

## Chapter

# PID Gain Tuning for Robust Control of PMDC Motor for External Disturbance Rejection with Constrained Motor Parameter Variations through $H_\infty$

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## Abstract

This chapter describes the controller modeling for PID gain tuning against the external disturbances with constrained internal parameter variation of the PMDC motor based on an optimization technique of H-infinity. To fit the goals in the H infinite framework, auto tuning of the PID controller gains is used. Different performance goals for tracking are preset as design objectives. Researchers in literature have presented many Robust Control techniques for motor control applications. Methods like back-stepping algorithms, fuzzy and neural based control systems, model predictive control and SMC (Sliding Mode Control) are available in literature. In this chapter, SC (speed control) of PMDC-motor is addressed with variations in outer load disturbances and internal variations of the system parameters for a particular application. C-PID (conventional PID controllers) is preferred, and equivalent robustness characteristics are established using the H-infinity development procedures. The optimization effort is to get simultaneous fast-tracking response and better disturbance rejection.

**Keywords:** H-infinity, C-PID, PMDC motor, robust control, disturbance rejection

## 1. Introduction

This necessity of controller design modeling for proportional integral and derivative (PID) gain tuning against the external disturbances with constrained internal parameter variation of the permanent magnet direct current (PMDC) motor based on an optimization technique of H-infinity is highly recommended. Here, PID controller with auto tuning of gains are used to match the goals in H infinity framework. Different performance goals for tracking are preset as design objectives. The speed control of PMDC-motor has so far attracted the attention of the researchers in recent times and many approaches and improvements have been proposed for PMDC motor

drives. PMDC motors are used in electrical equipment, computer peripherals, and manipulators because of their precise speed control capabilities. C-PID controllers have been in use since several years for different applications such as motor-control. The classical tuning methods of PID controller, as well as response method of Zeigler-Nichols frequency considers the system in the mode of oscillation to analyze the tuning procedure [1]. Since the manual tuning of PID controller is not user friendly, as it tends to be a tedious process though being simple in structure.

We know that PID controller fails to address instant tracking / regulation to get robustness against disturbance rejection. The majority of the time, industrial controllers are not properly tuned. Traditionally, motor control applications have better performed in control execution for specific working conditions. Controller parameters can be tuned for exact working conditions with an underlying assumption that the conditions are ordered and defined. Basically, working conditions according to the framework that are prone to variations leads to undesirable results if the variations are not accurately introduced. As these controllers do not guarantee the robustness to inner and outer disturbances, the outer disturbances and the system parameter variations have huge impact on performance and degradation in the applications of motor control.

Researchers in literature have presented many Robust Control techniques for motor control applications. Methods like back-stepping algorithms, fuzzy and neural based control systems, model predictive control and sliding mode control (SMC) are available in literature. The SMC based methodology stimulates the chattering phenomenon due to switching function of the inherent discontinuity. In Eker [2], the authors have demonstrated SMC for various applications of PMDC motor control. In Mamani et al. [3] and Corradini et al. [4], semi-SMC, adaptive SMC and boundary layer control (BLC) have been introduced as the development to classical style of Controlling the SMC and reducing chattering impact. Until now, most of the heuristic engineering methods have been developed to achieve optimal tuning of PID.

PID control with GA optimization based for the DC motor is implemented in Pal et al. [5] and can be stimulated by the aid of normal development methods. These methods have proven that the goal functions have degraded [6].

The PSO (Parasitic Swarm Optimization) is considered as the method of ideal structure to control of BLDC motor by PID control in Nasri et al. [7]. The major advantages being ease of implementation and computational complexity, which facilitates meeting constraints for some of the parameters apart from exhibiting fast convergence.

Position and speed control applications have been implemented using H-infinity-based control [8] providing better robustness in performance against disturbances, making this as an attractive alternative. To develop the Robust Controller, the NN based H-infinity controller was presented in Premkumar et al. [9] that introduces the DC motor H-infinity controller and has addressed parameter uncertainties.

In this chapter, SC application of PMDC-motor is addressed with variations in outer load disturbances and internal variations of the system parameters. Controller is also opted as C-PID and equivalent robustness characteristics are established using the H-infinity development procedures. The optimization effort is to get simultaneous fast-tracking response with better disturbance rejection.

## **2. PMDC motor model**

Using supply current, motor speed and supply voltage relationship, we have the PMDC motor mathematical model as:

Parameter	Nominal value	Variation (tolerable limit)
$L$	0.00063 H	41%
$R$	2.07 Ohm	40%
Viscous-friction ( $K_f$ )	0.000049 Nms rad <sup>-1</sup>	51%
EMF constant ( $K_b$ )	0.053 Vs rad <sup>-1</sup>	[0.013 to 0.1]
Armature-constant ( $K_m$ )	0.053 NmA <sup>-1</sup>	[0.012 to 0.1]

**Table 1.**  
 The parameters of motor and their acceptable limits of variation.

$$\frac{di}{dt} = -\frac{R}{L}i - \frac{k_b}{L}\omega + \frac{1}{L}u \quad (1)$$

$$\frac{d\omega}{dt} = \frac{1}{J}k_m i - \frac{1}{J}k_f \omega \quad (2)$$

Here,  $R$  and  $L$  are resistance and inductance of motor coil,  $k_b$  is denoted as the constant of back Electromotive force,  $J$  is rotor inertia and  $k_f$  is friction constant.

The representation of SS (state space) matrix of similar set of the equations is given by:

$$\frac{d\omega}{dt} \begin{bmatrix} i \\ \omega \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_b}{L} \\ \frac{K_m}{J} & -\frac{K_f}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad (3)$$

$$y(t) = [0 \ 1] \begin{bmatrix} i \\ \omega \end{bmatrix} + [0]u(t) \quad (4)$$

The overall transfer function for the speed and input voltage, is expressed as:

$$\frac{\omega(s)}{u(s)} = \frac{K_m}{JLs^2 + (JR + LK_f)s + K_mK_b + RK_f} \quad (5)$$

The transfer function of PMDC motor model is considered for a specified output speed and input voltage provided. The unknown motor parameters are prone to the variations due to aging of motor and its wear and tear. The controller must be built to offer resilience for each motor parameter within acceptable variation limits. The parameters of nominal motor are chosen from the standard motor of Maxon-RE35. The variation limits and motors parameters are given in **Table 1**. The proposed controller design for disturbance rejection in this work is based on the presumption that the PMDC motor internal parameter variations are within tolerable limits as specified for Maxon-RE35. The proposed Robust Controller design is discussed in subsequent section.

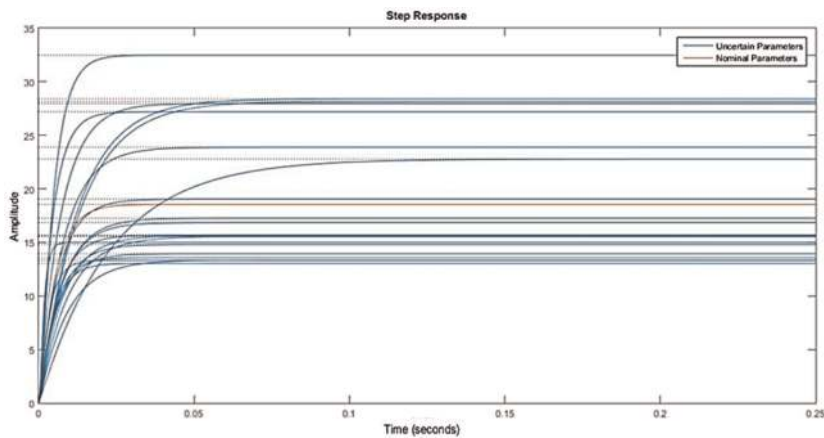
### 3. H $\infty$ based robust controller design for disturbance rejection

The proposed controller design model should meet the desired specifications within bounds of internal parameter variations. Figure below shows the variation in

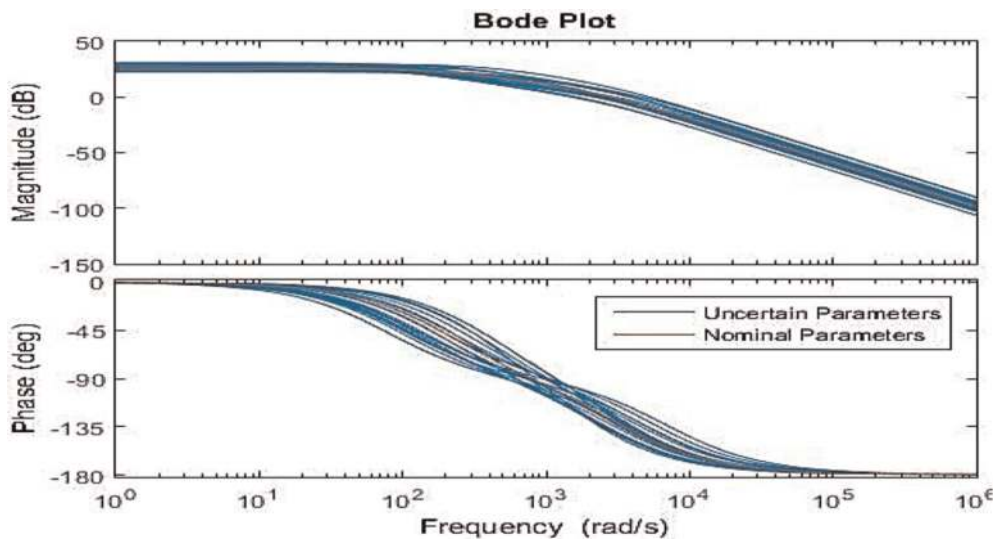
the gain of PMDC plant against variations of internal parameters. Similar variations are represented in the domain of frequency via bode plot, that are shown in **Figures 1** and **2**. Henceforth, PID controller is designed to address variation in the model and provide desired specifications for closed loop performance.

This chapter discusses the target specifications for better disturbance rejection and simultaneously set the point tracking. Here, we assume that the control bandwidth, exhibits increasing slope at the crossover frequency compared to disturbance rejection, allowing better gain within the bandwidth. The higher slope is described as lesser phase margin resulting in acceptable overshoot in response to set point. In order to match competing requirements of the tracking rejection and disturbance, the PID controller with 2-DOF (Degree of Freedom) is used as transfer function.

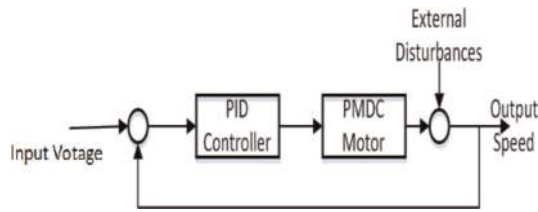
$$u = K_P(br - y) + \frac{K_i}{s}(r - y) + \frac{K_{Ds}}{1 + T_f s}(cr - y) \quad (6)$$



**Figure 1.**  
The variations in PMDC gain of motor plant for uncertain and nominal internal parameter variations.



**Figure 2.**  
Bode plot of plant model subjected to the internal parameter variations.



**Figure 3.**  
*Block diagram with external disturbances (load variations).*

2-DOF (Degree of Freedom) PID controllers contain weighing coefficients associated with derivative and proportional terms. These weighing coefficients facilitate effective disturbance rejection restraining maximization of the over-shoot under optimal conditions. The PID controller of 2-DOF works better in moderating the changes arising in the reference signal or control-signal.

The close loop control system is considered as shown in **Figure 3**. Here, the external disturbances such as load variations on motor shafts and its torque variations are added that influences on the motor parametric variations.

#### 4. The performance goals for the optimization of $H_{\infty}$

The 2-DOF for controller can be tuned by utilizing the approach of H infinity optimization. The open loop gain being a critical indicator of feedback loop behavior, gain of open loop should be more than one in control bandwidth to confirm better DR (Disturbance Rejection) and it must be lower than one outer of control bandwidth to confirm insensitivity to measurement noise of unmodelled dynamics. The ideal performance terms can be displayed regarding execution objectives. To accomplish a decent disturbance rejection and tracking, three execution objectives/limitations are forced on the tuning of controller gain (see discussion in previous section), which is as follows:

- i. "Tracking" – used to identify the RT (response time) to step input
- ii. "ML (Minimum loop) gain" – used to recognize loop gain before the frequency of crossover.
- iii. "ML (Maximum loop) gain" – used to to identify the control bandwidth at higher frequencies.

The CGs (Controller-gains) must be tuned with constraint i.e., function cost, connected with every specification subjected to minimization in H-infinity framework.

##### 4.1 Tracking as an execution objective

The frequency domain specification for monitoring between output and input is described in this performance target. This frequency domain constraint indicates the most extreme relative error as a FF (frequency function). The ME (maximum error) is given by:

$$error_{max} = \frac{(error_{peak})s + \omega_c(error_{dc})}{s + \omega_c} \quad (7)$$

where,  $\omega_c$  denotes cut-off frequency.

The scalar function  $f(x)$  describes the tracking goal, where  $x$  is denoted as a tuneable vector of entire parameters in the system. The target optimization (TO) is to modify the parameter function  $f(x)$ , which is optimized. The scalar function for tracking case is described using  $f(x)$ :

$$f(x) = \left\| \frac{1}{error_{max}} (T(s, x) - I) \right\|_{\infty} \quad (8)$$

where,  $T(s, x)$  is symbolized as closed loop transfer function between input and output.

#### 4.2 Min-LG (minimum) as execution limitation

The minimal gain on the open loop frequency response at specified frequencies is limited by this performance goal. The frequency dependent minimum gain constraint in turn gives the inverse sensitivity function of minimum gain limit. The min constraint gain characterizes the capacity of scalar function  $f(x)$  whereas the advancement procedure attempts to drive lower value of  $f(x)$ . The capacity of scalar  $f(x)$  is defined as:

$$f(x) = \|W_S(D^{-1}SD)\|_{\infty} \quad (9)$$

where,  $W_S$  is denoted as Min-LG profile and  $S$  is defined as sensitivity function.

#### 4.3 Max loop gain as execution limitation

This execution goal aims at highest gain of open loop at the determined frequencies in the given framework. The Max-LG can be characterized as the frequency domain element. This type of constraint restrains upper limit on the corresponding sensitivity. The maximum loop gain determines the scalar function  $f(x)$  given by:

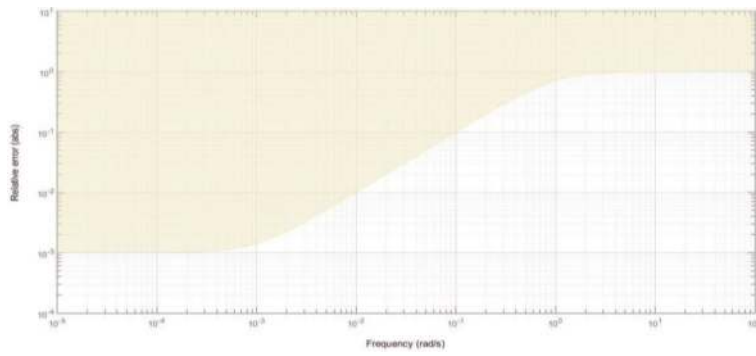
$$f(x) = \|W_T(D^{-1}TD)\|_{\infty} \quad (10)$$

where,  $W_T$  is symbolized as the reciprocal profile of the Max-LG.  $T$  is symbolized as the function of complementary sensitivity.

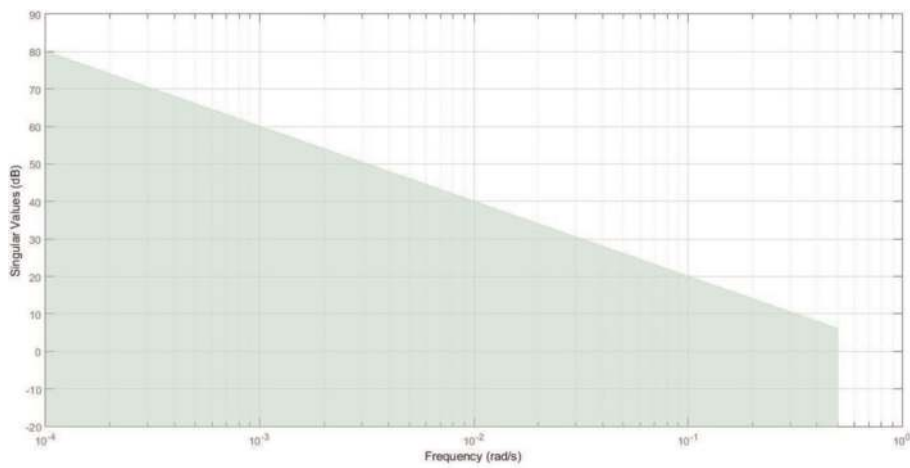
### 5. Performance goal description

The three performance goals for the H-infinity minimization such as tracking, maximum LG and minimum LG. The limits/range for these goals is expected as represented in **Figures 4–6** as:

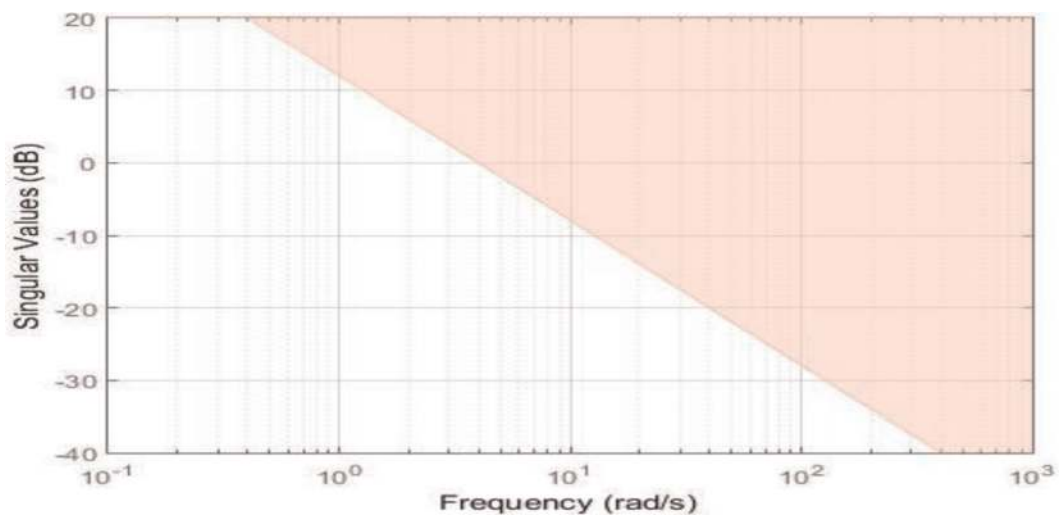
- Tracking better than 2 sec



**Figure 4.**  
Performance goal 1: The response for desired tracking.



**Figure 5.**  
Performance goal 2: The desired min-LG.



**Figure 6.**  
Performance goal 3: The desired max-LG.



- Min-LG: to be higher below 0.5 rad/s.
- Max-LG: to be less beyond 4 rad/s and the roll off with at least 20db/decade.

## 6. Simulations and results

The motor model transfer function as described in Eq. (5) along with the constraints from Eqs. (6)–(10) is simulated using MATLAB Simulink.

To model the uncertainty of plant parameters, the specified motor parameters can be used as variables. The PID controller of 2-DOF is characterized to have tunable gains. The analysis point is where disturbance torques are calculated and disturbance sensitiveness is determined. The performance goals as examined in Section 5 are selected and  $H_\infty$  minimization of the scalar function relating to the described performance goals is evolved in this work. This results in optimal tuning of controller gains and is tabulated as shown in **Table 2**.

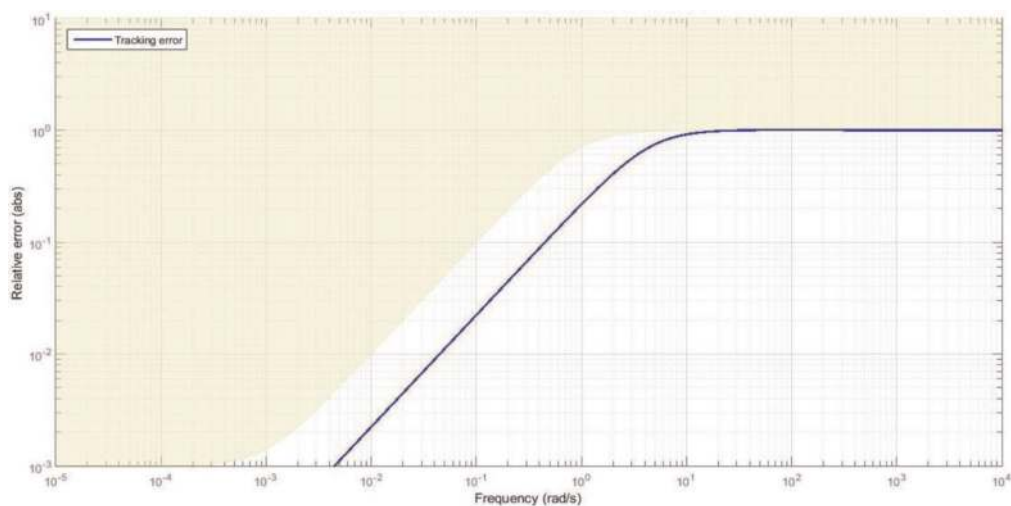
The outcomes are presented in the following figures.

The **Figure 7** represents response of TE (tracking error) of the tuned function of closed loop transfer function. This plot represents the tracking error that is

Parameter	Value
$K_p$	0.034
$T_f$	28.447
$K_d$	-0.942
$K_i$	0.341

**Table 2.**

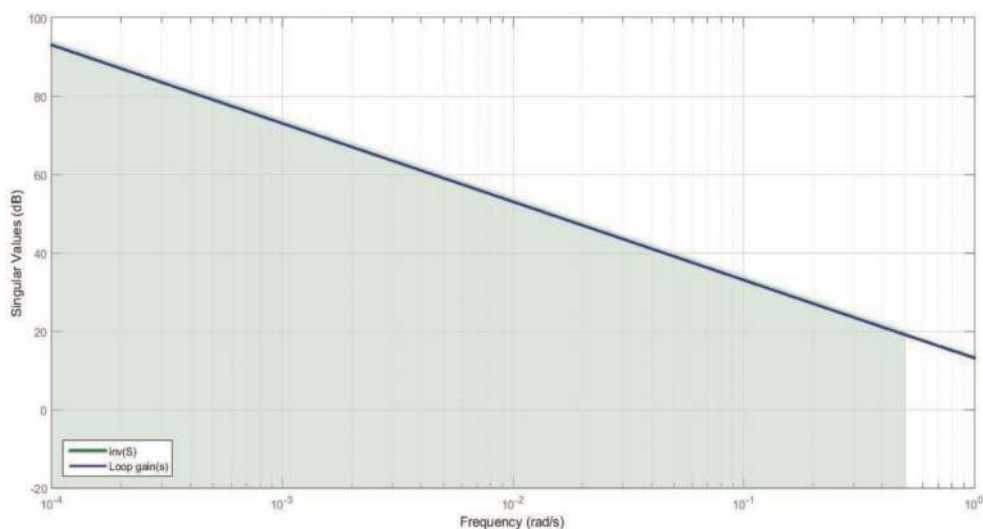
*The resulting tuned parameters of 2-DOF PID from simulation for optimization.*



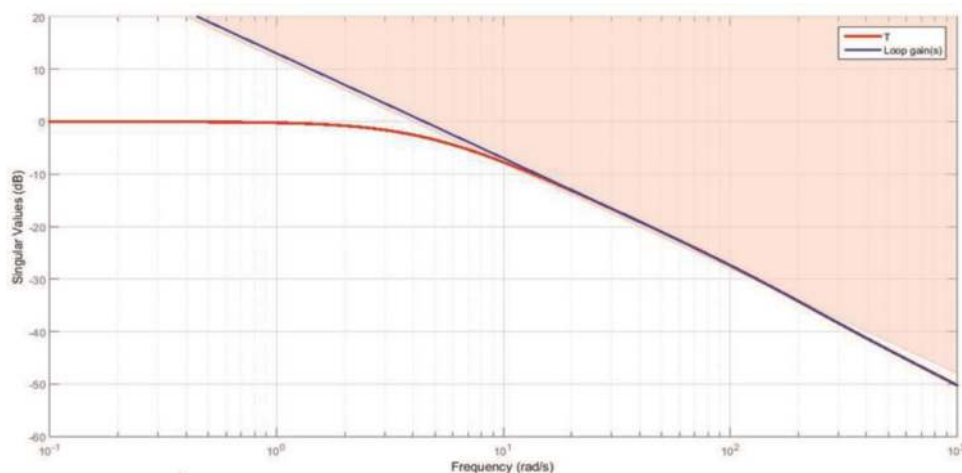
**Figure 7.**

*The performance goal 1: Desired v/s achieved.*





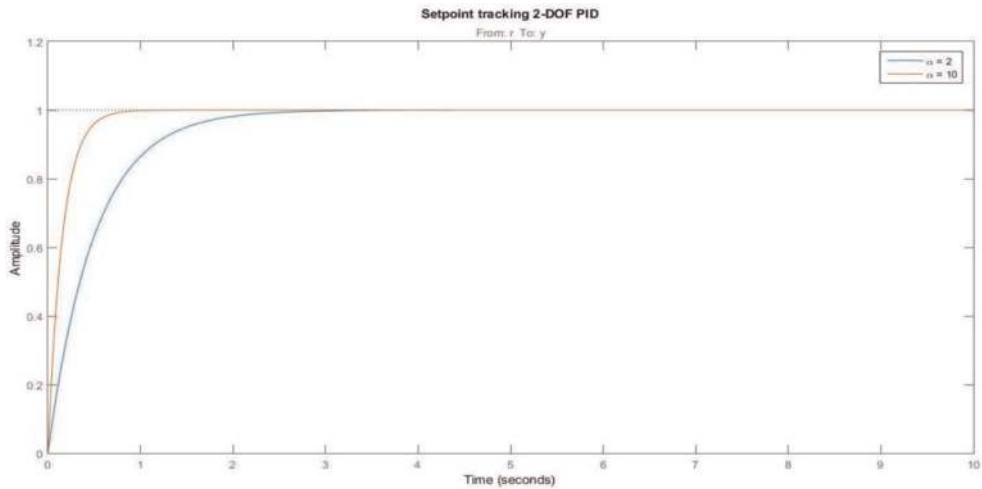
**Figure 8.**  
 The performance goal 2: Desired v/s achieved.



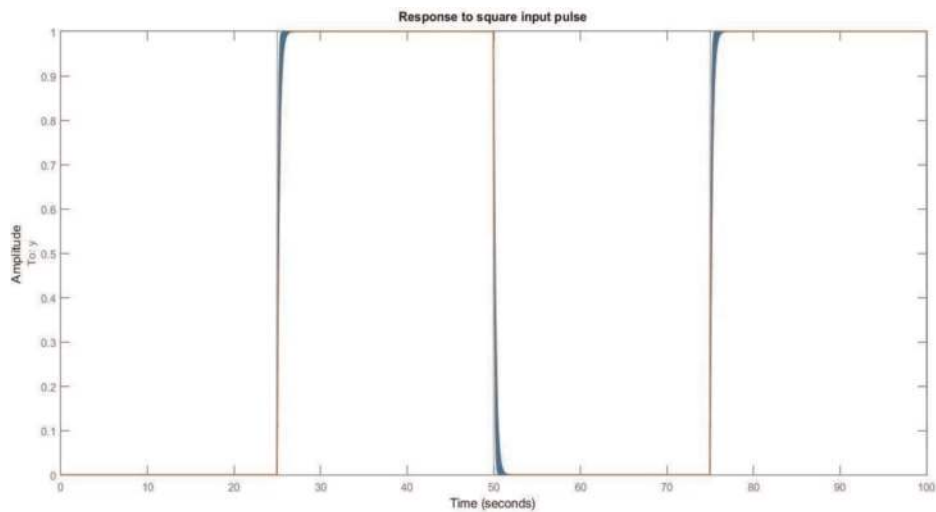
**Figure 9.**  
 The performance goal 3: Desired v/s achieved.

accomplished over all ranges of frequencies below the specified margin. Correspondingly, the max-LG and achieved min-loop v/s ideal values are plotted in **Figures 8** and **9**.

**Figure 8** shows the achieved transient duration of proposed model is much less that desired transient duration. **Figures 10** and **11** demonstrates tracking performance of the proposed model for any arbitrary input. In all cases, the set point of tracking can be accomplished well within the prescribed 2 seconds limitations in system response. From these plots, we infer that simulation of the proposed model guarantees motor internal parameters to remain within tolerable limits of variations for any applied arbitrary external disturbances. Further, the proposed model is tested for simulation with 30 different randomly chosen external disturbances.



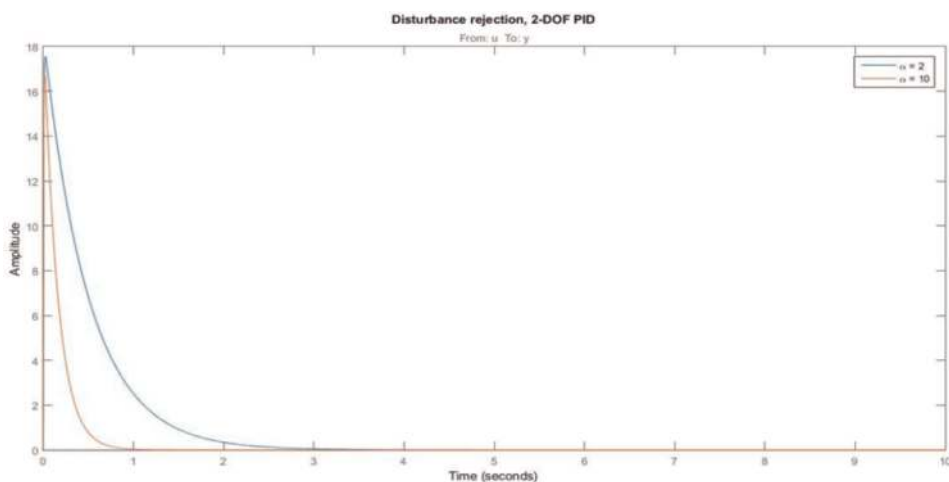
**Figure 10.**  
The performance of tracking with 2-DOF PID controller.



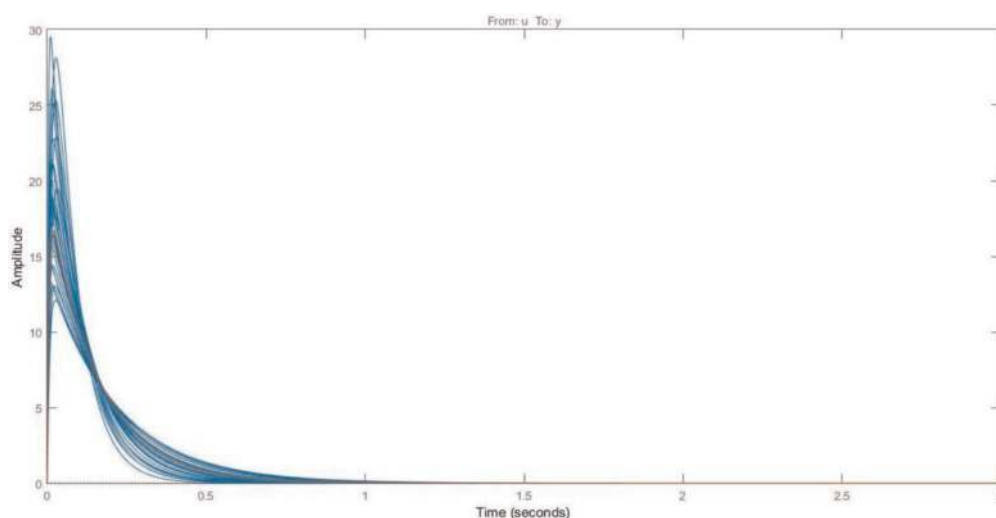
**Figure 11.**  
The performance of speed control to the pulse command (tracking better than the 2 sec).

It is significant to note that achieved performances are far superior to the focused objectives. Additionally,  $H_{\infty}$  advancement of targeted goals outperforms over expected performance. Out of several performance goals specified for PMDC motors, our work presents simulation results for only three objectives. However, other goals remain to be explored.

**Figure 12** represents the disturbance rejection achieved for two different values of proportional and derivative coefficients. The response for randomly chosen load disturbances (30 arbitrary signals) with implicit effect on internal parameters is presented in **Figure 13**. It can be observed that the response for the proposed robust model for controller is well within the accepted bounds. It is worth to observe that the disturbances attenuate well below the prescribed limit of 2 sec.



**Figure 12.**  
*Disturbance rejection with the controller of 2-DOF PID.*



**Figure 13.**  
*The response to 30 random disturbances with implicit parameter variations.*

## 7. Conclusion

This work introduces the MO-optimization (Multi Objective) with three performance goals in the structure of  $H_\infty$  for tuning parameters of PID controller for PMDC motor. The two simultaneous competing needs of disturbance rejection along with input tracking are achieved with Robust Controller design with 2-DOF over disturbance rejection using  $H_\infty$  framework. Hence, it is established that the PID controller can be tuned, and corresponding gains can be achieved for a specified constraints/performance capability. Further,  $H_\infty$  provides a better and acceptable framework for optimization.

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