Chapter

On the Zap Integral Operators over Fourier Transforms

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Abstract

We devote the current chapter to describe a class of integral operators with properties equivalent to a killer operator of the quantum mechanics theory acting over a determined state, literally killing the state but now operating over some kind of Fourier integral transforms that satisfies a certain Fredholm integral equation, we call this operators Zap Integral Operators (ZIO). The result of this action is to eliminate the inhomogeneous term and recover a homogeneous integral equation. We show that thanks to this class of operators we can explain the presence of two extremely different solutions of the same Generalized Inhomogeneous Fredholm equation. So we can regard the Generalized Inhomogeneous Fredholm Equation as a Super-Equation with two kinds of solutions, the resonant and the conventional but coexisting simultaneously. Also, we remember the generalized projection operators and we show they are the precursors of the ZIO. We present simultaneous academic examples for both kinds of solutions.

Keywords: integral operators, generalized inhomogeneous Fredholm equations, killer operators, evanescent waves, electromagnetic resonances

1. Introduction

Recently a new question about the solutions of integral Fredholm emerges, that is the question about the type of equation each of them solve. If we follow the steps or the clue marked by the linear second order differential equations the solutions of the inhomogeneous equation do not solve de homogeneous equation. But we have shown in a recent paper that both kind of solutions of the homogeneous and also the inhomogeneous Fredholm equations satisfy a third class of integral equation we named the Generalized Inhomogeneous Fredholm Equation (GIFE) which is only a bit different for the traditional inhomogeneous [1–3]. Even more, we can transform his appearance in a continuous form from homogeneous to inhomogeneous, but preserving his very extraordinary property: the two kinds of solutions are simultaneous solutions. This situation is quite different from differential equations but not the connection between eigenfunctions and solutions of inhomogeneous equations through the Green function [4–7]. And if we want to explain this behavior we find a founder: an integral operator which is hidden in the structure of the GIFE. There is no surprise in the fact that the new operator treats in different manner both kinds of solutions. Indeed, it seems to be natural that the new operators include the Green function and are close to the Fredholm operator [2, 3]. Before we define the ZIO operators we must underline the fact that in a broadcasting situation [8–10] we must take into accounts not only one kind of traveling waves but all the known ones because the complete description of the phenomena comes from the GIFE. Another important goal of this paper is to give an explanation of the simultaneous validity of two sets of boundary conditions that are very apart one to the other and the fact that there is a connection with other projection operators, the generalized projection operators (GPO) [11] that separates the constituents of a signal in orthogonal parts.

2. Remembering the GIFE

We remember that if we take the inhomogeneous vector integral Fredholm Eq. (1):

$$u^m(\mathbf{r},\omega) = u^{m(\circ)}(\mathbf{r},\omega) + \lambda(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')u^n(\mathbf{r}',\omega)dr' \tag{1}$$

Where the kernel $\mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')$, is the product of the interaction $A_t^m(\omega;\mathbf{r},\mathbf{s})$ (may be a non-local potential) with the free Green function $\mathbf{G}_n^{t(\circ)}(\omega;\mathbf{r},\mathbf{s})$.

And we make the ansatz of two successive approximations (a second order approach) [9], by the consideration that $\lambda(\omega)$ is a number with a very small absolute value ($|\lambda(\omega)| \ll 1$), we arrive to the integral equation we named the GIFE:

$$s^m(\mathbf{r},\omega) = s^{m(\circ)}(\mathbf{r},\omega) + \Theta^m(\mathbf{r},\omega) + \nu(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')s^n(\mathbf{r}',\omega)dr' \tag{2}$$

This last equation is the one that have the property of represent a complete panorama in a broadcasting problem, that is describes both the resonant and the conventional behavior of the electromagnetic field [12].

As we have commented, Eq. (2) carries a mechanism that allows simultaneously consider both types of solution. The so called generalized source is indeed a blend of integral operators as we will see with properties we want to visualize. But first we must present the Generalized Homogeneous Fredholm Equation (GHFE) [1]:

$$y_e^m(\mathbf{r};\omega) = \eta_e(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')y_e^n dr' \tag{3}$$

Eq. (3) has a special index e that mean a specific resonance [1, 4, 5, 8, 9]. Among the three Eqs. (1), (2), and (3) there are a common ingredient, for each equation we have used different names: λ, ν and η [1–3] but any of them can be incorporated to the kernel or used as an independent function or even an eigenvalue. In order to connect the homogeneous and inhomogeneous equation we must define some functions as we will see in the next sections.

3. Connection between the eigenvalues η_e and the functionλ

We know that because of the Hilbert-Schmidt theory [2, 3] and more recently by our previous results [1], the solutions of Eq. (3) that is all the $y_e^m(\mathbf{r};\omega)$, form a set of orthogonal functions and then a set of eigenvalues $\eta_e(\omega)$. Thus we can relate the functions appearing in Eqs. (1) and (3) as follows:

By means of the spectral representation of Green function, [2, 3] we have:

$$\mathbf{G}_m^{n(\circ)}(\omega;\mathbf{r},\mathbf{s}) = \sum_e C_e \frac{y_e^m(\mathbf{r})y_e^n(\mathbf{s})}{\lambda - \eta_e} \tag{4}$$

And also

$$u^m(\mathbf{r},\omega) = u^{m(\circ)}(\mathbf{r},\omega) - \sum_{e=1}^{\infty}\int_0^{\infty}\frac{y_e^m(\mathbf{r},\omega)y_e^n(\mathbf{r}',\omega)}{\lambda(\omega)-\eta_e(\omega)}u^{m(\circ)}(\mathbf{r}';\omega)dr' \tag{5}$$

The orthogonality relation is

$$\int_0^{\infty} y_e^m(\mathbf{r};\omega)A^{mn}y_i^n(\mathbf{r};\omega)dr = 0 \text{ if} i \neq e \tag{6}$$

4. Conditions imposed over the homogeneous Fredholm equations

In accordance with the theory of homogeneous Fredholm integral equations [1, 2, 13], the first Fredholm minor is a two point function, like a Green function, which must comply with an integral equation:

$$\mathcal{M}^m(\mathbf{r},\mathbf{r}_0;\omega) = \eta(\omega)\Delta(\eta,\omega) + \eta(\omega)\int_0^{\infty}\mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{s})\mathcal{M}^n(\mathbf{s},\mathbf{r}_0;\omega)ds \tag{7}$$

Two other conditions must be satisfied:

The first is that Fredholm determinant is zero

$$\Delta(\eta,\omega) = 0 \tag{8}$$

The second that the Fredholm eigenvalue equals to one:

$$\eta(\omega) = 1 \tag{9}$$

But thanks to our second order approximation Eq. (2) we can show that other interesting conditions are satisfied, for example if we define some particular functions (and operators):

$$\Psi(\mathbf{r};\omega) \equiv \mathcal{M}^m(\mathbf{r},\mathbf{r}_0;\omega) - \Delta(\eta,\omega)u^m(\mathbf{r};\omega) \tag{10}$$

And also

$$\Psi^{(\circ)}(\mathbf{r};\omega) \equiv \Delta(\eta,\omega)\left[\eta(\omega) - u^{m(\circ)}(\mathbf{r};\omega)\right] + \Delta(\eta,\omega)[\eta(\omega) - \nu(\omega)]\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')u^n(\mathbf{r}';\omega)dr' \tag{11}$$

We can see that the first Fredholm minor must satisfy through Ψ the inhomogeneous equation

$$\Psi(\mathbf{r};\omega) \equiv \Psi^{(\circ)}(\mathbf{r};\omega) + \eta(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{t})\Psi(\mathbf{t};\omega)dt \tag{12}$$

In order to write Eq. (2) in terms of the solutions of Eq. (1), we can define the operator:

$$\Theta^m(\mathbf{r};\omega) \equiv \nu^2(\omega)\varepsilon(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')\int_0^\infty \mathbf{K}_l^{n(\circ)}(\omega;\mathbf{r}',\mathbf{r}'')\mathrm{P}^m(\mathbf{r}'';\omega)dr''dr' \tag{13}$$

In Eq. (13) the function $\mathrm{P}^m(\mathbf{r};\omega)$ is an arbitrary negative exponential regulator.

Near a resonance the two small parameters $\nu(\omega)$ and $\varepsilon(\omega)$ makes $\Theta^m(\mathbf{r};\omega)$ lesser than a second order term, so can be neglected. Far of a resonance this later function sketches the behavior of the simultaneous existence of the resonant and non-resonant solutions because in terms of $\Theta^m(\mathbf{r};\omega)$ the conventional waves satisfy the inhomogeneous equation:

$$u^m(\mathbf{r};\omega) = u^{m(\circ)}(\mathbf{r};\omega) + \Theta^m(\mathbf{r};\omega) + \nu(\omega)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')u^n(\mathbf{r}';\omega)dr' \tag{14}$$

5. Defining a new class of integral operators

As we said in Section **1**, hidden in the structure of Eq. (2) there are some integral operators which allow the simultaneous existence of solutions with extremely different boundary conditions. So, let us define the Zap operators by the rules:

$$\mathrm{Z}u^m(\mathbf{r};\omega) \equiv \mathrm{Z}\left[\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right]u^m(\mathbf{r};\omega) = + \Delta(\eta)\left[\eta - u^{m(\mathrm{o})}(\mathbf{r};\omega)\right] + \eta\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')u^n(\mathbf{r}';\omega)dr' \tag{15}$$

That is, the Zap operator is associated to the integral Fredholm equation satisfied by the affected solution ($u^m(\mathbf{r};\omega)$ or $y_e^m(\mathbf{r};\omega)$), from which takes the source term and the free kernel.

DOI: http://dx.doi.org/10.5772/intechopen.94573

The same operator (15) acting over a homogeneous equation looks like

$$\begin{aligned} Zy_e^m(\mathbf{r};\omega) \equiv Z[\mathbf{r};\omega;0]y_e^m(\mathbf{r};\omega) = \\ +\Delta(\eta)[\eta-0] \\ +\eta\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')y_e^n(\mathbf{r}';\omega)dr' \end{aligned} \tag{16}$$

That is

$$Zy_e^m(\mathbf{r};\omega) \equiv Z[\mathbf{r};\omega;0]y_e^m(\mathbf{r};\omega) = \lambda y_e^m(\mathbf{r};\omega) \tag{17}$$

As we can see the effect of the Zap operator is to kill or eliminate the inhomogeneous term when applied to a resonant state. But this seems very artificial because we are giving indeed two parts for the complete rule. However we can build projection operators that can make the work we need.

Now, we define Zap projection operators in the next section.

6. The zap projection operators and their properties

On this section, we define the so named Zap projection operators (ZPO) which enable us to project a complex broadcasting system over a reduced resonant simplest one. The Zap operators acts over Fourier transforms [14, 15] related to integral operators.

Then, based on (15) and (17), we define de following operator:

$$\mathbb{Z}_P\left(\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right) = \lim_{\eta\to1} Z\left(\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right) \tag{18}$$

In order to get a display of the properties of this operator we propose a specific set of discrete antennas in the next example:

Suppose that we have p punctual sources that can be represented in the inhomogeneous term of the Fredholm equation like:

$$u^{m(\circ)}(\mathbf{r};\omega) = \sum_{i=1}^{p} \alpha_i^n \delta(\mathbf{r}-\mathbf{r}_i)\mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}_i) \tag{19}$$

Then, by applying the projection operator to Eq. (19) we have (remember that when $\eta = 1$ thus $\Delta = 0$):

$$\begin{aligned} \mathbb{Z}_P\left[\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right]u^m(\mathbf{r};\omega) = \lim_{\eta\to1}[\Delta(\eta)\left(\eta - \sum_{i=1}^{p}\alpha_i^n\delta(\mathbf{r}-\mathbf{r}_i)\mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}_i)\right) \\ +\eta\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')u^n(\mathbf{r}';\omega)dr'] \end{aligned} \tag{20}$$

Now, because $\eta = 1$ implies $\Delta = 0$, and because also $\nu = \lambda = 1$

$$\mathbb{Z}_P\left[\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right]u^m(\mathbf{r};\omega) = y_e^m(\mathbf{r};\omega) \tag{21}$$

In the last step we have used the fact that the solution of the remaining homogeneous equation is denoted by $y_e^m(\mathbf{r};\omega)$.

Eq. (21) says that if we take a blend of regular and resonant solutions we have:

$$\mathbb{Z}_P\left[\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right](u^m(\mathbf{r};\omega)+y_e^m(\mathbf{r};\omega))=2y_e^m(\mathbf{r};\omega) \tag{22}$$

So taken into account from Eqs. (18) until (22), we see that we have projected the original problem into a resonant one.

In analogy with $\mathbb{Z}_P$ we can define a projector over their complement:

Let us define the complementary Zap projection operator as

$$\begin{aligned}\mathbb{Z}_Q\left(\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right)u^m(\mathbf{r};\omega) &\equiv \lim_{\eta\to1} \mathrm{Z}^C\left(\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right)\\ &\equiv \lim_{\eta\to1}\{(\Delta(\eta)\eta+\nu)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')u^n(\mathbf{r}';\omega)dr'\\ &+\eta u^{m(\circ)}(\mathbf{r};\omega)\}=u^m(\mathbf{r};\omega)\end{aligned} \tag{23}$$

Even we apply $\mathbb{Z}_Q$ to a resonant state:

$$\begin{aligned}\mathbb{Z}_Q\left(\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right)y_e^m(\mathbf{r};\omega) &\equiv \lim_{\eta\to1} \mathrm{Z}^C\left(\mathbf{r};\omega;u^{m(\circ)}(\mathbf{r};\omega)\right)\\ &= \lim_{\eta\to1}\{(\Delta(\eta)\eta+\nu)\int_0^\infty \mathbf{K}_n^{m(\circ)}(\omega;\mathbf{r},\mathbf{r}')y_e^n(\mathbf{r}';\omega)dr'\\ &+\eta u^{m(\circ)}(\mathbf{r};\omega)\}=u^m(\mathbf{r};\omega)\end{aligned} \tag{24}$$

This is because the name of the solution of the remaining inhomogeneous equation is precisely $u^m(\mathbf{r};\omega)$.

7. An academic example for conventional traveling waves

In order to convince us of the utility of the $\mathbb{Z}_P$ and $\mathbb{Z}_Q$ operators we remember that in all of our developments the kernel always is $\mathbf{K}_n^{m(\circ)}$ that only contains the free Green function $\mathbf{G}_n^{t(\circ)}(\omega;\mathbf{r},\mathbf{s})$. But then, there is no difference between the kernels of the integral equations when are referred to conventional traveling waves or to evanescent or resonant waves. This last statement allows describing in an algebraic mode the application of the Zap projection operators. In this manner we can fix our kernel in accordance with a previous example that we have presented in some place as the matrix (27).

For the case of only two source points and omitting the three components of the field lifting only one, this matrix can be for example:

But first remember that

$$\int_V A(\mathbf{r}')G_\omega^{(\circ)}(\mathbf{r};\mathbf{r}')u(\mathbf{r}';\omega)dV' \equiv \mathbf{K}^{(\circ)}(\omega)u(\mathbf{r};\omega) \tag{25}$$

In Eq. (25) $A(\mathbf{r})$ is the interaction that in the general case may contain a non-local potential, but not in our example.

$$\mathbf{K}^{(\circ)}(\omega)u(\mathbf{r};\omega) \equiv \begin{pmatrix} \mathbf{K}_{11}^{(\circ)} & \mathbf{K}_{12}^{(\circ)} \\ \mathbf{K}_{21}^{(\circ)} & \mathbf{K}_{22}^{(\circ)} \end{pmatrix} \begin{pmatrix} u_1(\mathbf{r};\omega) \\ u_2(\mathbf{r};\omega) \end{pmatrix} \equiv \mathbf{K}^{(\circ)}(\omega) \begin{pmatrix} g_1(\omega) \\ g_2(\omega) \end{pmatrix} u(\mathbf{r}) \tag{26}$$

In Eq. (26) $u(\mathbf{r})$ is a scalar function.
And then, the kernel may be

$$\mathbf{K}^{(\circ)}(\omega) = \begin{pmatrix} \dfrac{\sin(\omega-\omega_p)d}{(\omega-\omega_p)d} & -i\dfrac{\cos(\omega-\omega_p)d}{(\omega-\omega_p)d} \\ i\dfrac{\cos(\omega-\omega_p)d}{(\omega-\omega_p)d} & \dfrac{\sin(\omega-\omega_p)d}{(\omega-\omega_p)d} \end{pmatrix} \tag{27}$$

So Eq. (19) takes the form:

$$u^{(\circ)}(\mathbf{r};\omega) = \sum_{i=1}^{2} \alpha_i \delta(\mathbf{r}-\mathbf{r}_i)\mathbf{K}^{(\circ)}(\omega)\mathbf{e}_i \tag{28}$$

Where

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{and} \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{29}$$

That is

$$u^{(\circ)}(\mathbf{r};\omega) = \alpha_1\delta(\mathbf{r}-\mathbf{r}_1)\mathbf{K}^{(\circ)}(\omega)\mathbf{e}_1 + \alpha_2\delta(\mathbf{r}-\mathbf{r}_2)\mathbf{K}^{(\circ)}(\omega)\mathbf{e}_2 \tag{30}$$

The conventional waves satisfy the scalar form of Eq. (1)

$$\begin{aligned} u(\mathbf{r},\omega) &= u^{(\circ)}(\mathbf{r},\omega) \\ &\quad + \lambda(\omega)\int_0^\infty \mathbf{K}^{(\circ)}(\omega;\mathbf{r},\mathbf{r}')u(\mathbf{r}',\omega)dr' \end{aligned} \tag{31}$$

Or in accordance with Eq. (25)

$$\begin{aligned} u(\mathbf{r},\omega) &= u^{(\circ)}(\mathbf{r},\omega) \\ &\quad + \lambda(\omega)\mathbf{K}^{(\circ)}(\omega)u(\mathbf{r},\omega) \end{aligned} \tag{32}$$

Where the form of $u(\mathbf{r},\omega)$ is unknown but possibly be sketched as

$$u(\mathbf{r},\omega) = \begin{pmatrix} \dfrac{\sin(\omega-\omega_p)d}{(\omega-\omega_p)d} \\ \dfrac{\sin(\omega-\omega_p+\beta)d}{(\omega-\omega_p+\beta)d} \end{pmatrix} u(\mathbf{r}) \tag{33}$$

Now we can apply the projection operator $\mathbb{Z}_P\left[\mathbf{r};\omega;u^{(\circ)}(\mathbf{r};\omega)\right]$ to Eq. (32) and obtain

$$\mathbb{Z}_P\left[\mathbf{r};\omega;u^{(\circ)}(\mathbf{r};\omega)\right]u(\mathbf{r};\omega) = \lim_{\eta\to 1}\left[\Delta(\eta)\left(\eta - \sum_{i=1}^{2}\alpha_i\delta(\mathbf{r}-\mathbf{r}_i)\mathbf{K}^{(\circ)}(\omega)\mathbf{e}_i\right)\right.$$
$$\left. +\eta\mathbf{K}^{(\circ)}(\omega)u(\mathbf{r};\omega)\right] \tag{34}$$

Then, by putting $\eta = 1$ and $\Delta = 0$ finally

$$\mathbb{Z}_P\left[\mathbf{r};\omega;u^{(\circ)}(\mathbf{r};\omega)\right]u(\mathbf{r};\omega) = y_e(\mathbf{r};\omega) \tag{35}$$

So it is irrelevant the part of the problem concerning the two sources, it is only a problem about resonances. Our problem is now to find the resonant frequencies by taking $\mathbf{K}^{(\circ)}(\omega)$ and impose the conditions $\eta = 1$ and $\Delta = 0$.

But, what is the real advantage of the $\mathbb{Z}_P$ and $\mathbb{Z}_Q$ operators?, the answer is that the Zap operator formalism may be viewed as a test for distinguish between an expression that cannot be transformed or yes, in whatever sense between the homogeneous and inhomogeneous equations under the rules established above; if not, we can ensure that some kind of irregular things are present. In case of the positive transformation, we have the confidence that both kinds of solutions can coexists, and then we can separate the solutions for convenience as if it was a problem of two steps: homogeneous and inhomogeneous.

Now the last condition over the Fredholm determinant is

$$\Delta\begin{pmatrix} \dfrac{\sin(\omega-\omega_p)d}{(\omega-\omega_p)d} - \eta & -i\dfrac{\cos(\omega-\omega_p)d}{(\omega-\omega_p)d} \\ i\dfrac{\cos(\omega-\omega_p)d}{(\omega-\omega_p)d} & \dfrac{\sin(\omega-\omega_p)d}{(\omega-\omega_p)d} - \eta \end{pmatrix} = 0 \tag{36}$$

The parameters d, η (the Fredholm eigenvalue) and ω_p (the plasma frequency) can take in principle, arbitrary values but for a specific media can be take numeric values. Now, we remember that we must also impose $\eta = 1$.

Then, Eq. (36) has two resonances:

$$\omega_1 = \frac{\pi}{4d} + \omega_p \tag{37}$$

And

$$\omega_1 = \frac{3\pi}{4d} + \omega_p \tag{38}$$

8. Forerunners of the zap projection operators

In Section **6** we defined a new class of integral operators we named Zap projection operators that literally cleans from a broadcasting problem the inhomogeneous part and leaves a projected homogeneous version. These operators act directly over an inhomogeneous Fredholm equation and are related to the Fredholm operators. But recently, we have defined another set of operators we called generalized projection operators (GPO) which projects a complete broadcasting signal (maybe described by a GIFE) not only into several independent mutually orthogonal signals but also can reverse the time direction as we wish. These GPO may be considered as the precursors of the Zap operators and we will see why. We remember their form:

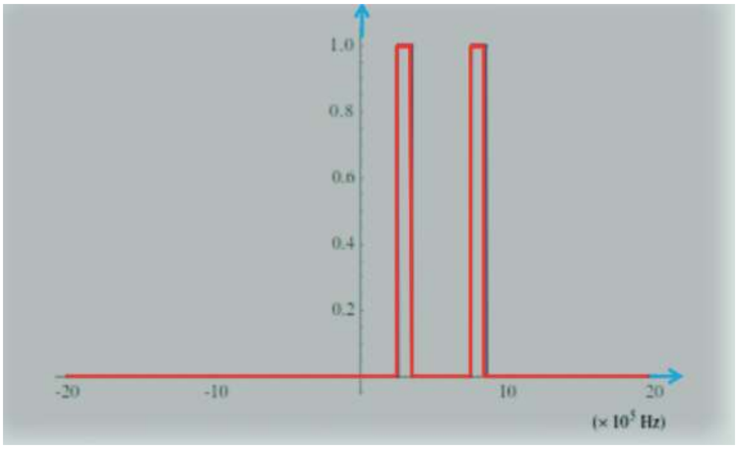

Figure 1.
The two rectangular functions $p_{4\pi\omega_e}(2(\omega-\omega_e)\omega_e)$ *and* $p_{4\pi\omega_u}(2(\omega-\omega_u)\omega_u)$.

$$\Omega^{(+)}_{e,T}S_a(t-T)=e^{i\omega_e t}\mathrm{P}^e_{S_a}(t) \tag{39}$$

And

$$\Omega^{(-)}_{e,T}S_a(t)=e^{i\omega_e(T-t)}\mathrm{P}^e_{S_a}(T-t) \tag{40}$$

In Eqs. (39) and (40) $\mathrm{P}^e_{S_a}(T-t)$ are simple projection operators [11].
Denoting the Fourier transform like

$$\mathcal{F}[f(t)]\equiv \mathrm{F}(\omega) \tag{41}$$

Then, the Fourier transform of the GPO is

$$\mathcal{F}\left[\mathrm{P}^e_{S_a}(t)e^{i\omega_e t}\right]=\sum_{n=-\infty}^{\infty}\mathrm{C}_{n,e}|2\omega_e|p_{4\pi\omega_e}(2(\omega-\omega_e)\omega_e)e^{-i(\omega-\omega_e)\pi n}\equiv F_{S_a}(\omega-\omega_e) \tag{42}$$

Where $p_{4\pi\omega_e}(2(\omega-\omega_e)\omega_e)$ is a rectangular function.
And for the convolution we have

$$\mathcal{F}\left[\mathrm{P}^e_{S_a}(t)e^{i\omega_e t}*\mathrm{P}^e_{S_b}(t)e^{i\omega_u t}\right]=F_{S_a}(\omega-\omega_e)F_{S_b}(\omega-\omega_u) \tag{43}$$

Then we see that the set of Fourier transforms of the GPOs behaves like a set of orthogonal basis functions for the frequency domain, that is, the resonant functions $y^m_e(\mathbf{r};\omega)$ as we can verify in **Figure 1**. So the GPO can be considered as the forerunners of the Zap projection operators.

9. Conclusions

We can conclude that the Zap projection operators (ZPO) can be used as an alternative approach to the generalized projection operators (GPO) that is like an alternative for clean the evanescent signals [10] from disturbances generated by the sources and at the same time to clean the source signals from resonant solutions. We can also use the two classes of projectors in a consecutively manner. The former vision suppose that the evanescent waves [10] can be considered as part of the conventional traveling waves like an everything and that we must take away the effect of the resonances with the application of the $\mathbb{Z}_Q$ operator. In any case we have shown the power of the Fourier transform applied to mathematical analysis in broadcasting problems and to physically characterize and solve them.

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