices $P \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{n \times n}$, matrices $H \in \mathbb{R}^{n \times p}$, $Y \in \mathbb{R}^{n \times m}$ and a positive scalar $\gamma \in \mathbb{R}$ such that*

$$P = P^T > 0, \quad Q = Q^T > 0, \quad \gamma > 0, \quad (44)$$

$$\begin{bmatrix} QA + A^T Q - YC - C^T Y^T & * & * & * \\ P - Q + \delta QA - \delta YC & -2\delta Q & * & * \\ 0 & \delta F^T Q & -\gamma I_p & * \\ C & 0 & 0 & -\gamma I_m \end{bmatrix} < 0, \quad (45)$$

$$QF = C^T H. \quad (46)$$

When the above conditions are affirmative the estimator gain matrix is given by the relation

$$J = Q^{-1}Y. \quad (47)$$

Proof. Using Krasovskii lemma, the Lyapunov function candidate can be considered as

$$v(e_q(t)) = e_q^T(t)Pe_q(t) + e_f^T(t)G^{-1}e_f(t) + \gamma^{-1} \int_0^t (e_y^T(r)e_y(r) - \gamma^2 e_f^T(r)e_f(r))dr > 0, \quad (48)$$

where $P = P^T > 0$, $G = G^T > 0$, $\gamma > 0$, and γ is an upper bound of H_∞ norm of the transfer function matrix $G_f(s)$. Then the time derivative of $v(e_q(t))$ has to be negative, i.e.,

$$\begin{aligned} \dot{v}(e_q(t)) &= \dot{e}_q^T(t)Pe_q(t) + e_q^T(t)P\dot{e}_q(t) + \dot{e}_f^T(t)G^{-1}e_f(t) + e_f^T(t)G^{-1}\dot{e}_f(t) \\ &\quad + \gamma^{-1}e_y^T(t)e_y(t) - \gamma e_f^T(t)e_f(t) < 0. \end{aligned} \quad (49)$$

If it is assumed that Eqs. (34) and (35) hold, then the substitution of Eq. (35) into Eq. (49) leads to

$$\begin{aligned} \dot{v}(e_q(t)) &= \dot{e}_q^T(t)Pe_q(t) + e_q^T(t)P\dot{e}_q(t) - e_q^T(t)C^THe_f(t) - e_f^T(t)H^T Ce_q(t) \\ &\quad + \gamma^{-1}e_y^T(t)e_y(t) - \gamma e_f^T(t)e_f(t) < 0. \end{aligned} \quad (50)$$

Since Eq. (41) implies

$$(A-JC)e_q(t) + Fe_f(t) - \dot{e}_q(t) = 0, \quad (51)$$

it is possible to define the following condition based on the equality (51)

$$(e_q^T(t)Q + \dot{e}_q^T(t)\delta Q)((A-JC)e_q(t) + Fe_f(t) - \dot{e}_q(t)) = 0, \quad (52)$$

where $Q \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix and $\delta \in \mathbb{R}$ is a positive scalar.

Then, adding Eq. (52) and its transposition to Eq. (50), the following has to be satisfied

$$\begin{aligned} \dot{v}(e_q(t)) &= \dot{e}_q^T(t)Pe_q(t) + e_q^T(t)P\dot{e}_q(t) - e_q^T(t)C^THe_f(t) - e_f^T(t)H^T Ce_q(t) \\ &\quad + (e_q^T(t)Q + \dot{e}_q^T(t)\delta Q)((A-JC)e_q(t) - \dot{e}_q(t)) + \gamma^{-1}e_y^T(t)e_y(t) \\ &\quad + ((A-JC)e_q(t) - \dot{e}_q(t))^T(Qe_q(t) + \delta Q\dot{e}_q(t)) - \gamma e_f^T(t)e_f(t) \\ &\quad + (e_q^T(t)Q + \dot{e}_q^T(t)\delta Q)Fe_f(t) + e_f^T(t)F^T(Qe_q(t) + \delta Q\dot{e}_q(t)) < 0. \end{aligned} \quad (53)$$

If the following requirement is introduced

$$e_f^T(t)(F^TQ - H^TC)e_q(t) + e_q^T(t)(QF - C^TH)e_f(t) = 0, \quad (54)$$

it is obvious that Eq. (54) can be satisfied when Eq. (46) is satisfied. Thus, the condition (54) allows to write Eq. (53) as follows

$$\begin{aligned} \dot{v}(e_q(t)) &= \dot{e}_q^T(t)Pe_q(t) + e_q^T(t)P\dot{e}_q(t) + e_q^T(t)\gamma^{-1}C^T Ce_q(t) - \gamma e_f^T(t)e_f(t) + \\ &\quad + (e_q^T(t)Q + \dot{e}_q^T(t)\delta Q)((A-JC)e_q(t) - \dot{e}_q(t)) + \\ &\quad + (e_q^T(t)(A-JC)^T - \dot{e}_q^T(t))(Qe_q(t) + \delta Q\dot{e}_q(t)) + \\ &\quad + \dot{e}_q^T(t)\delta QFe_f(t) + e_f^T(t)\delta F^T Q\dot{e}_q(t) < 0. \end{aligned} \quad (55)$$

Relying on Eq. (55), it is possible to write the observer stability condition as

$$\dot{v}(e_d(t)) = e_d^T(t)P_d e_d(t) < 0, \quad (56)$$

where the following notations

$$\mathbf{P}_d = \begin{bmatrix} \mathbf{Q}(A-JC) + (A-JC)^T \mathbf{Q} + \gamma^{-1} C^T C & \mathbf{P}-\mathbf{Q} + \delta(A-JC)^T \mathbf{Q} & \mathbf{0} \\ \mathbf{P}-\mathbf{Q} + \delta \mathbf{Q}(A-JC) & -2\delta \mathbf{Q} & \delta \mathbf{Q} \mathbf{F} \\ \mathbf{0} & \delta \mathbf{F}^T \mathbf{Q} & -\gamma \mathbf{I}_p \end{bmatrix} < 0, \quad (57)$$

$$\mathbf{e}_d^T(t) = [\mathbf{e}_q^T(t) \quad \dot{\mathbf{e}}_q^T(t) \quad \mathbf{e}_f^T(t)], \quad (58)$$

are exploited.

Introducing the substitution

$$\mathbf{Q} \mathbf{J} = \mathbf{Y} \quad (59)$$

and using the Schur complement property with respect to the item $\gamma^{-1} C^T C$, then Eq. (57) implies Eq. (45). This concludes the proof.

Theorem 3 *The adaptive fault observer (19) and (20) is stable if there exists a symmetric positive definite matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$, matrices $\mathbf{H} \in \mathbb{R}^{n \times p}$, $\mathbf{Y} \in \mathbb{R}^{n \times m}$ and a positive scalar $\gamma \in \mathbb{R}$ such that*

$$\mathbf{Q} = \mathbf{Q}^T > 0, \quad \gamma > 0, \quad (60)$$

$$\begin{bmatrix} \mathbf{Q} \mathbf{A} + \mathbf{A}^T \mathbf{Q} - \mathbf{Y} \mathbf{C} - \mathbf{C}^T \mathbf{Y}^T & * & * \\ \mathbf{F}^T \mathbf{Q} & -\gamma \mathbf{I}_p & * \\ \mathbf{C} & \mathbf{0} & -\gamma \mathbf{I}_m \end{bmatrix} < 0. \quad (61)$$

$$\mathbf{Q} \mathbf{F} = \mathbf{C}^T \mathbf{H}. \quad (62)$$

When the above conditions are affirmative the estimator gain matrix is given by the relation

$$\mathbf{J} = \mathbf{Q}^{-1} \mathbf{Y}. \quad (63)$$

Proof. Premultiplying the left side and postmultiplying the right side of Eq. (57) by the transformation matrix

$$\mathbf{T}_x = \text{diag}[\mathbf{I}_n \quad \delta^{-1} \mathbf{I}_n \quad \mathbf{I}_p \quad \mathbf{I}_m] \quad (64)$$

gives

$$\begin{bmatrix} \mathbf{Q}(A-JC) + (A-JC)^T \mathbf{Q} + \gamma^{-1} C^T C & \delta^{-1}(\mathbf{P}-\mathbf{Q}) + (A-JC)^T \mathbf{Q} & \mathbf{0} \\ \delta^{-1}(\mathbf{P}-\mathbf{Q}) + \mathbf{Q}(A-JC) & -2\delta^{-1} \mathbf{Q} & \mathbf{Q} \mathbf{F} \\ \mathbf{0} & \mathbf{F}^T \mathbf{Q} & -\gamma \mathbf{I}_p \end{bmatrix} < 0. \quad (65)$$

Considering that $\mathbf{P} = \mathbf{Q}$ and using the Schur complement property, then the inequality (65) can be rewritten as

$$\begin{aligned} & \mathbf{Q}(A-JC) + (A-JC)^T \mathbf{Q} + \gamma^{-1} C^T C \\ & + (A-JC)^T \mathbf{Q} \frac{1}{2} \delta \mathbf{Q}^{-1} \mathbf{Q}(A-JC) + \begin{bmatrix} \mathbf{0} \\ \mathbf{Q} \mathbf{F} \end{bmatrix} \gamma^{-1} \mathbf{I}_p \begin{bmatrix} \mathbf{0} & \mathbf{F}^T \mathbf{Q} \end{bmatrix} < 0. \end{aligned} \quad (66)$$

Since the first matrix element in the second row of Eq. (66) is zero matrix if $\delta = 0$ and considering that nonzero component unit of the last matrix element in this row is certainly positive semidefinite, it can claim that

$$Q(A-JC) + (A-JC)^T Q + \gamma^{-1} C^T C + QF\gamma^{-1} I_p F^T Q < 0. \tag{67}$$

Thus, applying the Schur complement property, it can be written as

$$\begin{bmatrix} Q(A-JC) + (A-JC)^T Q + \gamma^{-1} C^T C & QF \\ F^T Q & -\gamma I_p \end{bmatrix} < 0, \tag{68}$$

$$\begin{bmatrix} Q(A-JC) + (A-JC)^T Q & QF & C^T \\ F^T Q & -\gamma I_p & 0 \\ C & 0 & -\gamma I_m \end{bmatrix} < 0, \tag{69}$$

respectively. With the notation (59) then Eq. (69) gives Eqs. (61). This concludes the proof.

Comparing with Lemma 3, it can be seen that Eqs. (60)–(62) is an extended form of the bounded real lemma (BRL) structure, applicable in the design of proportional adaptive fault observers.

4. Observer dynamics with eigenvalues clustering in D-stability circle

Generalizing the approach covering decoupling of Lyapunov matrix from the observer system matrix parameters by using a slack matrix, with a good exposition of the given theorems, the observer eigenvalues placement in a circular D -stability region is proposed to enable wide adaptation to faults dynamics.

Theorem 4 *The adaptive fault observer (19) and (20) is D -stable if for given positive scalars $\delta, a, \rho \in \mathbb{R}$, $a > \rho$, there exist symmetric positive definite matrices $P \in \mathbb{R}^{n \times n}$, $Q \in \mathbb{R}^{n \times n}$, matrices $H \in \mathbb{R}^{n \times p}$, $Y \in \mathbb{R}^{n \times m}$ and a positive scalar $\gamma \in \mathbb{R}$ such that*

$$P = P^T > 0, \quad Q = Q^T > 0, \quad \gamma > 0, \tag{70}$$

$$\begin{bmatrix} -\rho Q & * & * & * & * \\ aQ + QA - YC & -\rho Q & * & * & * \\ P - Q + \frac{\delta a^2 - \rho^2}{\rho} Q + \frac{\delta}{\rho} QA - \frac{\delta}{\rho} YC & 0 & -2\delta Q & * & * \\ 0 & 0 & \frac{\delta}{\rho} F^T Q & -\gamma I_p & * \\ C & 0 & 0 & 0 & -\gamma I_m \end{bmatrix} < 0, \tag{71}$$

$$QF = C^T H. \tag{72}$$

When the above conditions are affirmative the estimator gain matrix can be computed as

$$J = \mathbf{Q}^{-1} Y \quad (73)$$

and the adaptive fault estimation algorithm is given by (27).

Proof. Choosing the Lyapunov function candidate as

$$\begin{aligned} v(\mathbf{e}_q(t)) = & \mathbf{e}_q^T(t) \mathbf{P} \mathbf{e}_q(t) + \mathbf{e}_f^T(t) \mathbf{G}^{-1} \mathbf{e}_f(t) + \gamma^{-1} \int_0^t (\mathbf{e}_y^T(r) \mathbf{e}_y(r) - \gamma^2 \mathbf{e}_f^T(r) \mathbf{e}_f(r)) dr \\ & + \varrho^{-1} \int_0^t \mathbf{e}_q^T(r) \mathbf{A}_e^T \mathbf{Q} \mathbf{A}_e \mathbf{e}_q(r) dr > 0, \end{aligned} \quad (74)$$

where $\mathbf{P} = \mathbf{P}^T > 0$, $\mathbf{G} = \mathbf{G}^T > 0$, $\mathbf{Q} = \mathbf{Q}^T > 0$, $\gamma > 0$, γ is an upper bound of H_∞ norm of the transfer function matrix (43) and where the generalized observer differential equation takes the form [28]

$$\dot{\mathbf{e}}_q(t) = \mathbf{A}_{er} \mathbf{e}_q(t) + \mathbf{F}_r \mathbf{e}_f(t), \quad (75)$$

while, with $a > 0$, $\varrho > 0$ such that $\varrho < a$, the matrices \mathbf{A}_{er} , \mathbf{F}_r are given as

$$\mathbf{A}_{er} = \frac{a}{\varrho} \mathbf{A}_e + \frac{a^2 - \varrho^2}{2\varrho} \mathbf{I}_n, \quad \mathbf{F}_r = \frac{1}{\varrho} \mathbf{F}. \quad (76)$$

Then, the time derivative of $v(\mathbf{e}_q(t))$ is

$$\begin{aligned} \dot{v}(\mathbf{e}_q(t)) = & \dot{\mathbf{e}}_q^T(t) \mathbf{P} \mathbf{e}_q(t) + \mathbf{e}_q^T(t) \mathbf{P} \dot{\mathbf{e}}_q(t) + \dot{\mathbf{e}}_f^T(t) \mathbf{G}^{-1} \mathbf{e}_f(t) + \mathbf{e}_f^T(t) \mathbf{G}^{-1} \dot{\mathbf{e}}_f(t) + \\ & + \mathbf{e}_q^T(t) \mathbf{A}_e^T \varrho^{-1} \mathbf{Q} \mathbf{A}_e \mathbf{e}_q(t) + \gamma^{-1} \mathbf{e}_y^T(t) \mathbf{e}_y(t) - \gamma \mathbf{e}_f^T(t) \mathbf{e}_f(t) < 0. \end{aligned} \quad (77)$$

Assuming that, with respect to Eqs. (34) and (35), the inequality (50) holds, then Eq. (77) gives

$$\begin{aligned} \dot{v}(\mathbf{e}_q(t)) = & \dot{\mathbf{e}}_q^T(t) \mathbf{P} \mathbf{e}_q(t) + \mathbf{e}_q^T(t) \mathbf{P} \dot{\mathbf{e}}_q(t) - \mathbf{e}_q^T(t) \mathbf{C}^T \mathbf{H} \mathbf{e}_f(t) - \mathbf{e}_f^T(t) \mathbf{H}^T \mathbf{C} \mathbf{e}_q(t) \\ & + \mathbf{e}_q^T(t) \mathbf{A}_e^T \varrho^{-1} \mathbf{Q} \mathbf{A}_e \mathbf{e}_q(t) + \gamma^{-1} \mathbf{e}_y^T(t) \mathbf{e}_y(t) - \gamma \mathbf{e}_f^T(t) \mathbf{e}_f(t) < 0. \end{aligned} \quad (78)$$

Generalizing the equation (75), the following condition can be set

$$(\mathbf{e}_q^T(t) \mathbf{Q} + \dot{\mathbf{e}}_q^T(t) \delta \mathbf{Q})(\mathbf{A}_{er} \mathbf{e}_q(t) + \mathbf{F}_r \mathbf{e}_f(t) - \dot{\mathbf{e}}_q(t)) = 0, \quad (79)$$

where $\mathbf{Q} \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix and $\delta \in \mathbb{R}$ is a positive scalar. Therefore, adding Eq. (79) and its transposition to Eq. (78) gives

$$\begin{aligned} \dot{v}(\mathbf{e}_q(t)) = & \dot{\mathbf{e}}_q^T(t) \mathbf{P} \mathbf{e}_q(t) + \mathbf{e}_q^T(t) \mathbf{P} \dot{\mathbf{e}}_q(t) + \mathbf{e}_q^T(t) \gamma^{-1} \mathbf{C}^T \mathbf{C} \mathbf{e}_q(t) - \gamma \mathbf{e}_f^T(t) \mathbf{e}_f(t) \\ & + (\mathbf{e}_q^T(t) \mathbf{Q} + \dot{\mathbf{e}}_q^T(t) \delta \mathbf{Q})(\mathbf{A}_{er} \mathbf{e}_q(t) - \dot{\mathbf{e}}_q(t)) + (\mathbf{e}_q^T(t) \mathbf{A}_{er}^T - \dot{\mathbf{e}}_q^T(t)) (\mathbf{Q} \mathbf{e}_q(t) + \delta \mathbf{Q} \dot{\mathbf{e}}_q(t)) \\ & + \mathbf{e}_q^T(t) \mathbf{A}_e^T \varrho^{-1} \mathbf{Q} \mathbf{A}_e \mathbf{e}_q(t) + \dot{\mathbf{e}}_q^T(t) \delta \mathbf{Q} \mathbf{F}_r \mathbf{e}_f(t) + \mathbf{e}_f^T(t) \delta \mathbf{F}_r^T \mathbf{Q} \dot{\mathbf{e}}_q(t) < 0. \end{aligned} \quad (80)$$

From Eq. (80), using the notation (58), the following stability condition can be obtained

$$\dot{v}(e_d(t)) = e_d^T(t)P_{de}e_d(t) < 0, \tag{81}$$

where

$$P_{de} = \begin{bmatrix} QA_{er} + A_{er}^T Q + q^{-1}A_e^T QA_e + \gamma^{-1}C^T C & P-Q + \delta A_{er}^T Q & 0 \\ P-Q + \delta QA_{er} & -2\delta Q & \delta QF \\ 0 & \delta F^T Q & -\gamma I_p \end{bmatrix} < 0. \tag{82}$$

It can be easily stated using Eq. (76) that

$$QA_{er} + A_{er}^T Q + q^{-1}A_e^T QA_e = \frac{a}{q}(QA_e + A_e^T Q) + \frac{a^2 - q^2}{q}Q + \frac{1}{q}A_e^T QA_e, \tag{83}$$

so, completing to square the elements in Eq. (83), it is immediate that

$$QA_{er} + A_{er}^T Q + q^{-1}A_e^T QA_e = (A_e + aI_n)^T q^{-1}Q(A_e + aI_n) - qQ. \tag{84}$$

Substituting Eqs. (76) and (84) in Eq. (82) gives

$$\begin{bmatrix} -qQ + (A_e + aI_n)^T q^{-1}Q(A_e + aI_n) + \gamma^{-1}C^T C & P-Q + \frac{\delta}{q}A_e^T Q + \frac{\delta a^2 - q^2}{q}Q & 0 \\ P-Q + \frac{\delta}{q}QA_e + \frac{\delta a^2 - q^2}{q}Q & -2\delta Q & \frac{\delta}{q}QF \\ 0 & \frac{\delta}{q}F^T Q & -\gamma I_p \end{bmatrix} < 0 \tag{85}$$

and using twice the Schur complement property, Eq. (85) can be rewritten as

$$\begin{bmatrix} -qQ & (A_e + aI_n)^T Q & P-Q + \frac{\delta}{q}A_e^T Q + \frac{\delta a^2 - q^2}{q}Q & 0 & C^T \\ Q(A_e + aI_n) & -qQ & 0 & 0 & 0 \\ P-Q + \frac{\delta}{q}QA_e + \frac{\delta a^2 - q^2}{q}Q & 0 & -2\delta Q & \frac{\delta}{q}QF & 0 \\ 0 & 0 & \frac{\delta}{q}F^T Q & -\gamma I_p & 0 \\ C & 0 & 0 & 0 & -\gamma I_m \end{bmatrix} < 0. \tag{86}$$

Thus, for A_e from Eq. (30) and with the notation (59) then Eq. (86) implies Eq. (71). This concludes the proof.

Theorem 5 (Enhanced BRL) *The adaptive fault observer (19) and (20) is \mathcal{D} -stable if for given positive scalars $a, q \in \mathbb{R}$, $a > q$, there exist a symmetric positive definite matrix $Q \in \mathbb{R}^{n \times n}$, matrices $H \in \mathbb{R}^{n \times p}$, $Y \in \mathbb{R}^{n \times m}$ and a positive scalar $\gamma \in \mathbb{R}$ such that*

$$\mathbf{Q} = \mathbf{Q}^T > 0, \quad \gamma > 0, \quad (87)$$

$$\begin{bmatrix} -\mathbf{Q} & * & * & * \\ \mathbf{a}\mathbf{Q} + \mathbf{Q}\mathbf{A} - \mathbf{Y}\mathbf{C} & -\mathbf{Q} & * & * \\ \mathbf{0} & \frac{1}{\epsilon}\mathbf{F}^T\mathbf{Q} & -\gamma\mathbf{I}_p & * \\ \mathbf{C} & \mathbf{0} & \mathbf{0} & -\gamma\mathbf{I}_m \end{bmatrix} < 0. \quad (88)$$

$$\mathbf{Q}\mathbf{F} = \mathbf{C}^T\mathbf{H}. \quad (89)$$

When the above conditions are affirmative the estimator gain matrix can be computed by Eq. (73).

Proof. Considering that in Eq. (86) $\mathbf{P} = \mathbf{Q}$, then premultiplying the left side and postmultiplying the right side of Eq. (86) by the transformation matrix

$$\mathbf{T}_y = \text{diag}[\mathbf{I}_n \quad \mathbf{I}_n \quad \delta^{-1}\mathbf{I}_n \quad \mathbf{I}_p \quad \mathbf{I}_m] \quad (90)$$

gives

$$\begin{bmatrix} -\mathbf{Q} & (\mathbf{A}_e + \mathbf{a}\mathbf{I}_n)^T\mathbf{Q} & \frac{1}{\epsilon}\mathbf{A}_e^T\mathbf{Q} + \frac{1}{\epsilon}\frac{\delta^2 - \epsilon^2}{2}\mathbf{Q} & \mathbf{0} & \mathbf{C}^T \\ \mathbf{Q}(\mathbf{A}_e + \mathbf{a}\mathbf{I}_n) & -\mathbf{Q} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \frac{1}{\epsilon}\mathbf{Q}\mathbf{A}_e + \frac{1}{\epsilon}\frac{\delta^2 - \epsilon^2}{2}\mathbf{Q} & \mathbf{0} & -2\delta^{-1}\mathbf{Q} & \frac{1}{\epsilon}\mathbf{Q}\mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{1}{\epsilon}\mathbf{F}^T\mathbf{Q} & -\gamma\mathbf{I}_p & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\gamma\mathbf{I}_m \end{bmatrix} < 0. \quad (91)$$

Then, using the Schur complement property, the inequality (91) can be rewritten as

$$\begin{aligned} & \begin{bmatrix} -\mathbf{Q} & (\mathbf{A}_e + \mathbf{a}\mathbf{I}_n)^T\mathbf{Q} \\ \mathbf{Q}(\mathbf{A}_e + \mathbf{a}\mathbf{I}_n) & -\mathbf{Q} \end{bmatrix} + \begin{bmatrix} \frac{1}{\epsilon}\mathbf{A}_e^T\mathbf{Q} + \frac{1}{\epsilon}\frac{\delta^2 - \epsilon^2}{2}\mathbf{Q} \\ \mathbf{0} \end{bmatrix} \delta^{-1} \mathbf{Q}^{-1} \begin{bmatrix} \frac{1}{\epsilon}\mathbf{Q}\mathbf{A}_e + \frac{1}{\epsilon}\frac{\delta^2 - \epsilon^2}{2}\mathbf{Q} & \mathbf{0} \end{bmatrix} \\ & + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \frac{1}{\epsilon}\mathbf{Q}\mathbf{F} \end{bmatrix} \gamma^{-1}\mathbf{I}_p \begin{bmatrix} \mathbf{0} & \mathbf{0} & \frac{1}{\epsilon}\mathbf{F}^T\mathbf{Q} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \gamma^{-1}\mathbf{I}_m \begin{bmatrix} \mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0. \end{aligned} \quad (92)$$

Since the second matrix element in Eq. (92) is zero matrix if $\delta = 0$ and nonzero components of the elements in the second row are positive semidefinite, it can claim that

$$\begin{bmatrix} -\mathbf{Q} & (\mathbf{A}_e + \mathbf{a}\mathbf{I}_n)^T\mathbf{Q} \\ \mathbf{Q}(\mathbf{A}_e + \mathbf{a}\mathbf{I}_n) & -\mathbf{Q} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \frac{1}{\epsilon}\mathbf{Q}\mathbf{F} \end{bmatrix} \gamma^{-1}\mathbf{I}_p \begin{bmatrix} \mathbf{0} & \frac{1}{\epsilon}\mathbf{F}^T\mathbf{Q} \end{bmatrix} + \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \gamma^{-1}\mathbf{I}_m \begin{bmatrix} \mathbf{C} & \mathbf{0} & \mathbf{0} \end{bmatrix} < 0 \quad (93)$$

and so Eq. (93) implies the linear matrix inequality

$$\begin{bmatrix} -\mathbf{Q} & (\mathbf{A}_e + \mathbf{a}\mathbf{I}_n)^T\mathbf{Q} & \mathbf{0} & \mathbf{C}^T \\ \mathbf{Q}(\mathbf{A}_e + \mathbf{a}\mathbf{I}_n) & -\mathbf{Q} & \frac{1}{\epsilon}\mathbf{Q}\mathbf{F} & \mathbf{0} \\ \mathbf{0} & \frac{1}{\epsilon}\mathbf{F}^T\mathbf{Q} & -\gamma\mathbf{I}_p & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & \mathbf{0} & -\gamma\mathbf{I}_m \end{bmatrix} < 0. \quad (94)$$

Thus, using Eq. (59) then Eq. (94) implies Eq. (88). This concludes the proof.

Theorem 6 The adaptive fault observer (19) and (20) is \mathcal{D} -stable if for given positive scalars $a, \rho \in \mathbb{R}$, $a > \rho$, there exist a symmetric positive definite matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$, matrices $\mathbf{H} \in \mathbb{R}^{n \times p}$, $\mathbf{Y} \in \mathbb{R}^{n \times m}$ such that

$$\mathbf{Q} = \mathbf{Q}^T > 0, \tag{95}$$

$$\begin{bmatrix} -\rho \mathbf{Q} & * \\ a\mathbf{Q} + \mathbf{Q}\mathbf{A} - \mathbf{Y}\mathbf{C} & -\rho \mathbf{Q} \end{bmatrix} < 0. \tag{96}$$

$$\mathbf{Q}\mathbf{F} = \mathbf{C}^T \mathbf{H}. \tag{97}$$

When the above conditions are affirmative the observer gain matrix can be computed by Eq. (73).

Proof. Considering only conditions implying from fault-free autonomous system (equivalent to $\mathbf{F} = \mathbf{0}$, $\mathbf{C} = \mathbf{0}$), then Eq. (88) implies directly Eq. (96). This concludes the proof.

Note, due to two integral quadratic constraints, setting the circle parameters to define \mathcal{D} -stable region is relatively easy only for systems with single input and single output.

5. Extended design conditions

In order to be able to formulate the fault observer equations incorporating the symmetric, positive definite learning weight matrix \mathbf{G} , Eqs. (21), (29), and (30) can be rewritten compositely as

$$\begin{bmatrix} \dot{e}_q(t) \\ \dot{e}_f(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{J}\mathbf{C} & \mathbf{F} \\ -\mathbf{G}\mathbf{H}^T \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} e_q(t) \\ e_f(t) \end{bmatrix}, \tag{98}$$

$$e_y(t) = [\mathbf{C} \quad \mathbf{0}] \begin{bmatrix} e_q(t) \\ e_f(t) \end{bmatrix}. \tag{99}$$

Since Eq. (98) can be rewritten as follows

$$\begin{bmatrix} \dot{e}_q(t) \\ \dot{e}_f(t) \end{bmatrix} = \left(\begin{bmatrix} \mathbf{A} & \mathbf{F} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{J} \\ \mathbf{H}^T \end{bmatrix} [\mathbf{C} \quad \mathbf{0}] \right) \begin{bmatrix} e_q(t) \\ e_f(t) \end{bmatrix}, \tag{100}$$

introducing the notations

$$\tilde{e}(t) = \begin{bmatrix} e_q(t) \\ e_f(t) \end{bmatrix}, \tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{F} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tilde{\mathbf{G}} = \begin{bmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix}, \tilde{\mathbf{J}} = \begin{bmatrix} \mathbf{J} \\ \mathbf{H}^T \end{bmatrix}, \tilde{\mathbf{C}} = [\mathbf{C} \quad \mathbf{0}], \tag{101}$$

where $\tilde{\mathbf{A}}, \tilde{\mathbf{G}} \in \mathbb{R}^{(n+p) \times (n+p)}$, $\tilde{\mathbf{J}} \in \mathbb{R}^{(n+p) \times m}$, $\tilde{\mathbf{C}} \in \mathbb{R}^{m \times (n+p)}$, $\tilde{e}(t) \in \mathbb{R}^{n+p}$, then it follows

$$\dot{\tilde{e}}(t) = (\tilde{\mathbf{A}} - \tilde{\mathbf{G}}\tilde{\mathbf{J}}\tilde{\mathbf{C}})\tilde{e}(t) = \tilde{\mathbf{A}}_e \tilde{e}(t), \tag{102}$$

$$e_y(t) = \tilde{\mathbf{C}}\tilde{e}(t), \tag{103}$$

where

$$\tilde{A}_e = \tilde{A} - \tilde{G}\tilde{J}\tilde{C}, \quad (104)$$

and $\tilde{e}(t)$ is the generalized fault observer error.

It is necessary to note that, in general, the elements of the positive definite symmetric matrix G are unknown in advance, and have to be interactive set to adapt the observer error to the amplitude of the estimated faults. Of course, even this formulation does not mean the elimination of the matrix equality from the design conditions, because the matrix structure of \tilde{A}_e in principle leads to the bilinear matrix inequalities.

Theorem 7. *The adaptive fault observer (19) and (20) is stable if for a given symmetric, positive definite matrix $G \in \mathbb{R}^{p \times p}$ there exist symmetric positive definite matrix $\tilde{P} \in \mathbb{R}^{(n+p) \times (n+p)}$ and matrices $\tilde{Z} \in \mathbb{R}^{(n+p) \times (n+p)}$, $\tilde{Y} \in \mathbb{R}^{(n+p) \times m}$ such that*

$$\tilde{P} = \tilde{P}^T > 0, \quad \tilde{P}\tilde{G} = \tilde{G}\tilde{Z}, \quad (105)$$

$$\tilde{P}\tilde{A} + \tilde{A}^T\tilde{P} - \tilde{G}\tilde{Y}\tilde{C} - \tilde{C}^T\tilde{Y}^T\tilde{G}^T < 0, \quad (106)$$

where $\tilde{A}, \tilde{G} \in \mathbb{R}^{(n+p) \times (n+p)}$, $\tilde{C} \in \mathbb{R}^{m \times (n+p)}$, $\tilde{J} \in \mathbb{R}^{(n+p) \times m}$ take the structures

$$\tilde{A} = \begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix}, \quad \tilde{G} = \begin{bmatrix} I_n & 0 \\ 0 & G \end{bmatrix}, \quad \tilde{C} = [C \quad 0], \quad \tilde{J} = \begin{bmatrix} J \\ H^T \end{bmatrix}. \quad (107)$$

When the above conditions hold, the observer gain matrix is given by

$$\tilde{J} = \tilde{Z}^{-1}\tilde{Y}. \quad (108)$$

Proof. Given $\tilde{A}, \tilde{G}, \tilde{C}$ such that (\tilde{A}, \tilde{C}) is observable, the Lyapunov function can be chosen as

$$v(\tilde{e}(t)) = \tilde{e}^T(t)\tilde{P}\tilde{e}(t) > 0, \quad (109)$$

where \tilde{P} is a positive definite matrix. Computing the first time derivative of Eq. (109), it yields

$$\dot{v}(\tilde{e}(t)) = \tilde{e}^T(t)\tilde{P}\dot{\tilde{e}}(t) + \dot{\tilde{e}}^T(t)\tilde{P}\tilde{e}(t) < 0, \quad (110)$$

which can be restated, using Eq. (102), as

$$\dot{v}(\tilde{e}(t)) = \tilde{e}^T(t)(\tilde{A}_e^T\tilde{P} + \tilde{P}\tilde{A}_e)\tilde{e}(t) < 0. \quad (111)$$

By the Lyapunov stability theory, the asymptotic stability can be achieved if

$$\tilde{A}_e^T\tilde{P} + \tilde{P}\tilde{A}_e < 0, \quad (112)$$

$$(\tilde{A} - \tilde{G}\tilde{J}\tilde{C})^T\tilde{P} + \tilde{P}(\tilde{A} - \tilde{G}\tilde{J}\tilde{C}) < 0, \quad (113)$$

respectively. It is evident that the matrix product $\tilde{P}\tilde{G}\tilde{J}\tilde{C}$ is bilinear with respect to the LMI variables \tilde{P} and \tilde{J} . To facilitate the stability analysis, it can be written as

$$\tilde{P}\tilde{G}\tilde{J}\tilde{C} = \tilde{P}\tilde{G}\tilde{Z}^{-1}\tilde{Z}\tilde{J}\tilde{C} = \tilde{P}\tilde{P}^{-1}\tilde{G}\tilde{Z}\tilde{J}\tilde{C} = \tilde{G}\tilde{Y}\tilde{C}, \quad (114)$$

$$\tilde{G}\tilde{Z}^{-1} = \tilde{P}^{-1}\tilde{G}, \quad \tilde{Z}\tilde{J} = \tilde{Y}. \quad (115)$$

Thus, Eqs. (113) and (115) imply Eqs. (105) and (106). This concludes the proof.

Theorem 8 *The adaptive fault observer (19) and (20) is D-stable if for a given symmetric, positive definite matrix $\tilde{G} \in \mathbb{R}^{p \times p}$ and positive scalars $a, \rho \in \mathbb{R}$, $a > \rho$, if there exist a symmetric positive definite matrix $\tilde{Q} \in \mathbb{R}^{(n+p) \times (n+p)}$ and matrices $\tilde{Z} \in \mathbb{R}^{(n+p) \times (n+p)}$, $\tilde{Y} \in \mathbb{R}^{(n+p) \times m}$ such that*

$$\tilde{Q} = \tilde{Q}^T > 0, \quad \tilde{Q}\tilde{G} = \tilde{G}\tilde{Z}, \quad \begin{bmatrix} \tilde{Q} & \tilde{Q}\tilde{Y} \\ a\tilde{Q} + \tilde{Q}\tilde{A} - \tilde{G}\tilde{Y}\tilde{C} & -\rho\tilde{Q} \end{bmatrix} < 0, \quad (116)$$

where $\tilde{A}, \tilde{G}, \tilde{C}, \tilde{J}$ are as in Eq. (107). When the above conditions are affirmative the observer gain matrix can be computed by Eq. (108).

Proof. Theorem 8, constructed as a generalization of the results giving stability conditions for adaptive fault observers, implies directly from Theorems 1 and 6. This concludes the proof.

6. Joint design strategy for FTC

It is assumed that the systems (16) and (17) are controllable, full state feedback control, combining with additive fault compensation from $f_e(t)$, is applied and an integral component part is added to eliminate steady tracking error. In this structure, the control law takes the form

$$u(t) = -\bar{K}\bar{q}(t), \quad (117)$$

$$\bar{q}^T(t) = [q^T(t) \quad f_e^T(t) \quad e_w^T(t)], \quad (118)$$

$$\bar{K} = [K_q \quad K_f \quad K_w], \quad (119)$$

$$e_w(t) = \int_0^t (w(\tau) - y(\tau)) d\tau, \quad (120)$$

where $w(t)$ is the reference output signal and $\bar{q}(t) \in \mathbb{R}^{n+p+m}$, $\bar{K} \in \mathbb{R}^{r \times (n+p+m)}$. Considering that in the fault-free regime

$$f_e^T(t) = GH^T C e_y(t) \doteq 0, \quad (121)$$

and Eq. (120) follows directly

$$\dot{e}_w(t) = w(t) - y(t) = w(t) - Cq(t), \quad (122)$$

the systems (16) and (17), the fault estimation equation (21) and (121) can be expanded as

$$\begin{bmatrix} \dot{q}(t) \\ \dot{f}_e(t) \\ \dot{e}_w(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{C} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} q(t) \\ f_e(t) \\ e_w(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} u(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I}_m \end{bmatrix} w(t), \quad (123)$$

$$y(t) = [\mathbf{C} \quad \mathbf{0} \quad \mathbf{0}] \begin{bmatrix} q(t) \\ f_e(t) \\ e_w(t) \end{bmatrix}, \quad (124)$$

where \mathbf{I}_m is the identity matrix of given dimension. Using the notations (118), (119), and

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{C} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{W}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{I}_m \end{bmatrix}, \quad \bar{\mathbf{C}}^T = \begin{bmatrix} \mathbf{C}^T \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (125)$$

$\bar{\mathbf{A}} \in \mathbb{R}^{(n+p+m) \times (n+p+m)}$, $\bar{\mathbf{B}} \in \mathbb{R}^{(n+p+m) \times r}$, $\bar{\mathbf{W}} \in \mathbb{R}^{(n+p+m) \times m}$ and $\bar{\mathbf{C}} \in \mathbb{R}^{m \times (n+p+m)}$, then

$$\dot{\bar{q}}(t) = \bar{\mathbf{A}}\bar{q}(t) + \bar{\mathbf{B}}u(t) + \bar{\mathbf{W}}w(t), \quad (126)$$

$$y(t) = \bar{\mathbf{C}}\bar{q}(t) \quad (127)$$

and applying the feedback control law (117) to the state space system in Eqs. (126) and (127), the expanded closed loop system becomes

$$\dot{\bar{q}}(t) = \bar{\mathbf{A}}_c\bar{q}(t) + \bar{\mathbf{W}}w(t), \quad (128)$$

$$y(t) = \bar{\mathbf{C}}\bar{q}(t), \quad (129)$$

where the closed-loop system matrix of the expanded system is

$$\bar{\mathbf{A}}_c = \bar{\mathbf{A}} - \bar{\mathbf{B}}\bar{\mathbf{K}}. \quad (130)$$

In order to design the system with reference attenuations γ_2 and γ_∞ , respectively, in the following is considered the transfer function matrix

$$\bar{\mathbf{G}}_w(s) = \bar{\mathbf{C}}(s\mathbf{I}_{n+p+m} - \bar{\mathbf{A}}_c)^{-1}\bar{\mathbf{B}}. \quad (131)$$

Proposition 1 (H_2 control synthesis) *The state feedback control (117) to the system (126) and (127) exists and $\|\bar{\mathbf{G}}_w(s)\|_2 < \gamma_2$ if for a given symmetric, positive definite matrix $\mathbf{G} \in \mathbb{R}^{p \times p}$ there exist symmetric positive definite matrices $\bar{\mathbf{V}} \in \mathbb{R}^{(n+p+m) \times (n+p+m)}$, $\bar{\mathbf{E}} \in \mathbb{R}^{m \times m}$, a matrix $\bar{\mathbf{Z}} \in \mathbb{R}^{r \times (n+p+m)}$ and a positive scalar $\eta \in \mathbb{R}$ such that*

$$\bar{\mathbf{V}} = \bar{\mathbf{V}}^T > 0, \quad \bar{\mathbf{E}} = \bar{\mathbf{E}}^T > 0, \quad \text{tr}(\bar{\mathbf{E}}) < \eta, \quad (132)$$

$$\begin{bmatrix} \bar{\mathbf{A}}\bar{\mathbf{V}} + \bar{\mathbf{V}}\bar{\mathbf{A}}^T - \bar{\mathbf{B}}\bar{\mathbf{Z}} - \bar{\mathbf{Z}}^T\bar{\mathbf{B}}^T & * \\ \bar{\mathbf{B}}^T & -\mathbf{I}_r \end{bmatrix} < 0, \quad (133)$$

$$\begin{bmatrix} \bar{V} & * \\ \bar{C}\bar{V} & \bar{E} \end{bmatrix} > 0, \tag{134}$$

where

$$\bar{A} = \begin{bmatrix} A & F & 0 \\ 0 & 0 & 0 \\ -C & 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \quad \bar{C} = [C \ 0 \ 0], \tag{135}$$

$$\bar{A} \in \mathbb{R}^{(n+p+m) \times (n+p+m)}, \bar{B} \in \mathbb{R}^{(n+p+m) \times r}, \bar{C} \in \mathbb{R}^{m \times (n+p+m)}.$$

When the above conditions hold, the control law gain is

$$\bar{K} = \bar{Z} \bar{V}^{-1}. \tag{136}$$

Proof. Replacing in the inequality (6), the couple (A, B) by the pair (\bar{A}, \bar{B}) from Eqs. (125) and (130), consequently redefines the linear matrix inequality (6) as

$$(\bar{A} - \bar{B}\bar{K})\bar{V} + \bar{V}(\bar{A} - \bar{B}\bar{K})^T + \bar{B}\bar{B}^T < 0 \tag{137}$$

and so using the Schur complement property and the notation

$$\bar{Z} = \bar{K} \bar{V}, \tag{138}$$

Eq. (137) implies Eq. (133).

Analogously, replacing in Eq. (7), the couple (C, V) by the pair (\bar{C}, \bar{V}) , the objective of H_2 control is now to minimize the constraint $\text{tr}(\bar{C} \bar{V} \bar{C}^T) < \gamma_2^2$.

Introducing the inequality

$$\bar{E} > \bar{C} \bar{V} \bar{C}^T = \bar{C} \bar{V} \bar{V}^{-1} \bar{V} \bar{C}^T, \quad \text{tr}(\bar{E}) = \eta, \tag{139}$$

with a new matrix variable \bar{E} being symmetric and positive definite, and using Schur complement property, then Eq. (139) implies directly Eq. (134). This concludes the proof.

Note, to obtain a feasible block structure of LMIs, the Schur complement property has to be used to rearrange Eq. (137) to obtain Eq. (133) while the dual Schur complement property is applied to modify Eq. (139) to obtain Eq. (134).

Proposition 2 (*H_∞ control synthesis*) *The state feedback control (117) to the systems (126) and (127) exists and $\|\bar{G}(s)\|_\infty < \gamma_\infty$ if for a given symmetric, positive definite matrix $G \in \mathbb{R}^{p \times p}$ there exist a symmetric positive definite matrix $S \in \mathbb{R}^{(n+p+m) \times (n+p+m)}$, a matrix $\bar{X} \in \mathbb{R}^{r \times (n+p+m)}$ and a positive scalar $\gamma_\infty \in \mathbb{R}$ such that*

$$\bar{S} = \bar{S}^T > 0, \quad \gamma_\infty > 0, \quad (140)$$

$$\begin{bmatrix} \overline{AS} + \overline{SA}^T - \overline{BX} - \overline{X}^T \overline{B}^T & * & * \\ \overline{B}^T & -\gamma_\infty \mathbf{I}_r & * \\ \overline{CS} & \mathbf{0} & -\gamma_\infty \mathbf{I}_m \end{bmatrix} < 0. \quad (141)$$

where

$$\bar{A} = \begin{bmatrix} A & F & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -C & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \bar{C} = [C \quad \mathbf{0} \quad \mathbf{0}], \quad (142)$$

$\bar{A} \in \mathbb{R}^{(n+p+m) \times (n+p+m)}$, $\bar{B} \in \mathbb{R}^{(n+p+m) \times r}$, and $\bar{C} \in \mathbb{R}^{m \times (n+p+m)}$.

When the above conditions hold, the control law gain is

$$\bar{K} = \bar{X} \bar{S}^{-1}. \quad (143)$$

Proof. Replacing in Eq. (9) the set of matrix parameters (A, C, D, I_w) by the foursome $(\bar{A}_c, \bar{C}, \bar{B}, I_r)$ and using the matrix variable \bar{U} , then Eq. (9) gives

$$\begin{bmatrix} \bar{U} \bar{A}_c + \bar{A}_c^T \bar{U} & \bar{U} \bar{D} & \bar{C}^T \\ \bar{B}^T \bar{U} & -\gamma_\infty \mathbf{I}_r & \mathbf{0} \\ \bar{C} & \mathbf{0} & -\gamma_\infty \mathbf{I}_m \end{bmatrix} < 0. \quad (144)$$

Defining the transform matrix

$$\bar{T} = \text{diag}[\bar{S} \quad \mathbf{I}_n \quad \mathbf{I}_m], \quad \bar{S} = \bar{U}^{-1}, \quad (145)$$

and premultiplying the left side and postmultiplying the right side of Eq. (144) by \bar{T} , it yields

$$\begin{bmatrix} \bar{A}_c \bar{S} + \bar{S} \bar{A}_c^T & \bar{B} & \bar{S} \bar{C}^T \\ \bar{B}^T & -\gamma_\infty \mathbf{I}_r & \mathbf{0} \\ \bar{C} \bar{S} & \mathbf{0} & -\gamma_\infty \mathbf{I}_m \end{bmatrix} < 0. \quad (146)$$

Substituting Eq. (130) modifies the linear matrix inequality (146) as follows

$$\begin{bmatrix} (\bar{A} - \bar{B} \bar{K}) \bar{S} + \bar{S} (\bar{A} - \bar{B} \bar{K})^T & \bar{B} & \bar{S} \bar{C}^T \\ \bar{B}^T & -\gamma_\infty \mathbf{I}_r & * \\ \bar{C} \bar{S} & \mathbf{0} & -\gamma_\infty \mathbf{I}_m \end{bmatrix} < 0 \quad (147)$$

and with the notation

$$\bar{X} = \bar{K} \bar{S} \quad (148)$$

Eq. (147) implies Eq. (141). This concludes the proof.

It is now easy to formulate a joint approach for integrated design of FTC, where $\bar{q}(t)$ is considered as in Eq. (118).

Theorem 9 *The state feedback control (117) to the systems (126) and (127) exists and $\|\bar{G}_w(s)\|_2 < \gamma_2$, $\|\bar{G}_d(s)\|_\infty < \gamma_\infty$ if for given symmetric, positive definite matrix $G \in \mathbb{R}^{p \times p}$ and positive scalars $a, \rho \in \mathbb{R}$, $a > \rho$, there exist symmetric positive definite matrices $\bar{V} \in \mathbb{R}^{(n+p+m) \times (n+p+m)}$, $\bar{Q} \in \mathbb{R}^{(n+p) \times (n+p)}$, matrices $\bar{X} \in \mathbb{R}^{r \times (n+p+m)}$, $\bar{E} \in \mathbb{R}^{m \times m}$, $\bar{Z} \in \mathbb{R}^{(n+p) \times (n+p)}$, $\bar{Y} \in \mathbb{R}^{(n+p) \times m}$, and a positive scalars $\gamma_\infty, \eta \in \mathbb{R}$ such that*

$$\bar{V} = \bar{V}^T > 0, \quad \bar{Q} = \bar{Q}^T > 0, \quad \gamma_\infty > 0, \quad \eta > 0, \tag{149}$$

$$\begin{bmatrix} a\bar{Q} + \bar{Q}\bar{A} - \bar{G}\bar{Y}\bar{C} & * \\ * & -\rho\bar{Q} \end{bmatrix} < 0, \tag{150}$$

$$\bar{Q}\bar{G} = \bar{G}\bar{Z}, \tag{151}$$

$$\begin{bmatrix} \bar{A}\bar{V} + \bar{V}\bar{A}^T - \bar{B}\bar{X} - \bar{X}^T\bar{B}^T & * & * \\ \bar{B}^T & -\gamma_\infty I_r & * \\ \bar{C}\bar{V} & \mathbf{0} & -\gamma_\infty I_m \end{bmatrix} < 0, \tag{152}$$

$$\begin{bmatrix} \bar{A}\bar{V} + \bar{V}\bar{A}^T - \bar{B}\bar{X} - \bar{X}^T\bar{B}^T & * \\ * & -I_r \end{bmatrix} < 0, \tag{153}$$

$$\begin{bmatrix} \bar{V} & * \\ \bar{C}\bar{V} & \bar{E} \end{bmatrix} > 0, \quad \text{tr}(\bar{E}) < \eta. \tag{154}$$

where are $\bar{A}, \bar{G}, \bar{C}, \bar{J}$ as in Eq. (107), $\bar{A}, \bar{B}, \bar{C}$ as in Eq. (142), and \bar{K} as in Eq. (119).

When the above conditions hold

$$\bar{K} = \bar{X}\bar{V}^{-1}, \quad \bar{J} = \bar{Z}^{-1}\bar{Y}. \tag{155}$$

Proof. Prescribing a unique solution of \bar{K} with respect to Eqs. (136) and (143), that is

$$\bar{V} = \bar{S}, \quad \bar{X} = \bar{Z}, \tag{156}$$

then Eqs. (132)–(134) and (140) and (141) in the joint sense imply Eqs. (152)–(154).

The design conditions are complemented by the inequalities (150) and (151), the same as Eq. (116). This concludes the proof.

Note, the introduced H_2H_∞ control maximizes the H_2 norm over all state-feedback gains \bar{K} while the H_∞ norm constraint is optimized. The set of LMIs (152)–(154) is generally well conditioned and feasible and, since \bar{A}_c is a convergent matrix, it follows that the state of the closed-loop system converges uniformly to the desired value.

The main reason for the use of D-stability principle in the fault observer design is to adapt the fault observer dynamics to the dynamics of the fault tolerant control structure and the expected dynamics of faults. But the joint FTC design may not be linked to this principle.

7. Illustrative example

To illustrate the proposed method, a system whose dynamics is described by Eqs. (16) and (17) is considered with the matrix parameters [43]

$$A = \begin{bmatrix} 1.380 & -0.208 & 6.715 & -5.676 \\ -0.581 & -4.290 & 0.000 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}, B = \begin{bmatrix} 0.000 & 0.000 \\ 5.679 & 0.000 \\ 1.136 & -3.146 \\ 1.136 & 0.000 \end{bmatrix}, F = \begin{bmatrix} 1.400 \\ 1.504 \\ 2.233 \\ 0.610 \end{bmatrix}, C^T = \begin{bmatrix} 4 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

To test the effectiveness and performance of the proposed estimators, the computations are carried out using the Matlab/Simulink environment and additional toolboxes, while the observer and controller design is performed by the linear matrix inequalities formulation using the functions of SeDuMi package [44]. The evaluation is performed in a standard condition, where the model to design the observer and the model for evaluation are the same and the simulations are performed according to the presented configuration of inputs and outputs.

Solving Eqs. (70)–(72), the fault observer design problem is solved as feasible where, with the prescribed stability region parameters $a = 7$, $\rho = 5$ and the tuning parameter $\delta = 2$, the resulted matrix parameters are

$$P = \begin{bmatrix} 11.1225 & 0.4148 & -3.7932 & -0.3068 \\ 0.4148 & 4.8026 & -2.6791 & -1.6972 \\ -3.7932 & -2.6791 & 4.2310 & -1.2725 \\ -0.3068 & -1.6972 & -1.2725 & 6.2685 \end{bmatrix}, Q = \begin{bmatrix} 6.4684 & 0.3600 & -3.1434 & -0.1831 \\ 0.3600 & 6.7121 & -4.6619 & -0.3100 \\ -3.1434 & -4.6619 & 5.6540 & -0.9782 \\ -0.1831 & -0.3100 & -0.9782 & 2.7161 \end{bmatrix},$$

$$\gamma = 27.9325, H_4 = \begin{bmatrix} 0.6166 \\ -1.2500 \end{bmatrix}, Y = \begin{bmatrix} 18.4698 & -53.1426 \\ -1.7560 & -25.4990 \\ -6.6739 & 38.4146 \\ 0.5201 & 15.2858 \end{bmatrix}, J_4 = \begin{bmatrix} 3.6529 & -4.8333 \\ 0.7482 & 1.4554 \\ 1.6614 & 6.6685 \\ 1.1215 & 7.8699 \end{bmatrix},$$

$$\rho(A_e) = \{-9.2971, -10.3524, -8.0807 \pm 0.5938i\}.$$

where $\rho(A_e)$ is the observer system matrix eigenvalues spectrum. Using the same optional parameters (if necessary), there are obtained the observer gains for the design conditions introduced in Theorems 1–3 and 5–6, respectively, while

$$H_1 = \begin{bmatrix} 0.1198 \\ -0.0017 \end{bmatrix}, J_1 = \begin{bmatrix} 3.6529 & -4.8333 \\ 0.7482 & 1.4554 \\ 1.6614 & 6.6685 \\ 1.1215 & 7.8699 \end{bmatrix}, \rho(A_e) = \left\{ \begin{array}{l} -3.4150, -5.2667 \\ -12.4523 \pm 20.2938i \end{array} \right\},$$

$$H_2 = \begin{bmatrix} 0.1809 \\ -0.5370 \end{bmatrix}, J_2 = \begin{bmatrix} 4.8866 & 3.6054 \\ 1.7146 & 2.5548 \\ 3.4303 & 15.4668 \\ 3.0854 & 8.3991 \end{bmatrix}, \rho(A_e) = \left\{ \begin{array}{l} -1.0022, -7.4681 \\ -10.4435, -24.1301 \end{array} \right\},$$

$$\mathbf{H}_3 = \begin{bmatrix} 1.3248 \\ 0.3732 \end{bmatrix}, \quad \mathbf{J}_3 = \begin{bmatrix} 0.7403 & 0.6309 \\ 4.2089 & 9.4205 \\ 9.0063 & 15.8599 \\ -0.3253 & 0.7215 \end{bmatrix}, \quad \rho(A_e) = \left\{ \begin{array}{l} -3.4908 \pm 0.7441i \\ -8.6876 \pm 15.3550i \end{array} \right\},$$

$$\mathbf{H}_5 = \begin{bmatrix} 0.4511 \\ -0.7967 \end{bmatrix}, \quad \mathbf{J}_5 = \begin{bmatrix} 3.4655 & -5.0754 \\ 0.6678 & 0.8357 \\ 1.5777 & 5.1656 \\ 1.3275 & 4.4003 \end{bmatrix}, \quad \rho(A_e) = \left\{ \begin{array}{l} -6.4021 \pm 1.6720i \\ -9.3518 \pm 0.4953i \end{array} \right\},$$

$$\mathbf{H}_6 = \begin{bmatrix} 0.0232 \\ -0.0440 \end{bmatrix}, \quad \mathbf{J}_6 = \begin{bmatrix} 3.4682 & -4.8675 \\ 0.6900 & 1.0472 \\ 1.5956 & 5.4720 \\ 1.3283 & 4.5180 \end{bmatrix}, \quad \rho(A_e) = \left\{ \begin{array}{l} -6.4178 \pm 1.6979i \\ -9.4094 \pm 0.6999i \end{array} \right\}.$$

Using an extended approach presented in Theorems 7 and 8, the effect of the learning weight on the dynamic performance of the adaptive fault observer is analyzed. Setting the weight $G = 7.5$ and using the optional factors as above, the resulted fault observer parameters are

$$\mathbf{H}_7 = \begin{bmatrix} 0.0530 \\ 0.1439 \end{bmatrix}, \quad \mathbf{J}_7 = \begin{bmatrix} -0.3895 & -1.7257 \\ 0.4619 & 2.3599 \\ 2.8130 & 4.5175 \\ -0.9425 & -0.3893 \end{bmatrix}, \quad \rho(A_e) = \left\{ \begin{array}{l} -2.8840 \pm 7.2479i \\ -3.3828 \pm 0.6281i \end{array} \right\},$$

$$\mathbf{H}_8 = \begin{bmatrix} 0.2053 \\ 0.3222 \end{bmatrix}, \quad \mathbf{J}_8 = \begin{bmatrix} 2.8037 & -2.6847 \\ 0.3667 & 2.0494 \\ 1.0796 & 7.2600 \\ 0.5665 & 6.6486 \end{bmatrix}, \quad \rho(A_e) = \left\{ \begin{array}{l} -6.6626 \pm 1.3897i \\ -8.6430 \pm 2.3545i \end{array} \right\}.$$

Separated simulations of fault estimation observer outputs are realized for system under the force mode control, with the control law given as

$$\mathbf{u}(t) = -\mathbf{K}_n \mathbf{q}(t) + \mathbf{W}_w \mathbf{w}(t). \tag{157}$$

Since separation principle holds and (A, B) is controllable, the eigenvalues of the closed-loop system matrix $A_c = A - BK$ can be placed arbitrarily. Using the MATLAB function *place.m*, the gain matrix K is chosen that A_c has the eigenvalues $\{-1, -2, -3, -4\}$, i.e.,

$$\mathbf{K}_n = \begin{bmatrix} -0.1014 & -0.2357 & 0.0147 & 0.1030 \\ -1.1721 & -0.2466 & 0.1472 & -0.4907 \end{bmatrix}$$

and the signal gain matrix \mathbf{W}_w is computed using the static decoupling principle as [45]

$$W = -(C(A-BK)^{-1}B)^{-1} = \begin{bmatrix} 0.0024 & 0.1055 \\ -0.0957 & 0.0401 \end{bmatrix}. \quad (158)$$

To evaluate the validity of the proposed compensation control scheme, weighted sinusoidal fault signals are considered. Since a weighted sinusoidal fault is suitable for evaluating the tracking performance and the robustness of the control scheme because it reflects more than slow changes in the fault magnitude, the faults in simulations are generated using the scenario

$$f(t) = g(t) \sin(\omega t), \quad g(t) = \begin{cases} 0, & t \leq t_{sa}, \\ \frac{1}{t_{sb}-t_{sa}}(t-t_{sa}), & t_{sa} < t_{sb}, \\ 1, & t_{sb} \leq t_{ea}, \\ -\frac{1}{t_{ea}-t_{sb}}(t-t_{eb}), & t_{ea} < t_{eb}, \\ 0, & t \geq t_{eb}, \end{cases} \quad (159)$$

where it is adjusted $\omega = 1$ rad/s, $t_{sa} = 10$ s, $t_{sb} = 15$ s, $t_{ea} = 35$ s, $t_{eb} = 40$ s.

Then, with the desired system output vector, the initial system condition and the external disturbance are chosen as follows

$$w^T(t) = [1 \quad 2], \quad q(0) = \mathbf{0}, \quad D^T = [0.610 \quad 2.233 \quad 1.504 \quad 1.400], \quad \sigma_d^2 = 0.01,$$

the faults estimates, obtained using the conditions from Theorems 1 to 6, are plotted in **Figures 1–6**. In all cases, the learning weight is set iteratively as $G = 7.5$. Simulation results obtained under the same simulation conditions, but realized by applying Theorems 7 and 8 with the prescribed weight $G = 7.5$, are given in **Figures 7 and 8**.

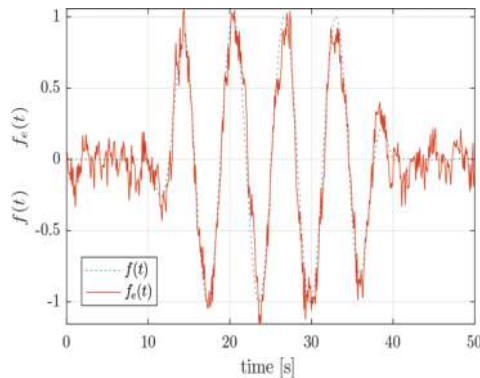


Figure 1. Estimation applying Theorem 1.

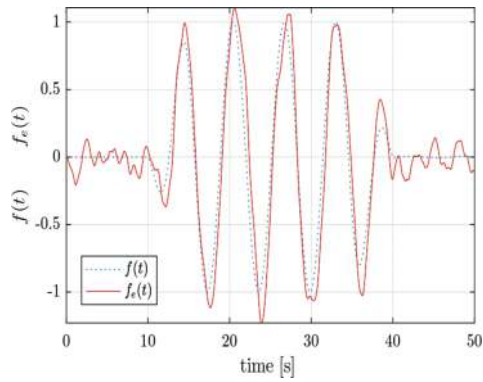


Figure 2. Estimation applying Theorem 2.

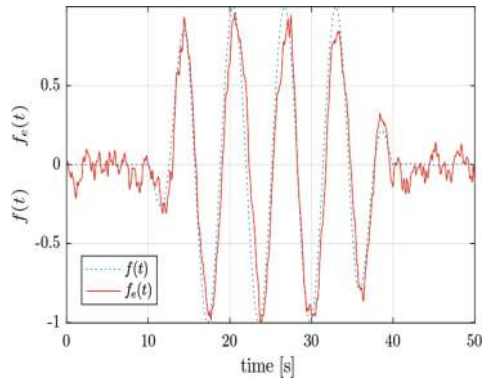


Figure 3. Estimation applying Theorem 3.

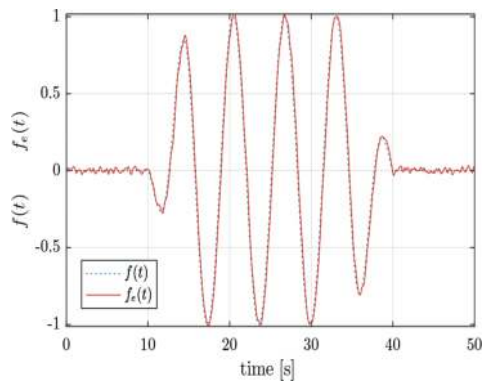


Figure 4. Estimation applying Theorem 4.

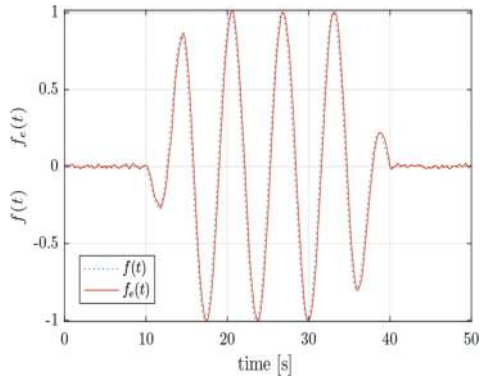


Figure 5. Estimation applying Theorem 5.

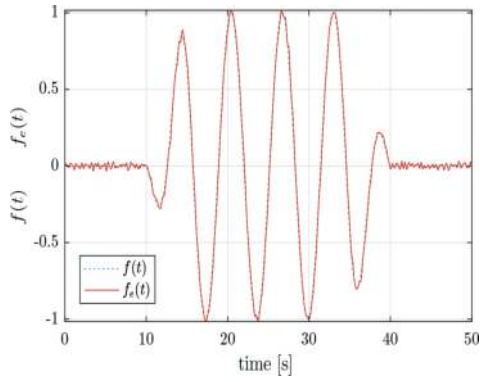


Figure 6. Estimation applying Theorem 6.

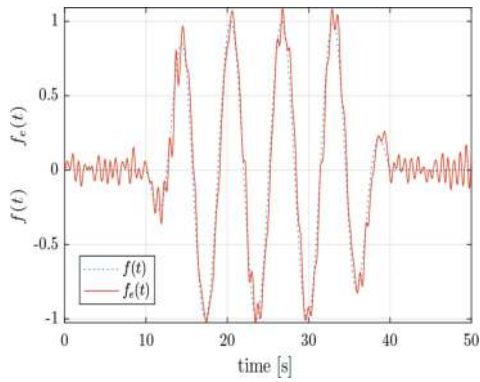


Figure 7. Estimation applying Theorem 7.

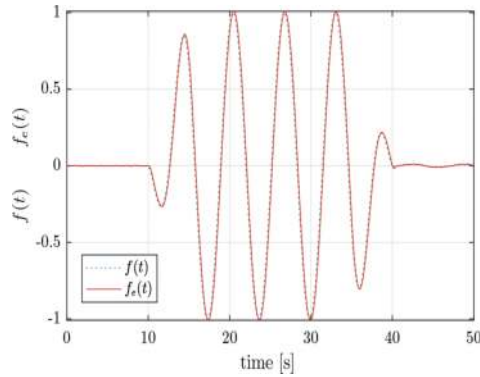


Figure 8. Estimation applying Theorem 8.

From these figures, it can be seen that fault estimation errors fast enough converge using an adaptive fault observer. Further, the extended approach with a prescribed circle \mathcal{D} -stability region is also effective in suppressing the disturbance noise effect on fault estimates.

Considering in the following an unforced system (126) and (127) and solving the set of linear matrix inequalities (132)–(135) to design FTC system parameters, the solution is obtained as follows

$$\bar{V} = \begin{bmatrix} 0.1995 & 0.0196 & -0.2602 & -0.1462 & 0.0794 & 0.2031 & -0.0932 \\ 0.0196 & 1.4771 & 0.1384 & 0.2529 & -0.0064 & 0.0036 & 0.3429 \\ -0.2602 & 0.1384 & 1.4776 & 0.6864 & -0.3175 & 0.1439 & 0.5436 \\ -0.1462 & 0.2529 & 0.6864 & 0.9270 & -0.0000 & 0.0696 & 0.6344 \\ 0.0794 & -0.0064 & -0.3175 & -0.0000 & 1.4436 & 0.0080 & -0.0224 \\ 0.2031 & 0.0036 & 0.1439 & 0.0696 & 0.0080 & 2.0837 & 0.0695 \\ -0.0932 & 0.3429 & 0.5436 & 0.6344 & -0.0224 & 0.0695 & 2.1627 \end{bmatrix},$$

$$\bar{Z} = \begin{bmatrix} -0.1746 & -0.9058 & 0.8639 & 1.1472 & 0.3790 & -0.0483 & -0.2186 \\ -1.9830 & -0.8055 & 1.9320 & -0.0805 & -1.5775 & 0.2255 & -0.3164 \end{bmatrix},$$

$$\bar{E} = \begin{bmatrix} 3.5753 & 0.0965 \\ 0.0965 & 2.2941 \end{bmatrix}, \quad \text{tr}(\bar{E}) = 5.8694, \quad \text{tr}(\bar{C}\bar{V}\bar{C}^T) = 3.5155 < \gamma_2^2.$$

Then, the set of control law matrix parameters is

$$\mathbf{K}_q = \begin{bmatrix} 0.5396 & -0.8207 & 0.1959 & 1.7572 \\ -13.1540 & 0.0167 & -0.2836 & -1.9893 \end{bmatrix}, \mathbf{K}_f = \begin{bmatrix} 0.2652 \\ -0.4418 \end{bmatrix}, \mathbf{K}_w = \begin{bmatrix} -0.1308 & -0.5055 \\ 1.4821 & -0.1132 \end{bmatrix},$$

while the eigenvalue spectrum of the closed-loop system matrix is

$$\rho(\bar{A}_c) = \{0, -0.2917, -0.4757-1.1533 \pm 6.7834i, -3.5221 \pm 16.1696i\}.$$

It is easy to see that the closed-loop system eigenvalues of the extended system strictly reflect the integral part of the control law that is, the set of inequalities (132)–(135) can be directly applied.

When solving the design conditions (140) and (141) for the equations of the unforced systems (126) and (127), the result is the set of matrix variables

$$\bar{\mathbf{S}} = \begin{bmatrix} 2.9032 & 0.2418 & -2.0149 & -0.4081 & 0.4638 & 0.2910 & -0.1012 \\ 0.2418 & 8.2299 & 1.0172 & 1.0940 & -0.0375 & 0.0847 & 1.4138 \\ -2.0149 & 1.0172 & 8.6558 & 3.9494 & -1.8552 & 1.3284 & 2.0586 \\ -0.4081 & 1.0940 & 3.9494 & 5.3411 & 0.0000 & 0.5492 & 2.6407 \\ 0.4638 & -0.0375 & -1.8552 & 0.0000 & 8.4354 & 0.3368 & -0.2675 \\ 0.2910 & 0.0847 & 1.3284 & 0.5492 & 0.3368 & 11.1517 & 0.3447 \\ -0.1012 & 1.4138 & 2.0586 & 2.6407 & -0.2675 & 0.3447 & 12.1897 \end{bmatrix},$$

$$\bar{\mathbf{X}} = \begin{bmatrix} -0.3425 & -4.5974 & 4.3400 & 6.8529 & 2.2149 & -0.1473 & -1.0174 \\ -13.2597 & -4.3184 & 11.1859 & -1.6190 & -9.2177 & 1.7119 & -2.1494 \end{bmatrix},$$

$$\gamma_\infty < 16.3245.$$

Based on these matrices, the closed-loop system matrix eigenvalues and the controller parameter (118) can be written out as

$$\mathbf{K}_q = \begin{bmatrix} 0.1556 & -0.7190 & 0.0494 & 1.5844 \\ -4.1380 & -0.3402 & 0.6571 & -1.0205 \end{bmatrix}, \mathbf{K}_f = \begin{bmatrix} 0.2545 \\ -0.7353 \end{bmatrix}, \mathbf{K}_w = \begin{bmatrix} -0.0928 & -0.3421 \\ 0.2609 & -0.0846 \end{bmatrix},$$

$$\rho(\bar{\mathbf{A}}_c) = \{0, -0.2054, -0.3514 - 1.2258 \pm 6.3796i, -2.1825 \pm 8.3875i\}.$$

Finally, the design conditions are designed in such a way that the upper bounds of H_2 and H_∞ norm of the system transfer function are incorporated and the parameters of the feedback controllers (117) and (118) are computed from the following set of matrix variables satisfying Eqs. (152)–(155)

$$\bar{\mathbf{V}} = \begin{bmatrix} 1.9774 & 0.2903 & -2.9899 & -1.0427 & 0.8321 & 0.9891 & -0.2776 \\ 0.2903 & 14.9964 & 1.0058 & 2.3203 & -0.0673 & -0.3526 & 2.3586 \\ -2.9899 & 1.0058 & 14.8053 & 5.7641 & -3.3284 & 1.0363 & 3.0673 \\ -1.0427 & 2.3203 & 5.7641 & 7.6521 & -0.0000 & 0.6069 & 4.0448 \\ 0.8321 & -0.0673 & -3.3284 & -0.0000 & 15.1342 & 0.1550 & -0.3263 \\ 0.9891 & -0.3526 & 1.0363 & 0.6069 & 0.1550 & 19.6050 & 0.4331 \\ -0.2776 & 2.3586 & 3.0673 & 4.0448 & -0.3263 & 0.4331 & 20.8419 \end{bmatrix},$$

$$\bar{\mathbf{X}} = \begin{bmatrix} -2.0516 & -7.0574 & 9.0166 & 12.1638 & 3.9738 & -0.1756 & -1.7169 \\ -23.6734 & -9.4744 & 21.5697 & -0.5497 & -16.5377 & 1.7345 & -3.7863 \end{bmatrix},$$

$$\bar{\mathbf{E}} = \begin{bmatrix} 33.4960 & 1.2672 \\ 1.2672 & 22.6309 \end{bmatrix}, \quad \text{tr}(\bar{\mathbf{C}}\bar{\mathbf{V}}\bar{\mathbf{C}}^T) = 30.1764 < \gamma_2^2, \quad \gamma_\infty < 22.8396,$$

while the controller matrix parameters are

$$\mathbf{K}_q = \begin{bmatrix} 0.0531 & -0.7236 & 0.0327 & 2.0030 \\ -14.6891 & -0.0607 & -1.2295 & -1.2123 \end{bmatrix}, \mathbf{K}_f = \begin{bmatrix} 0.2561 \\ -0.5650 \end{bmatrix}, \mathbf{K}_w = \begin{bmatrix} -0.0818 & -0.3876 \\ 0.9351 & 0.0175 \end{bmatrix}$$

and the spectrum of the closed-loop system matrix eigenvalues is

$$\rho(\overline{A}_c) = \{0, -0.1770, -0.3009-1.5837 \pm 7.1369i, -5.0472 \pm 16.5305i\}.$$

Considering the same fault generation method as above, but with $\omega = 0.5$ rad/s, then for the desired system output vector, the initial system condition and the external disturbance chosen are as follows

$$\mathbf{w}^T(t) = [1 \quad 2], \quad \overline{\mathbf{q}}(0) = \mathbf{0}, \quad \tilde{\mathbf{q}}_e(0) = \mathbf{0}, \quad \mathbf{D}^T = [0.610 \quad 2.233 \quad 1.504 \quad 1.400], \quad \sigma_d^2 = 0.01,$$

the output variable responses of the closed-loop system, obtained using the conditions from Proposition 2 and Theorem 9, are shown in **Figures 9** and **10** and are stable. To the structures (141), (142), and (152)–(155), the fault estimation is designed by Eq. (116).

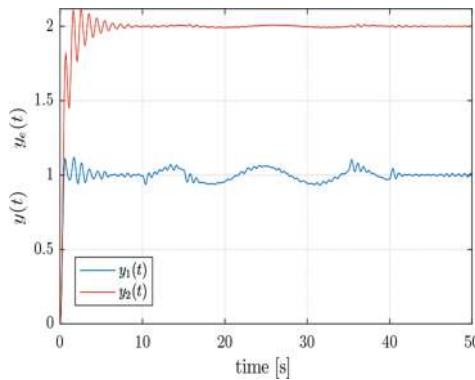


Figure 9. Compensation applying Proposition 2.

Summarizing the obtained simulation results it can be concluded that the adaptive fault estimators, designed by the standard estimation algorithm, has the worst properties (**Figure 1**) that are not significantly improved even though the conditions of synthesis are enhanced by a symmetric learning weight matrix G (**Figure 7**). Somewhat better results can be achieved when the synthesis conditions incorporate the H_∞ norm of the fault transfer function (**Figure 3**), even if they are combined with the use of an untying slack matrix Q (**Figures 2** and **5**). The best obtained results in accuracy and noise robustness are with the design conditions combining LMIs with constraints implying from \mathcal{D} -stability principle (**Figures 4, 6, and 8**).

The efficiency of the proposed algorithm to compensate the effect of an additive fault on the system output variables can be also observed. **Figures 9** and **10** show that the proposed H_2/H_∞ method increases control robustness due to the joint mixed LMI optimization that guarantees

system stability as well as the sufficient precision of compensation for a given class of slowly warring faults. Since the additive fault profile does not satisfy strictly the condition (22), its estimated time profile do not perfectly cover the actual values of the fault and where the variation of the amplitudes of $f(t)$ exceed its upper limit, there can be seen small fluctuations in compensation.

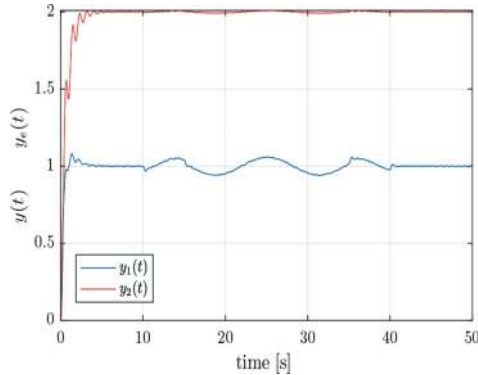


Figure 10. Compensation applying Theorem 9.

8. Concluding remarks

In this chapter, a modified approach for designing the adaptive fault observers is presented, and the \mathcal{D} -stability circle principle into fault observer design to outperform the two-stage known design approach in the fault observer dynamics adaptation is addressed. The design conditions are established as feasible problem, accomplishing under given quadratic constraints. Taking into consideration the slack updating effect, to cope with realistic operating conditions, the fault observer dynamics may be in the first case shifted to a stability region by exploiting the value of the tuning parameter. Integrated with the fault tolerant structures, H_2 and H_∞ norm-based analysis is carried out for compensated FTC structure to conclude about convergence of the fault compensation errors, and to derive the FTC design conditions. Using the LMI technique, the exploited mixed H_2H_∞ control design is possible to regularize the potential marginal feasibility of H_∞ -norm-based conditions. Presented illustrative example confirms the effectiveness of the proposed design alternative to construct the control structure with sufficient approximation of given class slowly warring faults and compensation of their impact on the system output variables.

Acknowledgements

The work presented in this chapter was supported by VEGA, the Grant Agency of the Ministry of Education and the Academy of Science of Slovak Republic, under Grant No. 1/0348/14. This support is very gratefully acknowledged.

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