Chapter 3

Effect of Various Edge Conditions on Free-Vibration Characteristics of Isotropic Square and Rectangular Plates

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Additional information is available at the end of the chapter

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Abstract

In this chapter, vibrations of isotropic rectangular plates have been analyzed by applying the wave propagation approach. The plate problem has been expressed in integral form by considering the strain and kinetic energies. The Hamilton's principle has been applied to transform the integral form into the partial differential equation of second order. The classical method namely product method has been used to separate independent variables. The partial differential equation has converted into the ordinary differential equations. The axial wave numbers are associated with particular boundary conditions. This is an approximate technique, which is based on eigenvalues of characteristic beam functions. The natural frequencies of plates are investigated versus modal numbers by varying the length and width of the plates with simply supported-simply supported (SS-SS), clamped-clamped (CC-CC), and simply supported-clamped (SS-CC) boundary conditions. The frequencies of the plates increase by increasing the modal number, and CC-CC frequencies are greater than the frequencies of other boundary conditions. Computational computer software MATLAB is engaged to characterize the frequencies. The results are compared with the earlier simulation work in order to test the accuracy and efficiency of the present method.

Keywords: rectangular plates, wave propagation approach, edge conditions, vibration

1. Introduction

Beams, plates, and shells are three main structural elements. Plates are widely used in various areas of engineering science like aerodynamics, civil and hydraulic instruments, vessels, mechanical structures, etc. Dynamical loading is applied on plates when they are involved in a physical system. Their dynamical behavior is studied theoretically. Vibration is an important feature of plates, having a critical role in applied field of engineering, which is investigated by researchers. A classical study of flexural vibrations of rectangular plates was performed by Mindlin et al. [1]. Leissa [2] compiled a book on vibration and buckling problems of the rectangular plates. He derived their natural frequencies. In the present study, vibrations of rectangular plates are analyzed by applying the wave propagation approach. This is an approximate technique which is based on eigenvalues of characteristic beam functions. Plates are structural elements that are frequently subjected to vibration, and controlling the frequency at which a plate vibrates is very important to structural designers. Free-vibration analyses of thin rectangular plates that are clamped on all edges have been studied by many researchers in the past with the aim of calculating its natural frequencies, and they have done using numerical approaches. Plates are important elements in various fields of engineering science and technology. Their extensive uses are found in aerodynamics, civil structures, hydraulic structures, containers, ships, instruments, and machine parts. Dynamical loadings are exerted on plates when they are involved in a physical system.

The theoretical vibration analysis is performed to approximate experimental values to evade any future complications. Many researchers [3–10] have worked on vibration characteristics of plates. For several decades, numerous measurements of vibrational properties and the vibrational behavior issues of plates have been studied in addition to different theories [10–12].

Natural frequencies for vibrating rectangular plates are obtained for various boundary conditions. Sakiyama and Huang [13] presented the vibration study of rectangular plates. The thickness of plates was variable. They based their analysis on the Green function method and obtained frequency spectra. They obtained some preliminary results for vibrations of plates. A best theoretical review on plates and shells has been presented by Ventsel and Krauthammer [14] using Galerkin method to carry out free-vibration analysis of SS-SS plate. Zhang et al. [15] presented a coupled structural-acoustic analysis for cylindrical shells having fluid inside them. They applied the wave propagation method for solving shell motion equations.

Werfalli and Karoud [16] conducted a vibration analysis of thin isotropic rectangular plates for number of end conditions. This study involved determination of their natural frequencies by applying the Galerkin method. Frequencies were obtained for a number of aspect ratios. They have discussed the formation of differential equations for plates and their analytical solutions. Hsu [17] applied a new version of differential quadrature technique to analyze vibrations of rectangular plates which rested on material foundations of elastic nature and carried any number of sprung masses. It was inferred that vibration characteristics of these types of plates can be studied for carrying any number of sprung masses and resting on the elastic foundations. Zhou and Ji [12] studied vibrations of rectangular plates associated with distributions of springs in uniform and continuous manner in a domain of rectangular nature. The Chebyshev polynomials in a series were taken to represent trial functions. Comparisons of natural frequencies verified accuracy and extensive applicability of the applied approach. From the results, it was noted that the natural frequencies and modes were exited in couples.

Mansour et al. [18] gave a theoretical wave propagation method to study vibrations of rectangular Kirchhoff plates. The direct exact solutions were obtained by Xing and Liu [11]. They

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applied method of separation of variables to solutions of the plate eigenvalue equation in exact shape. This solution procedure met the motion equation in the eigenvalues form and could be used for any kind of end conditions. Lal et al. [19] presented an analysis and numerical results for vibration characteristics of hetero-homogeneous rectangular plates with uniform thickness. The Rayleigh-Ritz technique method was used to solve the plate equation. Characteristic orthogonal polynomials were used for four conditions of clamped, simply supported, and free ends with mixed forms. The Gram-Schmidt procedure was used to produce orthogonal polynomials meeting end conditions. Sun et al. [20] used wave propagation approach to analyze vibrations of thin rotating circular cylinders. Njoku et al. [21] used Taylor series peculiar shape functions for clamped-clamped isotropic thin rectangular plates and applied Galerkin functional to determine the fundamental frequencies of the vibrating plate.

Recently, the vibrational behaviors of plates have been reported in Refs. [22–26]. However, up to now, little is known about the vibrational properties of plates and moreover the effects of the geometrical/material parameters by using extensive wave propagation approach (WPA).

In this chapter, the WPA proposed by Zhang et al. [15] for computing the vibrations is extended to plates, which is our particular motivation. Increasingly, numerical calculations have been used to investigate vibrational properties of plates through different models [5, 9, 12, 13, 21]. The method of choice is based on WPA that allows for the study of fundamental frequencies of plates over various combinations of geometric parameters, and this approach has become increasingly popular in the numerical solution of engineering applications. There are many theoretical and numerical techniques that have been used for vibration problems of plates such as Rayleigh-Ritz [19], differential quadrature method [17], Galerkin's technique [16, 21, 27], WPA [15, 28], finite element method [23], and structural element method (SEM) [29]. Despite of its conceptual simplicity, the continuum models and Galerkin's technique are subject to several computational problems which have to be addressed. The WPA was found to be a very popular tool to compute the vibrational properties of plates. Recently, the strong formulation of WPA has been applied for investigations fundamental frequencies of single-walled carbon nanotubes and detailed discussion is given in our earlier published work [30–32]. The present model based on WPA is, therefore, another choice of powerful numerical technique, whose results are appropriate in the limit of acceptable statistical errors than the earlier used Raleigh-Ritz [19] and Galerkin's techniques [16, 21].

The main objective of the present work is to generalize a modified model based on WPA first time and is determine how to calculate the frequencies of plates under various boundary conditions. In our case, the WPA is applied to solve the presented dynamical equations. The natural frequencies of plates are investigated versus modal numbers by varying the length and width of the plates with simply supported-simply supported (SS-SS), clamped-clamped (CC-CC), and simply supported-clamped (SS-CC) boundary conditions. The frequencies of the plates increase by increasing the modal number and CC-CC frequencies are greater than the frequencies of other boundary conditions. Computational computer software MATLAB is engaged to characterize the frequencies. The results are compared with the earlier simulation/method in order to test the accuracy and efficiency of the present method.

2. Formulation

Consider **Figures 1** and **2** representing a rectangular plate made with length 'a,' width 'b' and thickness 'h' as geometrical parameters and Young's modulus E, Poisson's ratio υ and mass density ρ as material parameters. A coordinate system has set at middle reference of the rectangular plate with 'x' and 'y' as coordinates along x- and y-axis, respectively, in xy-plane for the cases of simply supported-simply supported (SS-SS), clamped-clamped: clamped-clamped boundary conditions as shown in **Figures 1** and **2**.

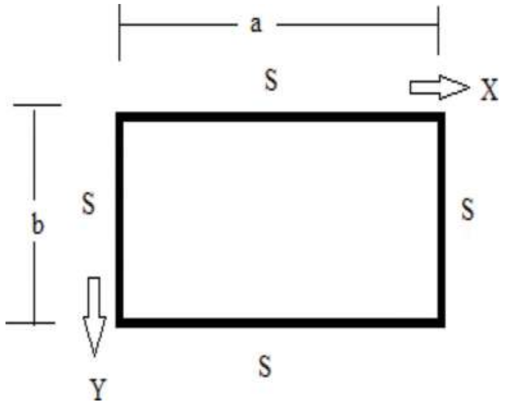

Figure 1. SS-SS plate.

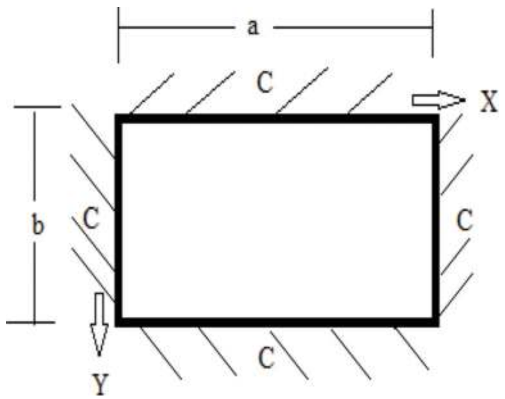

Figure 2. CC-CC plate.

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2.1. Governing equation of thin plate model

Consider a rectangular plate whose geometrical dimensions are '*a*' and '*b*.' Its thickness is denoted by *h*. Then, *a*, *b*, and *h* are called its geometrical parameters. Its material parameters are *E*, modulus of elasticity; *h*, plate thickness; *ν*, Poisson's ratio; *ρ*, density; Young's modulus, *E*; *v*, Poisson's ratio; and *ρ*, mass density. Suppose that *w (x,y,t)* designates the deformation displacement out of the plane of motion in the transverse direction. The strain energy, *U*, of this rectangular plate when it is vibrating, is expressed as:

$$U = \frac{1}{2}\int_0^b\int_o^a D\left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2}\right)^2 dxdy \tag{1}$$

The expression for kinetic energy, *T* for the rectangular plate is written as:

$$T = \frac{1}{2}\rho h\int_0^b\int_0^a \left(\frac{\partial^2 w(x,y,t)}{\partial t}\right)^2 dxdy \tag{2}$$

Here t denotes the time variable, $D = \frac{Eh^3}{12(1-\nu^2)}$ designates the flexural rigidity, E is Young's modulus, ρ is the density of the plate material, and h is the rectangular plate thickness. The Lagrangian energy variational functional is formulated by considering the expressions for the strain and kinetic energies of the vibrating rectangular pate and is written as:

$$\prod = T - U \tag{3}$$

For deriving, pate governing equation is obtained by applying the Hamiltonian variational principle [33]. This principle states that during very short interval of time, the change in the Lagrange functional is minimized. So using this principle to the expression (3), we get the following:

$$\int_{t_1}^{t_2} \delta\left(\prod\right) dt = 0 \tag{4}$$

Further, it can be written as:

$$\int_{t_1}^{t_2} \delta(T - U)dt = 0 \tag{5}$$

This process furnishes the governing equation that states the flexural vibration for the rectangular plates as:

$$\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y,t)}{\partial y^4} + \frac{\rho h}{D}\frac{\partial^2 w(x,y,t)}{\partial t^2} = 0 \tag{6}$$

Usually, energy variational methods are applied to investigate the vibration characteristics of structural elements namely: beams, plates, and shells. These methods consist of the Rayleigh-Ritz method [19] and the Galerkin method [16, 21]. When a vibrating problem is written in the integral form, the Rayleigh-Ritz technique is applied. When this problem is in the form of differential equations, the Galerkin procedure is applied. In both these techniques, axial modal dependence is assumed by different of mathematical functions which meet the boundary conditions described on the ends of structural elements. Frequently, beam functions are exploited for this purpose. These functions are obtained from the solutions of the beam differential equation for various end conditions.

The differential equation for a vibrating beam is written as:

$$\frac{\partial^4 w}{\partial x^4} + \alpha^4 \frac{\partial^2 w}{\partial t^2} = 0 \tag{7}$$

The general solution of the equation for the wave deformation displacement ω of the beam vibration can be written as:

$$w = \left\{A_1 e^{k_m x} + A_2 e^{-k_m x} + A_3 e^{ik_m x} + A_4 e^{-ik_m x}\right\} e^{i\omega t} \tag{8}$$

where k_m is the axial wave number whose value depends upon boundary conditions applied at the beam ends. ω is the natural angular frequency of the beam. In the solution (8), there are four terms which are functions of the axial variable, x. They represent the negatively decaying evanescent wave, the positively wave, the negatively propagating wave, and the positively propagating wave. Determination of values of $A_i^{'}s$ and the axial wave mode k_m are associated with edge conditions. Moreover, they are related to the eigenvalues of characteristic beam functions.

Expression (8) may further be written in the following:

$$w = \{B_1 \cos(k_m x) + B_2 \sin(k_m x) + B_3 coh(k_m x) + B_4 \sinh(k_m x)\} e^{i\omega t} \tag{9}$$

For wave propagation approach, this expression is truncated and is taken as:

$$w = e^{-ik_m x} e^{i\omega t} \tag{10}$$

This is illustrated by the example of boundary conditions viz., simple supported—simply supported. Application of these boundary conditions generates the trigonometric equation: $\sin(k_m d) = 0$. Infinite solution of this equation gives $k_m d = m\pi$, where m is the number of the axial standing waves and d is the dimension of the rectangular plate. Hence for a complete simply supported rectangular plate on four ends, $k_m = \frac{m\pi}{a}$ and $k_n = \frac{n\pi}{b}$ are used to evaluate natural frequency of the plate for the vibration parameters, m and n. Here, a and b are the dimensions, that is, length and width of the rectangular plate.

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2.2. Application of wave propagation approach

Area of determination of solutions of plate equation describing vibration phenomenon has been remained an attractive interests of mathematicians and engineers for their applied aspects. A comprehensive material on plate solutions has been compiled by Leissa [2]. The present era has been said to be the era of computer and its applications. The invention of computer has made the mathematical computational process very simple, and complicated expressions are simplified by applying computer software packages like Mathematica, Matlab, Maple, etc. Various numerical techniques are available to solve the differential equations found in engineering fields. With new developments in the world of computer science, complicated problems encountered in the areas of engineering and technology have been solved very easily and in efficient way. Linear and nonlinear differential equations have solved by the finite difference method, Rayleigh-Ritz method, the Galerkin method, finite element method, Fourier series method, and boundary element method. Ultimately, there are numerous commercially developed software packages. It is the basic interest of a researcher to apply a method which implicate less time and labor. This concept leads to develop a new technique which is more efficient and simple and provides accurate results. It has been seen that in the recent years, the wave propagation approach has been employed successfully to solve a number of shell and tube problems [28, 30–32]. Application of this approach reduces the differential equations in simple algebraic equations. For the present plate problem, this procedure is used to get the plate eigenvalue equation.

Modal displacement functions. For classical solutions of partial differential equations, method of separation of variables is employed to split the independent variables. In the governing differential equation of motion for rectangular plates, three independent variables are involved viz., two space variables x, y and one time variable, t. For splitting variables, the following modal displacement function forms are adopted:

$$w(x, y, t) = X(x)Y(y)\sin \omega t \tag{11}$$

or

$$w(x, y, t) = X(x)Y(y)\cos \omega t \tag{12}$$

or

$$w(x, y, t) = X(x)Y(y)e^{-i\omega t} \tag{13}$$

where $X(x)$ and $Y(y)$ are unknown functions. They are taken from algebraic functions and assumed to meet boundary conditions. A trigonometric function or an exponential complex function represents harmonic response. When modal form (7) or (8) or (9) is substituted in the equation of motion of plates,

$$\frac{\partial^4 X}{\partial x^4} + 2\frac{\partial^2 X}{\partial x^2}\frac{\partial^2 Y}{\partial y^2} + \frac{\partial^4 Y}{\partial y^4} = \frac{\rho h \omega^2}{D} \tag{14}$$

For wave propagation approach, the space modal functions $X(x)$ and $Y(y)$ are supposed to be the following forms:

$$X(x) = e^{-ik_m x} \tag{15}$$

and

$$Y(y) = e^{-ik_n y} \tag{16}$$

where k_m and k_n are axial mode wave numbers, and their values depend on the nature of boundary conditions specified at the plate four ends.

Making substitutions of expressions (14) and (15) in Eq. (13), we get

$$\frac{\partial^4 (e^{-ikmx})}{\partial x^4} + 2\frac{\partial^2 (e^{-ikmx})}{\partial x^2}\frac{\partial^2 (e^{-ikny})}{\partial y^2} + \frac{\partial 4 (e^{-ikn})}{\partial y^4} = \frac{\rho h \omega^2}{D} \tag{17}$$

or

$$k_m^4 + 2k_m^2 k_n^2 + k_n^4 = \frac{\rho h \omega^2}{D} \tag{18}$$

or

$$\frac{\rho h \omega^2}{D} = \left[k_m^4 + 2k_m^2 k_n^2 + k_n^4\right] \tag{19}$$

or

$$\frac{\rho h \omega^2}{D} = \left[k_m^2 + k_n^2\right]^2 \tag{20}$$

So the frequency equation for rectangular plates is obtained as:

$$\omega = \sqrt{\frac{D}{\rho h}}\left[k_m^2 + k_n^2\right] \tag{21}$$

2.3. Boundary conditions

By applying the Hamilton's principle [33] to the Lagrangian energy variational functional, the simply supported conditions are described as:

$$\Phi_m(x) = \frac{d^2\Phi_m(x)}{dx^2} = 0$$

at $x = 0$, *or* $x = L$

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and clamped condition is defined as:

$$\Phi_m(x) = \frac{d\Phi_m(x)}{dx} = 0$$

at $x = 0$ or $x = L$

and free condition is stated as:

$$\frac{\Phi^2{}_m(x)}{dx^2} = \frac{d^3\Phi_m(x)}{dx^3} = 0$$

at $x = 0$ or $x = L$.

Most of the vibration analysis of rotating functionally graded cylindrical shell with ring supports has been performed using the simply supported boundary conditions. In this case, the axial deformation displacement is estimated by the trigonometric functions, that is:

$$U(x) = \frac{d\Phi_m(x)}{dx} = \cos(m\pi x/L)$$

$$V(x) = W(x) = \Phi_m(x) = \sin(m\pi x/L)$$

Differential equations represent a physical problem and involve unknown functions. These functions are determined by applying some constraints on the boundary of solutions. These conditions are called boundary conditions. Plate vibration is an initial-boundary value problem and is transformed into the boundary value problem and four boundary conditions are described at four ends of a rectangular plate. For a rectangular plate with edges a and b, there are eight physical boundary conditions:

i. Fully simply supported end conditions

$$w(0, y, t) = 0, \frac{\partial^2 w(0, y, t)}{\partial x^2} = 0, w(a, y, t) = 0, \frac{\partial^2 w(a, y, t)}{\partial x^2} = 0 \tag{22}$$

$$w(x, 0, t) = 0, \frac{\partial^2 w(x, 0, t)}{\partial y^2} = 0, w(x, b, t) = 0, \frac{\partial^2 w(x, b, t)}{\partial y^2} = 0 \tag{23}$$

ii. Clamped-clamped end condition

$$w(0, y, t) = 0, \frac{\partial w(0, y, t)}{\partial x} = 0, w(a, y, t) = 0, \frac{\partial w(a, y, t)}{\partial x} = 0 \tag{24}$$

$$w(x, 0, t) = 0, \frac{\partial w(x, 0, t)}{\partial y} = 0, w(x, b, t) = 0, \frac{\partial w(x, b, t)}{\partial y} = 0 \tag{25}$$

iii. Two opposite ends simply supported and other two ends clamped

$$w(0,y,t)=0,\ \frac{\partial^2 w(0,y,t)}{\partial x^2}=0,\ w(a,y,t)=0,\ \frac{\partial^2 w(a,y,t)}{\partial x^2}=0 \tag{26}$$

$$w(x,0,t)=0,\ \frac{\partial w(x,0,t)}{\partial y}=0,\ w(x,b,t)=0,\ \frac{\partial w(x,b,t)}{\partial y}=0 \tag{27}$$

iv. Fully free end conditions

$$\frac{\partial^2 w(0,y,t)}{\partial x^2}=0,\ \frac{\partial^3 w(0,y,t)}{\partial x^3}=0,\ \frac{\partial^2 w(a,y,t)}{\partial x^2}=0,\ \frac{\partial^3 w(a,y,t)}{\partial x^3}=0 \tag{28}$$

$$\frac{\partial^2 w(x,0,t)}{\partial y^2}=0,\ \frac{\partial^3 w(x,0,t)}{\partial y^3}=0,\ \frac{\partial^2 w(x,b,t)}{\partial y^2}=0,\ \frac{\partial^3 w(x,b,t)}{\partial x^3}=0 \tag{29}$$

v. Clamped at two ends and free other two ends

$$w(0,y,t)=0,\ \frac{\partial w(0,y,t)}{\partial x}=0,\ w(a,y,t)=0,\ \frac{\partial w(a,y,t)}{\partial x}=0 \tag{30}$$

$$\frac{\partial^2 w(x,0,t)}{\partial y^2}=0,\ \frac{\partial^3 w(x,0,t)}{\partial y^3}=0,\ \frac{\partial^2 w(x,b,t)}{\partial y^2}=0,\ \frac{\partial^3 w(x,b,t)}{\partial y^3}=0 \tag{31}$$

2.4. Frequency equation for various boundary conditions

Using axial wave numbers, various frequency formulas can be formed for a number of boundary conditions.

1. SS-SS

$$\omega=\frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}\left[m^2+\frac{n^2a^2}{b^2}\right] \tag{32}$$

2. SS-SS and CC-CC

$$\omega=\frac{\pi^2}{a^2}\sqrt{\frac{D}{\rho h}}\left[m^2+\left(\frac{2n+1}{2}\right)^2\frac{a^2}{b^2}\right] \tag{33}$$

3. CC-CC and CC-CC

$$\omega=\frac{\pi^2}{4a^2}\sqrt{\frac{D}{\rho h}}\left[(2m+1)^2+(2n+1)^2\frac{a^2}{b^2}\right] \tag{34}$$

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4. SS-C and SS-C

$$\omega = \frac{\pi^2}{16a^2}\sqrt{\frac{D}{\rho h}}\left[(4m+1)^2 + (4n+1)^2\frac{a^2}{b^2}\right] \tag{35}$$

3. Results and discussion

According to various support conditions (CC-CC, SS-CC, and SS-SS), the fundamental frequencies (Hz) of the plate have been studied using wave propagation approach. The obtained results are discussed and compared with earlier theoretical results and simulation methods using same sets of material and geometrical parameters. In addition to these, as a final case study, the effect of CC-CC, SS-CC, and SS-SS for two sorts of plates (square and rectangular) is calculated and investigated. Their material properties, E, ν, and ρ, for isotropic plate are $2.07788 \times 10^{11} N/m^2$, 0.317756, and $8166 Kg/m^3$ [23], and boundary condition is specified in Ref. [31]. Here, a number of results are presented for vibrating rectangular isotropic plates. The vibration frequency equation for the plate has been obtained in terms of vibration, geometrical, and material parameters. The wave propagation approach has been applied for various boundary conditions. For the accuracy and stability of the present method, the findings are in good agreement with the existing results.

Tables 1 and **2** show the comparison of natural frequencies of simply supported square plate with FEM [23] and SEM [29]. As the number of modes increases, the frequencies also increase. This comparison shows that present approach is efficient to find the vibration of plates.

In **Tables 3–5**, the frequencies for a vibrating rectangular plate have been evaluated for modal parameters (m, n). It is observed that as m is kept fixed, n is allowed to vary, the frequency for the square plate is increased. Here, behavior of natural frequencies has been shown for a SS-SS,

	Frequencies (THz)		
Modes	**Present**	**FEM [23]**	**SEM [30]**
(1, 1)	4.845	4.857	4.866
(1, 2)	12.11	12.14	12.16
(2, 2)	19.28	19.43	19.46
(1, 3)	24.19	24.28	24.28
(2, 3)	31.41	31.57	31.6
(4, 1)	41.10	41.26	41.3
(3, 3)	43.61	43.71	43.75

Table 1. Convergence of natural frequencies (Hz) with FEM and SEM of simply supported square plate.

	Frequencies (THz)		
Modes	**Present**	**FEM [31]**	**SEM [32]**
(1, 1)	55.10	55.13	55.15
(1, 2)	55.52	55.73	55.75
(2, 2)	56.42	56.62	56.67
(1, 3)	65.50	65.52	65.59
(2, 3)	69.68	69.60	69.68
(4, 1)	77.12	77.19	77.30
(3, 3)	83.04	83.18	83.33

Table 2. Convergence of natural frequencies (Hz) with FEM and SEM of clamped square plate.

CC-CC, and SS-CC rectangular plate with regard to geometrical and material parameters. The frequencies of CC-CC plate are greater than that of SS-CC and SS-SS.

Figures 3 and **4** show the variation of frequencies versus modal wave number. **Figure 3** is drawn for the square plate when we take $a = b = 3$ m and $m = 1, 2$, $n = 1 \sim 5$ with various boundary conditions and show the influence of these conditions. As the constraints in the end conditions are applied more, the frequencies increase. It has been seen that the frequency is

Modal parameter (*m*, *n*)	ω	*f*(Hz)	Modal parameter (*m*, *n*)	ω	*f*(Hz)	Modal parameter (*m*, *n*)	ω	*f*(Hz)
(1, 1)	152.66	24.2972	(2, 1)	381.66	60.743	(3, 1)	763.32	121.486
(1, 2)	381.65	60.743	(2, 2)	610.656	97.1889	(3, 2)	992.316	157.932
(1, 3)	763.31	121.486	(2, 3)	992.316	157.932	(3, 3)	1374	218.675
(1, 4)	1297.6	206.526	(2, 4)	1526.6	242.972	(3, 4)	1908.3	303.715
(1, 5)	1984.6	315.864	(2, 5)	2213.6	352.31	(3, 5)	2595.3	413.053

Table 3. Variations of natural frequencies (Hz) of simply supported plate with modal parameter *(m, n)* ($a = 1$ m, $b = 1$ m, $E = 2.052 \times 10^{11}$ N/m^2; $\upsilon = 0.3$, $\rho = 7850$ kg/m^3).

Modal parameter (*m*, *n*)	ω	*f*(Hz)	Modal parameter (*m*, *n*)	ω	*f*(Hz)	Modal parameter (*m*, *n*)	ω	*f*(Hz)
(1, 1)	248.079	39.483	(2, 1)	477.075	75.9288	(3, 1)	858.735	136.672
(1, 2)	553.407	88.0774	(2, 2)	782.403	124.523	(3, 2)	1164.1	185.266
(1, 3)	1011.4	160.969	(2, 3)	1240.4	197.415	(3, 3)	1622.1	258.158
(1, 4)	1622.1	258.158	(2, 4)	1851.1	294.604	(3, 4)	2232.7	355.347
(1, 5)	2385.4	379.644	(2, 5)	2614.4	416.089	(3, 5)	2996	476.833

Table 4. Variations of natural frequencies (Hz) of simply supported-clamped plate with modal parameter *(m, n)* ($a = 1$ m, $b = 1$ m, $E = 2.052 \times 10^{11}$ N/m^2; $\upsilon = 0.3$, $\rho = 7850$ kg/m^3).

Modal parameter (*m*, *n*)	ω	*f*(Hz)	Modal parameter (*m*, *n*)	ω	*f*(Hz)	Modal parameter (*m*, *n*)	ω	*f*(Hz)
(1, 1)	343.494	54.6687	(2, 1)	648.822	103.263	(3, 1)	1106.8	176.155
(1, 2)	648.822	103.263	(2, 2)	954.15	151.858	(3, 2)	1412.1	224.749
(1, 3)	1106.8	176.155	(2, 3)	1412.1	224.749	(3, 3)	1870.1	297.641
(1, 4)	1717.5	273.344	(2, 4)	2022.8	321.938	(3, 4)	2480.8	394.83
(1, 5)	2480.8	394.83	(2, 5)	2786.1	443.424	(3, 5)	3244.1	516.316

Table 5. Variations of natural frequencies (Hz) of clamped-clamped plate with modal parameter *(m, n)* (a = 1 m, b = 1 m, $E = 2.052 \times 10^{11}$ N/m^2; υ = 0.3, ρ = 7850 kg/m^3).

almost two times for m = 1 and m = 2 for CC-CC, SS-CC, and SS-SS boundary condition. **Figure 4** is drawn for the square plate when we take $a = b = 4$ m and m = 3, 4, $n = 1 \sim 5$ with various boundary conditions and show the influence of these conditions. As the constraints in the end conditions are applied more, the frequencies increase. The frequency gapes for these boundary conditions at m, n = (1, 2), (2, 1) at $a = b$ = 3, 4 are very close to each other, and as we proceed the modal number, then the frequency gape is higher.

Figures 5 and **6** show the variation of natural frequencies (Hz) of rectangular plates versus the vibration modal wave number (m, n) for the boundary conditions on four edges viz., SS-SS, CC-CC, and SS-CC square plate. **Figure 5** is drawn for the rectangular plate when we take a = 3, b = 2 m. As the constraints in the end conditions are applied more, the frequencies increase. Here, we fix the value of m = 1 but vary the value of n from $1 \sim 5$. It is observed that frequency increases by increasing the value of n. It has been seen that the frequency is almost two times for m = 1 and m = 2 for SS-SS, SS-CC, and CC-CC boundary condition. The frequency curves are observed closed to each other for modal wave number, (m, n) = (1, 1), (1, 2), (2, 1), (2, 2).

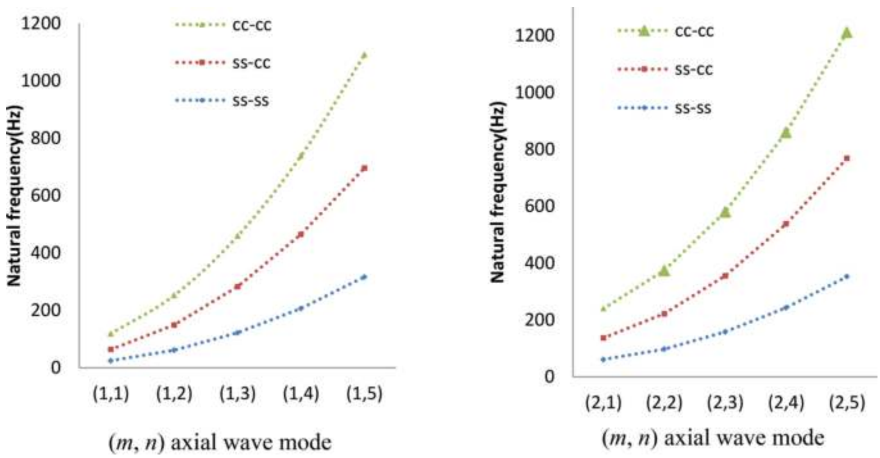

Figure 3. The influence of natural frequencies of a square plate for SS-SS, SS-CC, and CC-CC boundary conditions (a = 3 m, b = 3 m, $E = 2.052 \times 10^{11}$ N/m^2; υ = 0.3, ρ = 7850 kg/m^3).

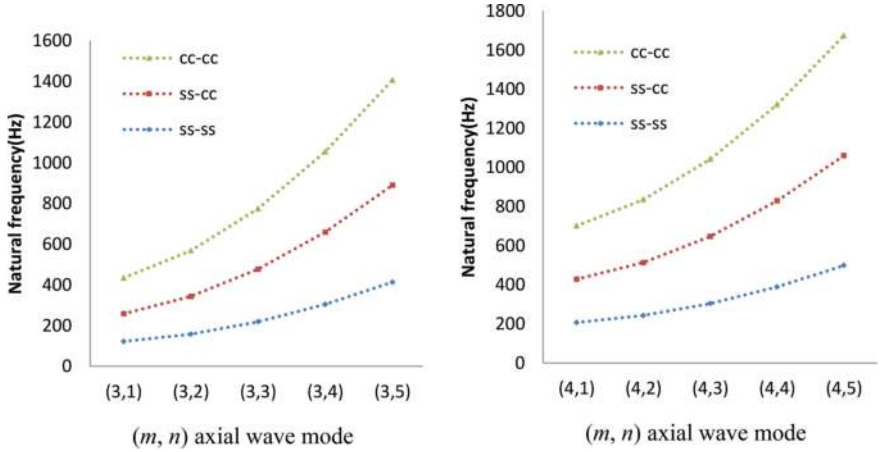

Figure 4. The influence of natural frequencies of a square plate for SS-SS, SS-CC, and CC-CC boundary conditions ($a = 4$ m, $b = 4$ m, $E = 2.052 \times 10^{11}$ N/m^2; $\upsilon = 0.3$, $\rho = 7850$ kg/m^3).

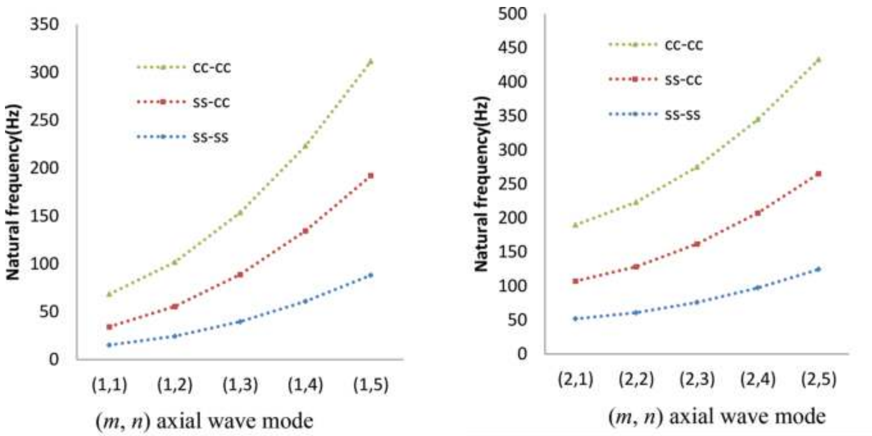

Figure 5. The influence of natural frequencies of a rectangular plate for SS-SS, SS-CC, and CC-CC boundary conditions ($a = 3$ m, $b = 2$ m, $E = 2.052 \times 10^{11}$ N/m^2; $\upsilon = 0.3$, $\rho = 7850$ kg/m^3).

Figure 6 is drawn for the rectangular plate when we take $a = 4$, $b = 3$ m. It can be perceived from above discussion that the CC-CC boundary conditions have the height frequency curves of rectangular plates and other boundary condition followed as SS-CC and SS-SS. It is also concluded that the frequency curves with SS-SS boundary condition are the lowest for varying the modal wave number. For these boundary conditions, on increasing the length a and width b, the frequencies also increase (m, n). In these figures, it can be seen that the gap between the CC-CC and SS-CC is greater than that of SS-SS boundary condition. **Figures 5** and **6** also show that the trend of frequency is same for the symmetry of (m, n).

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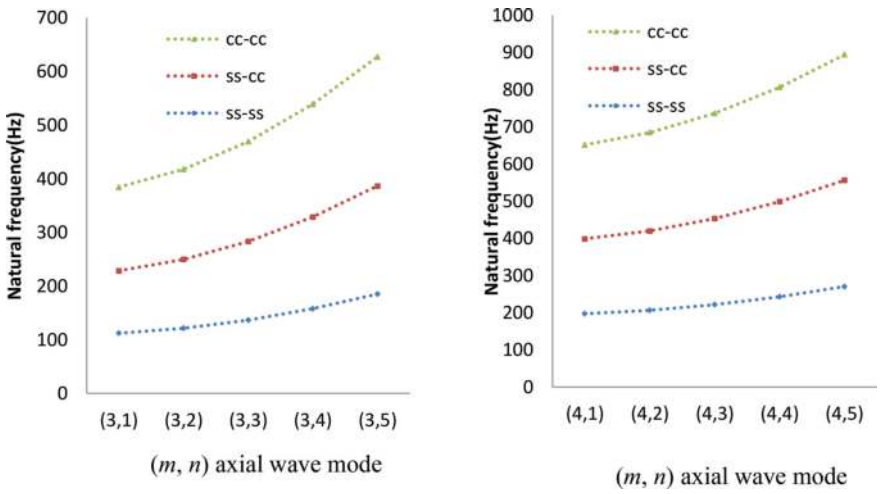

Figure 6. The influence of natural frequencies of a rectangular plate for SS-SS, SS-CC, and CC-CC boundary conditions ($a = 4$ m, $b = 3$ m, $E = 2.052 \times 10^{11}$ N/m^2; $\upsilon = 0.3$, $\rho = 7850$ kg/m^3).

4. Concluding remarks

In this study, vibrations of isotropic square and rectangular plates have been investigated for modal parameters. Wave propagation approach has been engaged to solve this problem. The axial deformations along axial variables are approximated by the complex exponential functions. The axial wave numbers are associated with particular boundary conditions. As the modal wave numbers are enhanced, the frequencies for the plates indefinitely. Moreover, the influence of the boundary conditions has been studied for by changing the axial wave modes. Dynamical loadings are exerted on plates when they are involved in a physical system. Their dynamical behavior is studied theoretically. In the present study, vibrations of square and rectangular plates are analyzed by applying the wave propagation approach. This is an approximate technique related to the axial wave modes obtained characteristic beam functions. These axial wave modes represent boundary conditions specified at four ends of a rectangular plate. Natural frequencies for vibrating square and rectangular plates are obtained for various boundary conditions. The natural frequencies of plates are investigated versus modal numbers by varying the length and width of the plates with simply supported- simply supported (SS-SS), clamped-clamped (CC-CC), and simply supported-clamped (SS-CC) boundary conditions. The frequencies of the plates increase by increasing the modal number, and CC-CC frequencies are greater than the frequencies of other boundary conditions. If we change the nature of material of plate or other physical parameters applied to maintain motion in radial direction, then a new problem can be formed. These problems can be solved for different set of boundary conditions. This analysis can be applied to examine the vibrations of functionally graded material plates.

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