Chapter

Multi-Parameter Estimation of Uncertain Systems Based on the Extended PID Control Method

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Abstract

Parameter estimation is an important step in the identification of systems. With the extension of systems, there needs the multi-parameter estimation of systems. The estimation of multi parameters of complex systems based on the extended PID controllers is considered in this chapter. As the related references proved that the integral item of the nonlinear PID controller could deal with the uncertain part of the complex system (which can also be called new stripping principle, simple notes as NSP). Based on this theory, new multi-parameter estimation method is given. Firstly, the unknown parameters are expanded to new states of the system. Two cases, parameters are constant or changing with time, are separately analyzed. In the time-variant case, the unknown parameters are extended to functions which actual forms are uncertain. Secondly the method NSP could be applied to cope with the uncertain part, and then reconstruction state observation to estimate the states. If the states are observed, the unknown parameters are obtained at the same time. Finally the convergence analysis of the error systems and some simulations will be given in this chapter to indicate the effectiveness of the proposed method.

Keywords: multi-parameter, parameter estimation, complex system, extended PID control, convergence analysis

1. Introduction

Dynamic process model is the basis to study the uncertain systems. Generally speaking, the establishment of dynamic process model for the research object is the first step to solve the problem, and the parameter estimation of the established dynamic process model is the next key procedure to solve the problem. So the identification of dynamic processes is of great significance.

The design of the state observer in the control theory is to construct a dynamic system artificially, to make it approximate the real state of the dynamic system by selecting a certain form of the observer. The criterion for designing the state observer is to make the error system asymptotically converge to the origin, that is to say, as time goes by, the error will asymptotically converge to zero. It is based on this design idea we use in the parameter estimation problem. In the identification of the model, the dynamic process model is often accompanied with unknown disturbances. In the analysis of the estimation of multiple time-varying parameters, when the parameters are expanded to the states, there are also unknown parts in the dynamic process model. So this chapter will study a method for estimating multiple time-varying parameters based on the combination of disturbance stripping principle with state observer.

The reference [1] proposed a general form of establishing the state observer of the nonlinear system, and gave a direct method to deal with the nonlinear control system [2]. On the basis of the references [1–3], several specific state observers was provided to realize the estimation of a single time invariant parameter, and appropriate design parameters were selected according to the relevant results in the book [4]. By analyzing the stability the error system, a design method that made the error system asymptotically converge to zero was obtained. The simulation results showed that this method can estimate the parameters effectively [3].

For the estimation of time-varying parameters, the article [5] analyzed a system with one time-varying parameter. The design of the state observer in this article used the combination of binary control with PID control, which can handle the unknown items in the extended states. Although there was no rigorous theoretical proof in this article, the effect of parameter estimation did have excellent characteristics of fast convergence with less chatter. The reference [6] gave a method of combining binary control with nonlinear PID controller, and conducted a rigorous theoretical proof. Then it was extended to the regulation of high-level systems, and the principle of disturbance stripping [7] for the regulation of complex network systems. This laid the foundation for the theoretical analysis of the estimation methods of multiple time-varying parameters below. So this chapter is based on [5–7] and other references. The method of estimating a time-varying parameter in the nonlinear system in [5] is extended to the estimation of multiple time-varying parameters in a dynamic system by using the principle of disturbance stripping in the article [7]. The simulation studies showed that this method was also suitable for the estimation of time-invariant parameters.

The content of this chapter is arranged as follows: The Section 2 simply introduces the main idea of NSP and gives detail proof of it. The Section 3 puts forward an estimation method that contains multiple time-varying parameters in a nonlinear system. It describes the applicable objects of this kind of parameter estimation method, and gives a design of a specific state observer. Theoretical analysis and simulation research verifies the feasibility of the method. Section 4 summarizes the research methods and results presented in this chapter.

2. The main idea of NSP

The PID control method applies the error $\epsilon(t)$ between the reference input and observation. The PID control is the linear combination of the error, its differential and its integration. That is

$$u(t) = k_P\epsilon(t) + k_I\int_{t_0}^{t}\epsilon(\tau)d\tau + k_D\dot{\epsilon}(t) \tag{1}$$

where k_P, k_I, k_D are design parameters, $\epsilon(t)$ is the error, $\dot{\epsilon}(t)$ is the differential of the error, $\int_{t_0}^{t}\epsilon(\tau)d\tau$ is the integration of the error, t_0 is the intial time.

The theory analysis and large applications showed that the PID control u often had the conflict in fast and overshoot. Luckily, the nonlinear PID could solve this problem [8], which used the nonliner function such as sat funtion, fal function. It was the nonlinear combination of the error, its difference and its integration. At the same time, it also applied the nonlinear tracking-differentiator to filter the noise of the observation, and got the differential of the signal which may be not differentiable. The detail of this nonlinear PID controller can be seen in [8].

Basd on the idea of the extended PID controller, the NSP thought was proposed [6, 10, 12, 13]. They found that the integration of the error in the extended PID controller could stripping the unknown item in the complex systems. So we could use the NSP to deal with the system with unknown parts. The basic conclusion to be used in the following analysis, which is the most important thought in NSP involved in [6, 10, 12, 13]. The core idea will be simplified here, given in the form of a lemma, and with detailed proof.

Lemma 1 If the dynamic process $\mu(t)$ takes the following form:

$$\dot{\mu}(t) = \begin{cases} -\gamma sign(\sigma(t)), & |\mu(t)| \leq 1, |\mu(t_0)| \leq 1 \\ -\omega\mu(t), & |\mu(t)| > 1 \end{cases} \tag{2}$$

where $\sigma(t) = g(t) + k\mu(t)\int_{t_0}^{t}|e(\tau)|d\tau$, $e(t)$ is the difference between the state observer system and the original system (which is $\overline{x}$ as mentioned below), $g(t)$ is the unknown quantity with the known variation range, $\gamma > 0$, $\omega > 0$ is the undetermined constant. When the condition

$$k > \sup_{t \geq t_0}\left|\frac{g(t)}{\int_{t_0}^{t}|e(\tau)|d\tau}\right| \tag{3}$$

is satisfied, there will be a finite time t', if $t > t'$, then $\sigma(t) \equiv 0$. ■

Proof: Let us prove it by contradiction method. It is supposed that when $t > t'$, $\sigma(t)$ is not always 0.

Assuming that there is a certain moment $\sigma(t) \neq 0$, we might set $\sigma(t) > 0$ by the local scope. From the formula (2), there is $\dot{\mu}(t) = -\gamma$. The integral on both sides about time t is calculated, and we obtained:

$$\begin{aligned} \int_{t'}^{t}\dot{\mu}(\tau)d\tau &= \int_{t'}^{t}(-\gamma)d\tau \\ \mu(t) - \mu(t') &= -\gamma(t - t') \\ \mu(t) &= \mu(t') - \gamma(t - t') \end{aligned} \tag{4}$$

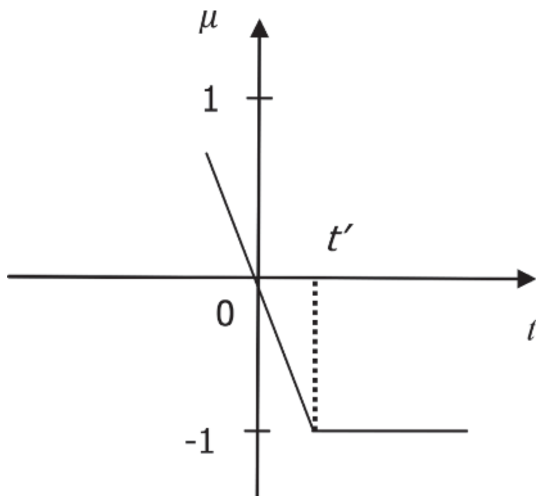

Figure 1.
Local changes of $\mu(t)$ over time.

Knowing from the definition of $\mu(t)$ that $|\mu(t)| \leq 1$, and $\dot{\mu}(t) = -\gamma \leq 0$, so at a certain moment t_1, as shown in the **Figure 1**, once $\mu(t)$ reaches the value $-sign(\sigma(t))$, there is $\mu(t) \equiv -sign(\sigma(t))$ (i.e. $\dot{\mu}(t) = 0$). Otherwise, it contradicts $\dot{\mu}(t) \leq 0$.

Here, t_1 is found in the following method. Let $t = t_1$, we have

$$\mu(t_1) = \mu(t') - \gamma(t_1 - t') = -1 \tag{5}$$

Then there is

$$\begin{aligned} t_1 &= t' + \frac{1 + \mu(t')}{\gamma} \\ &\leq t' + \frac{2}{\gamma} \end{aligned} \tag{6}$$

So when $t > t' + \frac{2}{\gamma}$, there is $t > t_1$, there must be $\mu(t) \equiv -sign(\sigma(t))$. By $\sigma(t) = g(t) + k\mu(t)\int_{t_0}^{t} |e(\tau)|d\tau$, then

$$\begin{aligned} \sigma(t) &= g(t) + k\mu(t)\int_{t_0}^{t} |e(\tau)|d\tau \\ &= g(t) - ksign(\sigma(t))\int_{t_0}^{t} |e(\tau)|d\tau \end{aligned} \tag{7}$$

Then,

$$\begin{aligned} \sigma^2(t) &= \sigma(t)\left[g(t) - ksign(\sigma(t))\int_{t_0}^{t} |e(\tau)|d\tau\right] \\ &= \sigma(t)g(t) - k|\sigma(t)|\int_{t_0}^{t} |e(\tau)|d\tau \\ &\leq |\sigma(t)||g(t)| - k|\sigma(t)|\int_{t_0}^{t} |e(\tau)|d\tau \end{aligned} \tag{8}$$

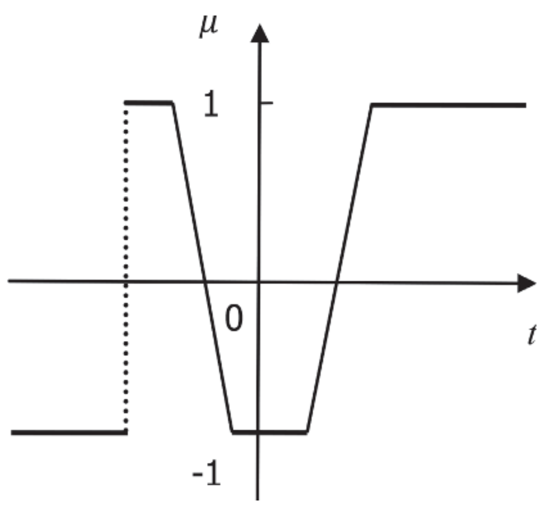

Figure 2.
Overall changes of $\mu(t)$ over time.

From the condition (3), we know that $\sigma(t)^2 < 0$, it leads a contradictory. When $\sigma(t) < 0$, the contradiction can be derived in the same way, and the overall change of $\mu(t)$ will be shown in **Figure 2**. In summary, the conclusion is established. ■

3. Estimation of multiple time-varying parameters based on the new stripping principle

The new stripping principle (NSP) in control theory can be effectively to deal with the interactive influence of nodes in complex network systems. Based on this, we use it to strip the unknown disturbance problem in the extended state observer in the time-varying parameter estimation. Therefore, this section proposes an estimation method for multiple time-varying parameters based on the combination of the NSP with the state observer.

3.1 The statement of the problem

The following system with multiple parameters, and the system itself is highly coupled, as shown in the following system:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2, \ldots, x_n, \theta_1) \\ \dot{x}_2 = f_2(x_1, x_2, \ldots, x_n, \theta_2) \\ \quad \vdots \\ \dot{x}_n = f_n(x_1, x_2, \ldots, x_n, \theta_n) \\ y = x = [x_1, x_2, \ldots, x_n]^T \end{cases} \tag{9}$$

where $\dot{x}$ means the derivate function of x with respect to time t. For this type of coupling problem, the method of NSP was proposed ([7, 9–11]). If x_i $(i = 1, 2, \cdots, n)$ is regarded as the interconnected nodes in the network, then the system (9) represents each different network node and the relationship among them. This kind of problems are common in real life, such as WeChat, QQ, Sina Weibo etc. in social networking tools. For example, a complex social system is formed through mutual attention and friendship between people, so here the individual x_i is one of them, $f_i(x_1, x_2, \ldots, x_n, \theta_i)$ is the interaction (such as relations or research works) among x_i, there are also other network problems like this.

There are always more or less unknowns in the modeling of such problems, which need to be estimated by using known information, which is the multi-parameter estimation problem to be analyzed in this section.

Assuming that $\frac{\partial f_i}{\partial \theta_i} \neq 0$, the following subsection uses the method of combining state observer with NSP to study the parameter estimation method. Specific parameter estimation methods, the convergence analysis and simulation research are described in detail in the following subsections respectively.

3.2 Parameter estimation method

This subsection discusses the parameter estimation method based on the combination of the state observer with the new stripping principle. We need to use the state observer to solve the parameter estimation problem, and we consider the time-varying parameters and time-invariant parameters as well.

Firstly, we extend the unknown parameters in the system (9) to states:

$$\begin{cases} \dot{x}_i = f_1(x_1, x_2, \dots, x_n, x_{n+i}) \\ \dot{x}_{n+i} = g_i(t), (i = 1, \dots, n) \end{cases} \quad (10)$$

That is to say, the parameter θ_i is extended to the system state x_{n+i}, $i = 1, \dots, n$. Then a state observer is built for the extended system (10):

$$\begin{cases} \dot{\hat{x}}_i = f_1(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n, \hat{x}_{n+i}) + l_i(t) \\ \dot{\hat{x}}_{n+i} = l_{n+i}(t), (i = 1, \dots, n) \end{cases} \quad (11)$$

where $l_i(t)$ $(i = 1, \dots, n)$ is the function to be designed. Let the error $\overline{x} = x - \hat{x}$, then the error system of the established state observer is as follows:

$$\begin{cases} \dot{\overline{x}}_i = f_1(x_1, x_2, \dots, x_n, x_{n+i}) - f_1(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n, \hat{x}_{n+i}) - l_i(t) \\ \dot{\overline{x}}_{n+i} = g_i(t) - l_{n+i}(t), (i = 1, \dots, n) \end{cases} \quad (12)$$

Next, we need to design $l_i(t)$ to make the error system (12) asymptotically stable. The choice of $l_i(t)$ equivalents to the control problem of the uncertain system (12). We design the control items $l_i(t)(i = 1, \dots, 2n)$ according to the error of the states, so that the error system (12) is asymptotically stable to zero.

Regarding the relevant conclusions of the estimation problem with multiple time-varying parameters in a nonlinear system, we present it in the form of the following theorem and give a stability analysis.

Theorem 1 If the system (10) satisfies f_i (i=1,2,...,n) is differentiable, and $\frac{\partial f_i}{\partial x_{n+i}} \neq 0$ (i=1,2,...,n), take the error feedback $l_i(t)$ in the following form:

$$\begin{cases} l_i(t) = \sum_{j=1}^{n} k_{ij} sign(\overline{x}_j) \\ l_{n+i}(t) = l_{n+i,1}(t) + l_{n+i,2}(t) \\ l_{n+i,1}(t) = k_i \mu_{1i}(t) \int_{t_0}^{t} |\overline{x}_i(\tau)| d\tau \\ l_{n+i,2}(t) = k_{n+i,i} |u_i(t)| \overline{x}_i sign\left(\frac{\partial f_i}{\partial x_{n+i}}\right) \\ i = 1, \dots, n \end{cases} \quad (13)$$

where $u_i(t) = \mu_i(t)(b_{1i}|\epsilon_{i1}(t)|^{\alpha_i} + b_{2i}|\epsilon_{i2}(t)|^{\alpha_i})$, $\mu_{1i}(t)$ is determined by binary control as follows:

$$\dot{\mu}_{1i}(t) = \begin{cases} -\gamma_{1i} sign(\sigma_{1i}(t)), & |\mu_{1i}(t)| \le 1, \ |\mu_{1i}(t_0)| \le 1 \\ -\omega_{1i}\mu_{1i}(t), & |\mu_{1i}(t)| > 1 \end{cases} \quad (14)$$

where $\sigma_{1i}(t) = g_i(t) - l_{n+i,1}(t) = \dot{\overline{x}}_{n+i}(t) + l_{n+i,2}(t) \doteq k_{d_i}\ddot{x}_i(t) + l_{n+i,2}(t)$. $\mu_i(t)$ is determined by binary control as follows:

$$\dot{\mu}_i(t) = \begin{cases} -\gamma_i sign(\sigma_i(t)), & |\mu_i(t)| \le 1, |\mu_i(t_0)| \le 1 \\ -\omega_i \mu_i(t), & |\mu_i(t)| > 1 \end{cases} \quad (15)$$

where $\sigma_i(t) = \epsilon_{i1}(t) + c_i \epsilon_{i2}(t)$, $k_{(n+i),i}$ is greater than 0, so here it is set that the upper bound of $k_{(n+i),i}|u_i|/\left(\frac{\partial f_i}{\partial x_{n+i}}\right)$ is K_m. k_{d_i}, k_{ij}, ω_{1i}, ω_i, c_i are all greater than 0, γ_i, b_{1i} and b_{2i} are design parameters in the formula (5.16) and formula (5.17) in Ref. [6], $0 < \alpha_i \le 1$, $i, j = 1, \dots, n$. The design parameters k_i, γ_{1i} respectively satisfy:

$$k_i > \sup_{t \geq t_0} \left| \frac{g_i(t)}{\int_{t_0}^{t} |\overline{x}_i(\tau)| d\tau} \right| \quad i = 1, 2, \dots, n \tag{16}$$

$$\gamma_{1i} > \sup_{t \geq t_0} \left| \frac{k_i |\overline{x}_i(t)| + \dot{g}(t)}{k_i \int_{t_0}^{t} |\overline{x}_i(\tau)| d\tau} \right| \quad i = 1, 2, \dots, n \tag{17}$$

Then the system (11) can be used as an observer of the extended system (10), and there is the results as follows:

$$\lim_{t \to \infty} \hat{x}_i(t) = x_i(t), \quad i = 1, \dots, 2n. \tag{18}$$

■

Now we prove the theorem 1 according to the lemma 1.

If $f_i(x_1, x_2, \dots, x_n, x_{n+i}) - f_i(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n, \hat{x}_{n+i})$ can be approximately expanded to $\sum_{j=1}^{n} \frac{\partial f_i}{\partial x_j} \overline{x}_j + \frac{\partial f_i}{\partial x_{n+i}} \overline{x}_{n+i}$ by Taylor expansion. Select proper $l_i(t)$ $(i = 1, \dots, n)$ to restrain the main part of $\frac{\partial f_i}{\partial x_j} \overline{x}_j(\ j = 1, \dots, n)$. Then the error system (12) will become the following ones:

$$\begin{cases} \dot{\overline{x}}_i = \sum\limits_{j=1}^{n} \dfrac{\partial f_i}{\partial x_j} \overline{x}_j + \dfrac{\partial f_i}{\partial x_{n+i}} \overline{x}_{n+i} - l_i(t) \\ \dot{\overline{x}}_{n+i} = g_i(t) - l_{n+i}(t), \quad (i = 1, \dots, n) \end{cases} \tag{19}$$

At this point, the problem is transformed into a control problem of the system (19).

Known from the conditions that $\sigma_{1i}(t) = g_i(t) - l_{n+i,1}(t) = \dot{\overline{x}}_{n+i} + l_{n+i,2}(t)$ (i=1,2,...,n). Because the actual value of the parameter $\theta_i (i = 1, 2, \dots, n)$ is unknown, so $\dot{\overline{x}}_{n+i}$ $(i = 1, 2, \dots, n)$ are also unknown. In order to estimate unknown parameters, it is necessary to find the equivalent or related quantities of $\dot{\overline{x}}_{n+i}$. From the formula (19), $\overline{x}_{n+i}$ is related to $\dot{\overline{x}}_i$, And $\frac{\partial f_i}{\partial x_{n+i}}$ is bounded, the observer designed here with $k_{d_i} \ddot{\overline{x}}_i$ instead of $\dot{\overline{x}}_{n+i}$in $\sigma_{1i}(t)$, where k_{d_i} is a design parameter. $\dot{\overline{x}}_{n+i}$ can also be replaced with other function forms of $\ddot{\overline{x}}_i$.

According to the lemma 1, when the conditions (16) and (17) are satisfied, it can be obtained that $g_i(t) - l_{n+i,1}(t) \equiv 0$ $(i = 1, \dots, n)$ when the time reaches a certain moment, so the error system (12) is approximately equivalent to the following system without unknown $g(t)$:

$$\begin{cases} \dot{\overline{x}}_i = \dfrac{\partial f_i}{\partial x_{n+j}} \overline{x}_{n+j} \\ \dot{\overline{x}}_{n+i} = -l_{n+i,2}(t), (i = 1, \dots, n) \end{cases} \tag{20}$$

Since $\sigma_i(t) = \epsilon_{i1}(t) + c_i \epsilon_{i2}(t)$ (i=1,2,...,n), according to the lemma 1, when the design parameters b_{1i}, b_{2i}, γ_i (i=1,2,...,n) satisfied the conditions that the formula (5.16) and formula (5.17) in Ref. [6], and the time is greater than a certain moment, there will be $\sigma_i(t) = \epsilon_{i1}(t) + c_i \epsilon_{i2}(t) = 0$, namely:

$$\overline{x}_i + c_i \dot{\overline{x}}_i = 0 \quad (i = 1, \dots, n) \tag{21}$$

For analyzing the stability of the error systems, the following Lyapunov function for the system (20) were constructed:

$$V_i = \frac{1}{2}\left(K_m \overline{x}_i^2 + \overline{x}_{n+i}^2\right) \ \ (i = 1, \dots, n) \tag{22}$$

It is easy to know that, except for the origin, $V_i > 0 \ \ (i = 1, \dots, n)$. Let us analyze the derivative function of V_i with respect to time,

$$\begin{aligned}\dot{V}_i &= K_m \overline{x}_i \dot{\overline{x}}_i + \overline{x}_{n+i} \dot{\overline{x}}_{n+i} \\ &= K_m \overline{x}_i \dot{\overline{x}}_i + k_{n+i,i}|u_i| \overline{x}_i sign\left(\frac{\partial f_i}{\partial x_{n+i}}\right) \overline{x}_{n+i} \\ &= K_m \overline{x}_i \dot{\overline{x}}_i + k_{n+i,i}|u_i| \overline{x}_i \dot{\overline{x}}_i / \left(|\partial f_i / \partial x_{n+i}|\right) \\ &= \left[K_m - k_{n+i,i}|u_i| / \left(|\partial f_i / \partial x_{n+i}|\right)\right] \overline{x}_i \dot{\overline{x}}_i \\ &\ (i = 1, \dots, n)\end{aligned} \tag{23}$$

From the formula (21),

$$\dot{V}_i = -c_i \left[K_m - k_{n+i,i}|u_i| / |\frac{\partial f_i}{\partial x_{n+i}}|\right] \dot{\overline{x}}_i^2 \ \ (i = 1, \dots, n) \tag{24}$$

Known by the condition $k_{n+i,i}|u_i| / \frac{\partial f_i}{\partial x_{n+i}}$ has an upper bound K_m, then $\dot{V}_i < 0 \ \ (i = 1, \dots, n)$.

In summary, when the time is greater than a certain moment, $\hat{x}_i, \hat{x}_{n+i}$ can be used as the estimation of $x_i, \theta_i \ (i = 1, \dots, n)$ respectively. During this progress, there is nothing to do with the specific form of $g(t)$. ■

Remark 1 When the parameter is a time-invariant parameter, it is easy to prove that the theorem 1 still works. Because at this time the expanded states $g_i(t) = 0 (i = 1, \dots, n)$ in the (10), then we can take $l_{n+i,1}(t) = 0 \ (i = 1, \dots, n)$ in our control law.■

The subsection focuses on the estimation problem of multiple time-varying parameters in general nonlinear systems. A parameter estimation method based on the combination of the state observer with the new stripping principle is given. Stability analysis is also carried out. The following simulation studies further verify the effectiveness of the parameter estimation method proposed in this subsection.

3.3 Simulation analysis

This subsection simulates the parameter estimation method proposed in the previous subsection. We have studied the estimation of a single time-varying and time-invariant parameter, and the estimation of multiple time-varying and time-invariant parameters in a dynamic system respectively. We also consider whether the observation contains observation noise or not. Further verify the robustness of the parameter estimation method.

3.3.1 Single parameter estimation simulation analysis

Example 1 We choose the nonlinear system as follow (that is, example 2 in Ref. [5]):

$$\dot{x} = -|x|\theta + x + cos\theta \tag{25}$$

Here, we assume that the true value of the unknown parameter changes with time $\theta = 1 + sin\,(2t)$, and the initial state of the system is $x(0) = 2$.

According to the system (25), there is $\frac{\partial f}{\partial \theta} = -|x| - sin\theta$. The situation that x and θ are both 0 almost never exists, so it can be considered that the condition $\frac{\partial f}{\partial \theta} \neq 0$ is established.

Next, the extended state system based on the parameter estimation method is described as below:

$$\begin{cases} \dot{x}_1 = -|x_1|x_2 + x_1 + cosx_2 = f(x_1, x_2, t) \\ \dot{x}_2 = g_1(t) \end{cases} \tag{26}$$

We design its observer as follows:

$$\begin{cases} \dot{\hat{x}}_1 = f(\hat{x}_1, \hat{x}_2, t) + k_{11}sign(y - \hat{x}_1) \\ \dot{\hat{x}}_2 = k_1\mu_{11}\int_{t_0}^{t} |\overline{x}_1(\tau)|d\tau + k_{21}|u_1|\overline{x}_1 sign\left(\frac{\partial f}{\partial x_2}\Big|_{\hat{x}_1, \hat{x}_2}\right) \end{cases} \tag{27}$$

The design of μ_{11} and u_1 is shown in the theorem 1, here we will not repeat them again.

Firstly, we consider the case where the observation does not contain noise. Set the design parameters in the simulation analysis as $k_1 = 0.1$, $k_{d_1} = 0.1$, $k_{11} = 40$, $k_{21} = 10$, $b_{11} = 1$, $b_{12} = 10$, $\alpha_1 = 0.5$, $c_1 = 1$, $\omega_{11} = \omega_1 = 3$, $\gamma_{11} = \gamma_1 = 10$, $\mu_{11}(0) = \mu_1(0) = 0$. Suppose that the initial state of the state observer is $(0, 0)$. We get the estimation of the states and parameters and the estimation errors are shown in the **Figure 3**, the estimation results are satisfied. And after a certain period of time (for example, this simulation is about 7 seconds), the estimated errors of the states and parameters can be controlled within 10^{-2}.

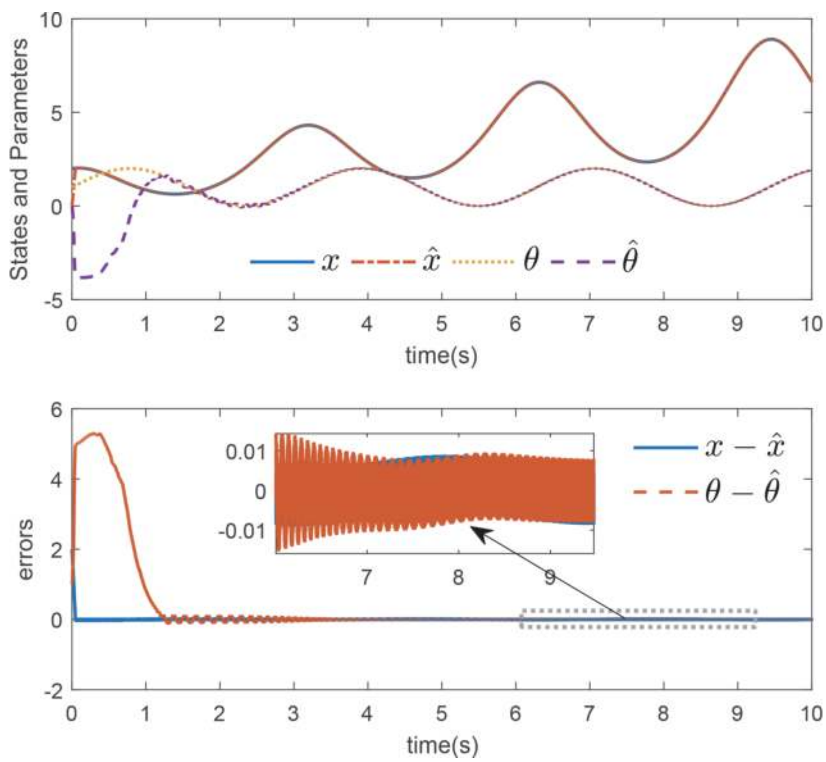

Figure 3.
The case with single parameter and the observation without noise: states, time-varying parameters estimation and estimation errors based on NSP.

Consider when the observation of the system (25) contains noise, for example, there is noise in the observation that obeys uniformly distributed in $[-0.001, 0.001]$, that is, $y(t) = x(t) + \epsilon(t)$, where $\epsilon(t) \sim U[-0.001, 0.001]$. Under these circumstances, design parameters are still taken as $k_1 = 0.001$, $k_{d_1} = 0.01$, $k_{11} = 40$, $k_{21} = 10$, $b_{11} = 1$, $b_{12} = 10$, $\alpha_1 = 0.5$, $c_1 = 1$, $\omega_{11} = \omega_1 = 3$, $\gamma_{11} = \gamma_1 = 10$, $\mu_{11}(0) = \mu_1(0) = 0$, and suppose that the initial state of the state observer is $(0, 0)$. The estimation errors of the state and parameter are shown in the **Figure 4**. Where the estimation error of the state is 10^{-3}, which is larger than the estimation error without noise. The parameter estimation error controlled within 2×10^{-2} is larger than the parameter estimation error without noise as well.

Previously, we studied the estimation problem based on the principle of disturbance stripping for the estimation of a single time-varying parameter, and then we will analyze the situation that the unknown parameter does not change with time.

Example 2 This example is still focusing on the nonlinear system of the system (25):

$$\dot{x} = -|x|\theta + x + cos\theta \tag{28}$$

It is assumed here that the true value of θ is a constant $\theta = 1$ that does not change with time, and the initial state value is $x(0) = 2$.

In the simulation analysis, the parameters are time-invariant, so $g_1(t) = 0$, the feedback item l_{21} can be ignored, and the design parameters $k_1 = 0$, $k_{d_1} = 0$, $k_{11} = 40$, $k_{21} = 10$, $b_{11} = 1$, $b_{12} = 10$, $\alpha_1 = 0.5$, $c_1 = 1$, $\omega_{11} = \omega_1 = 3$, $\gamma_{11} = \gamma_1 = 10$, $\mu_{11}(0) = \mu_1(0) = 0$. Suppose that the initial state of the state observer is $(0, 0)$. The state and parameter estimation and estimation errors are shown in the **Figure 5**.

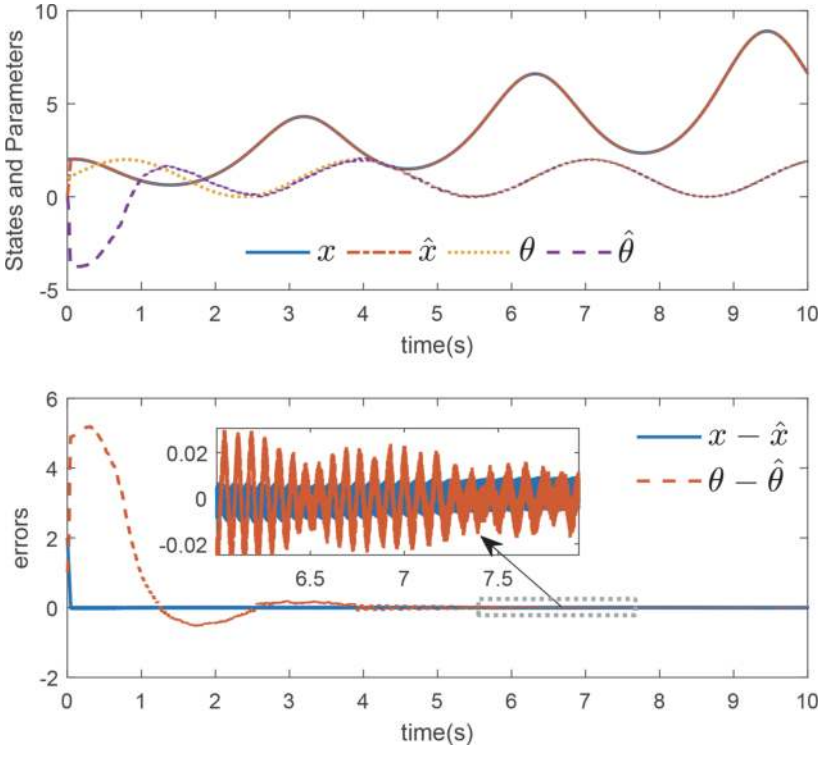

Figure 4.
The case with single parameter and the observation with noise: states, time-varying parameters estimation and estimation errors based on NSP.

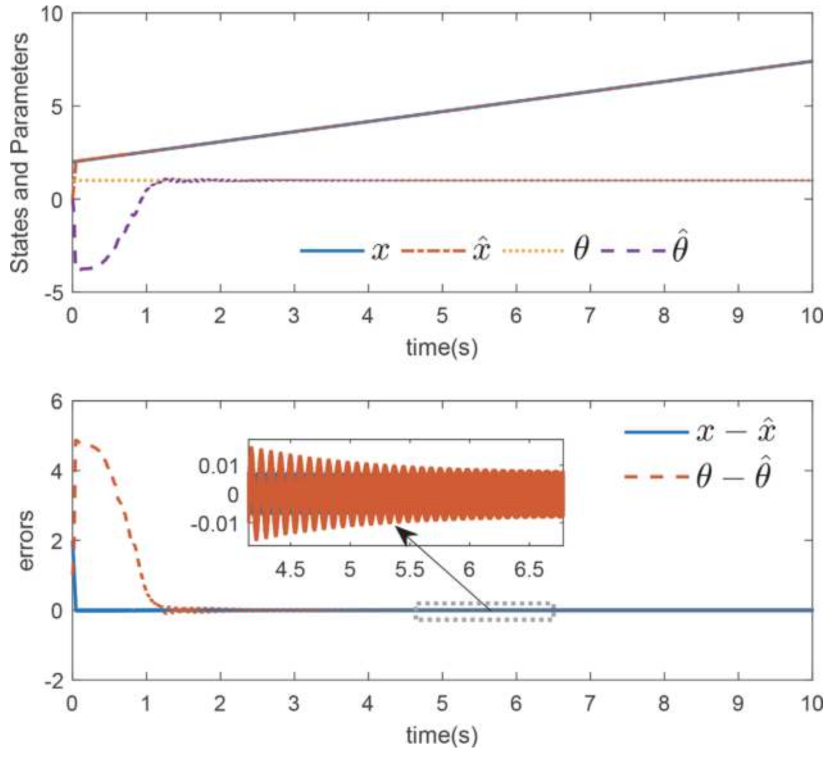

Figure 5.
The case with single parameter and the observation without noise: states, time-varying parameters estimation and estimation errors based on NSP.

The simulation shows that the state and parameters have close to the true value within 1 second, the estimation error can be controlled within 10^{-2} within 5 seconds, and the state estimation converges to the true state value faster due to the effect of error feedback.

When there is noise in the observation of the system (25), for example, the observation contains uniformly distributed noise that obeys $[-0.001, 0.001]$. That is, $y(t) = x(t) + \epsilon(t)$, where $\epsilon(t) \sim U[-0.001, 0.001]$. Under these circumstances, the design parameters in simulation analysis are still taken as $k_1 = 0$, $k_{d_1} = 0$, $k_{11} = 40$, $k_{21} = 10$, $b_{11} = 1$, $b_{12} = 10$, $\alpha_1 = 0.5$, $c_1 = 1$, $\omega_{11} = \omega_1 = 3$, $\gamma_{11} = \gamma_1 = 10$, $\mu_{11}(0) = \mu_1(0) = 0$. Suppose the initial state of the state observer is $(0, 0)$. The estimated errors of the parameters and states are shown in **Figure 6**, the estimation error of the state is 10^{-3}, which is more than the estimation error without noise. However, the parameter estimation error is larger than the parameter estimation without noise, but the estimation error can still be controlled within 2×10^{-2}.

In summary, this subsection studies the application of parameter estimation methods based on the combination of NSP with state observer in the estimation of single parameters of nonlinear systems. This subsection not only analyzed the two cases of time-invariant and time-varying parameters through simulation, but also analyzed the situation that the observations of the system include observation noise. In these simulation studies, based on the preliminary adjusted design parameters, when analyzing the time-varying and time-invariant parameters, and the presence or absence of observation noise, the design parameters were basically not changed, but the simulation results show that the state and parameters in the observer (27) can asymptotically converge to the true value. These studies show the feasibility and robustness of the combination of the state observer with the stripping principle in the single parameter estimation of nonlinear systems.

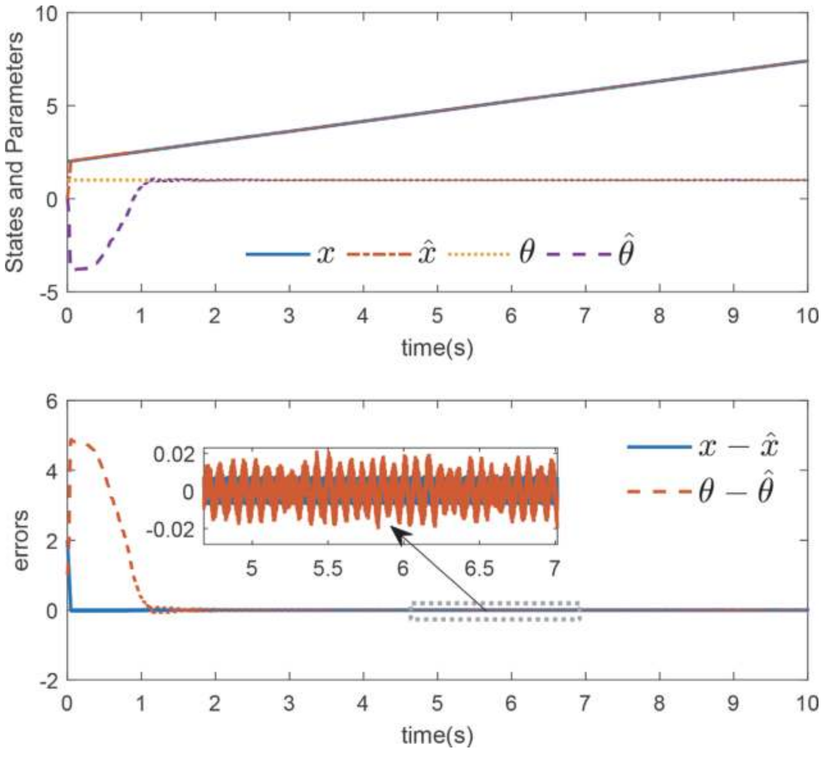

Figure 6.
The observation contains noise: Statse, time-invariant parameters estimation and estimation errors based on NSP.

3.3.2 Multiple parameter estimation simulation analysis

This subsection will study the simulation results with multiple parameter estimates in the dynamic process.

Example 3 Consider the following nonlinear system with two unknown parameters:

$$\begin{cases} \dot{x}_1 = 5\theta_1 x_2 \\ \dot{x}_2 = -\theta_2 cos x_2^2 - 3x_1 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases} \tag{29}$$

Here, we assume that the true value of the unknown parameter changes with time $\theta_1 = sin\,(2t)$, $\theta_2 = cos\,(2t)$, and $\theta_1(0) = 1$, $\theta_2(0) = 1$, the initial state of the system is $x(0) = (5.4, -1.4)$. In this example, θ_1, θ_2 are unknown parameters. According to the parameter estimation method, the unknown parameters are extended to the state, and we obtain the following extended state system:

$$\begin{cases} \dot{x}_1 = 5x_3 x_2 \\ \dot{x}_2 = -x_4 cos x_2^2 - 3x_1 \\ \dot{x}_3 = g_1(t) \\ \dot{x}_4 = g_2(t) \end{cases} \tag{30}$$

where $g_1(t), g_2(t)$ are unknown functions of time t.

The state observer of the above system is established as follows:

$$\begin{cases} \dot{\hat{x}}_1 = 5\hat{x}_3\hat{x}_2 + l_1 \\ \dot{\hat{x}}_2 = -\hat{x}_4 \cos \hat{x}_2^2 - 3\hat{x}_1 + l_2 \\ \dot{\hat{x}}_3 = l_3 = l_{31} + l_{32} \\ \dot{\hat{x}}_4 = l_4 = l_{41} + l_{42} \end{cases} \tag{31}$$

Let $\overline{x}_1 = y_1 - \hat{x}_1, \overline{x}_2 = y_2 - \hat{x}_2$, where l_i is set as follows:

$$\begin{cases} l_1 = k_{11}sign(\overline{x}_1) + k_{12}sign(\overline{x}_2) \\ l_2 = k_{21}sign(\overline{x}_1) + k_{22}sign(\overline{x}_2) \\ l_{31} = k_1\mu_{11}\int_0^t |\overline{x}_1(\tau)|d\tau \\ l_{32} = k_{31}|u_1|\overline{x}_1 sign\left(\dfrac{\partial f_1}{\partial x_3}\right) \\ l_{41} = k_2\mu_{12}\int_0^t |\overline{x}_2(\tau)|d\tau \\ l_{42} = k_{42}|u_2|\overline{x}_2 sign\left(\dfrac{\partial f_2}{\partial x_4}\right) \end{cases} \tag{32}$$

The design of μ_{11}, μ_{12}, u_1 and u_2 is shown in the theorem 1.

We consider the case that the observation does not contain noise first. By using the design of the aforementioned observer (31), design parameters in simulation analysis are as $k_1 = k_2 = 0.01$, $k_{d_1} = k_{d_2} = 0.1$, $\omega_{11} = \omega_{12} = 3$, $\gamma_{11} = \gamma_{12} = 10$, $k_{11} = 15$, $k_{21} = 0.1$, $k_{12} = 0.1$, $k_{22} = 1$, $k_{31} = 50$, $k_{42} = 50$, $b_{11} = b_{21} = 15$, $b_{12} = b_{22} = 25$, $\alpha_1 = \alpha_2 = 0.5$, $c_1 = c_2 = 5$, $\omega_1 = 5$, $\omega_2 = 0.55$, $\gamma_1 = 10$, $\gamma_2 = 150$, $\mu_1(0) = \mu_2(0) = 0$. Suppose the initial state of the state observer is $(0, 0)$. We obtain the following states, parameters estimation and estimation errors as shown in **Figure 7**.

When the observation of the system (29) contains noise, for example, the observation contains noise that obeys uniformly distribute in $[-0.001, 0.001]$, namely $y(t) = x(t) + \epsilon(t)$, where $\epsilon(t) \sim U[-0.001, 0.001]$. In this case, the design parameters are the same as the above, and the estimated error of the states and parameters are shown in **Figure 8**.

Simulation results in **Figures 7** and **8** show that the observer designed in this section is applicable to the estimation of time-varying parameters and it has certain robustness to noise. The estimation error of the state is similar either with or without observation noise. For parameter estimation, when there is no noise in the observation, the parameter estimation error is controlled within 5×10^{-2}, but when there is noise in the observation, the parameter estimation effect of θ_2 is not ideal, and the design parameters need to be adjusted appropriately to obtain the more accurate estimation value.

We have studied the estimation of multiple time-varying parameters based on the principle of disturbance stripping above. The following will analyze the situation where the unknown parameters do not change with time.

Example 4 This example is still researching the system (29):

$$\begin{cases} \dot{x}_1 = 5\theta_1 x_2 \\ \dot{x}_2 = -\theta_2 cos x_2^2 - 3x_1 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases} \tag{33}$$

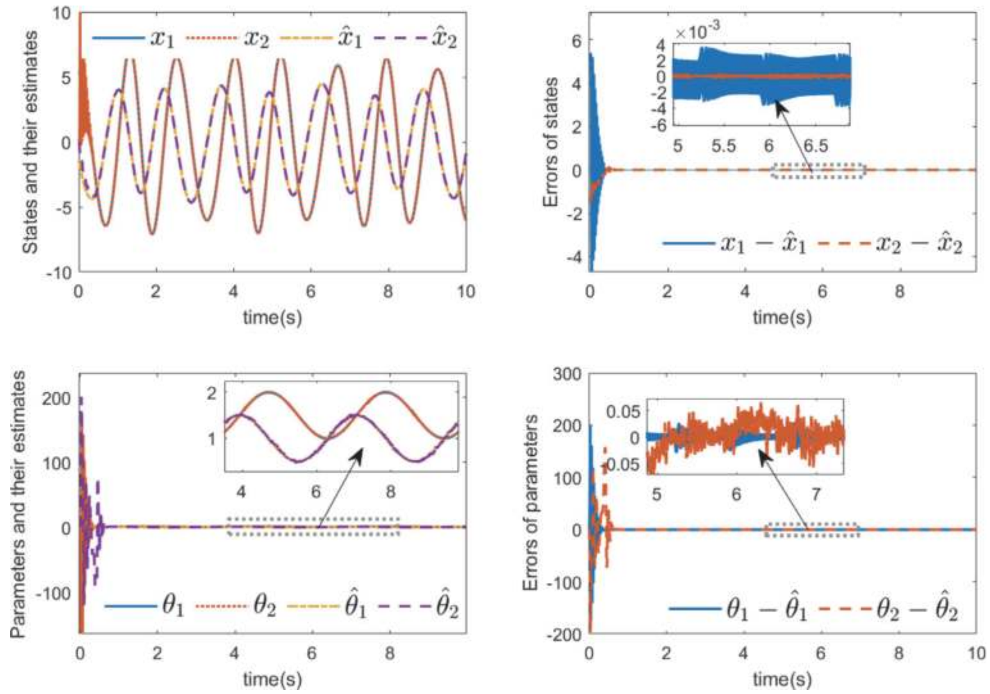

Figure 7.
The case with two parameters and the observation without noise: states, time-varying parameters estimation and estimation errors based on NSP.

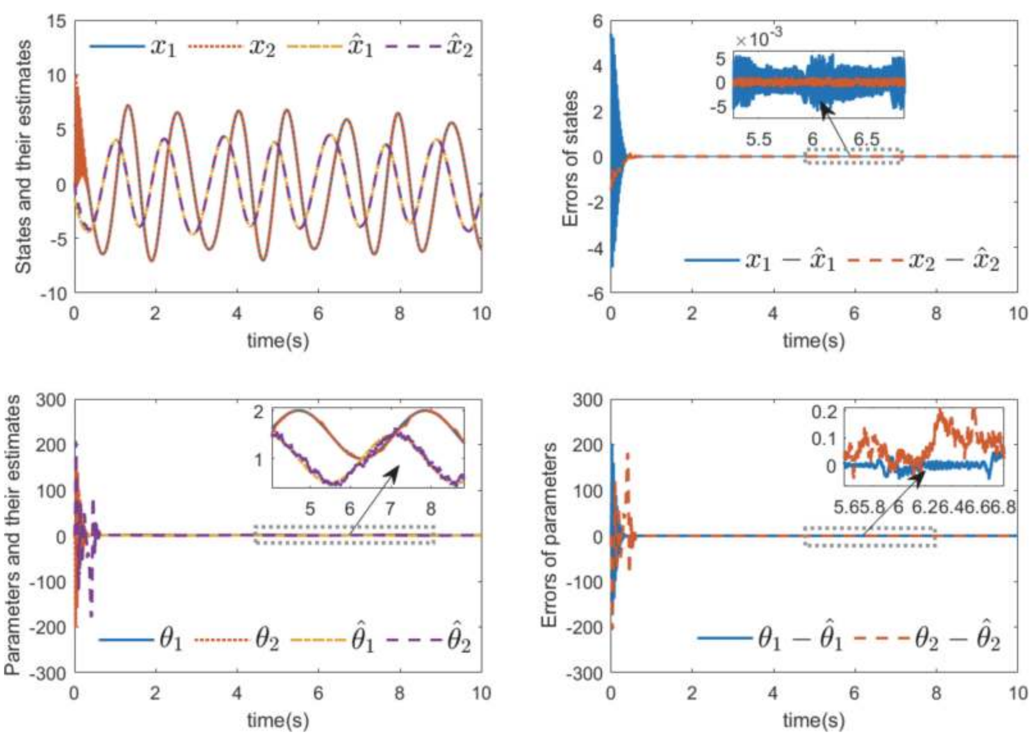

Figure 8.
The case with two parameters and the observation with noise: states, time-varying parameters estimation and estimation errors based on NSP.

Here we assume that the true value of the unknown parameter does not change with time. Suppose that $\theta_1 = 1.4$, $\theta_2 = -1.4$, the initial state of the system is $x(0) = (5.4, -1.4)$.

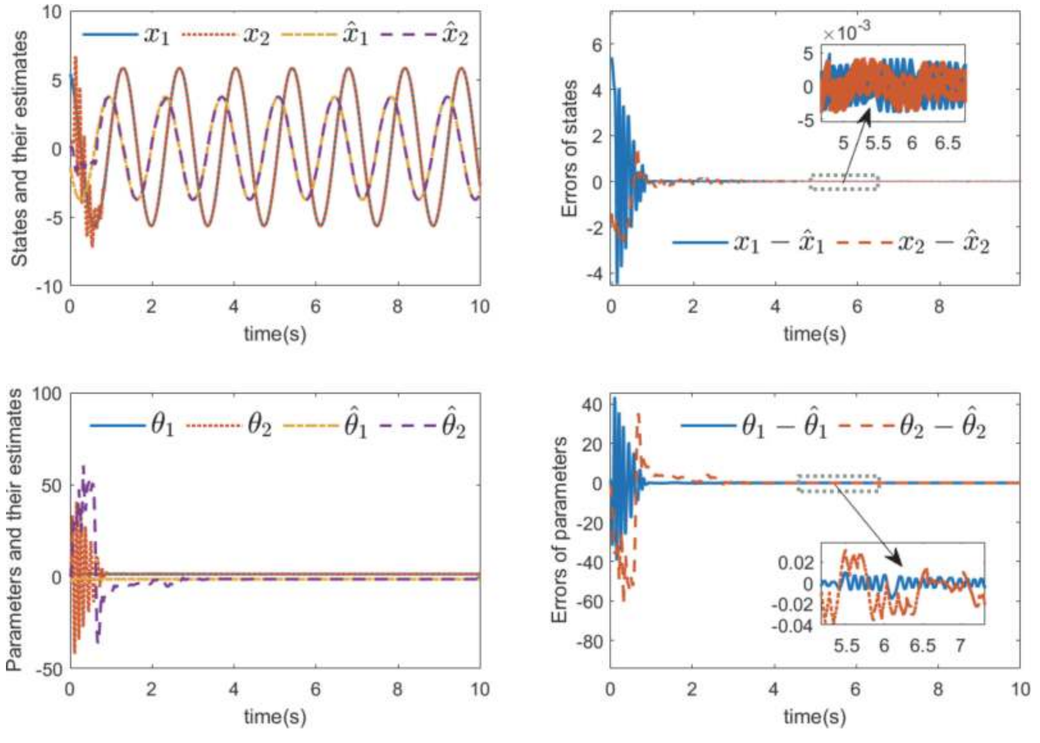

Figure 9.
The case with two parameters and the observation with noise: states, time-varying parameters estimation and estimation errors based on NSP.

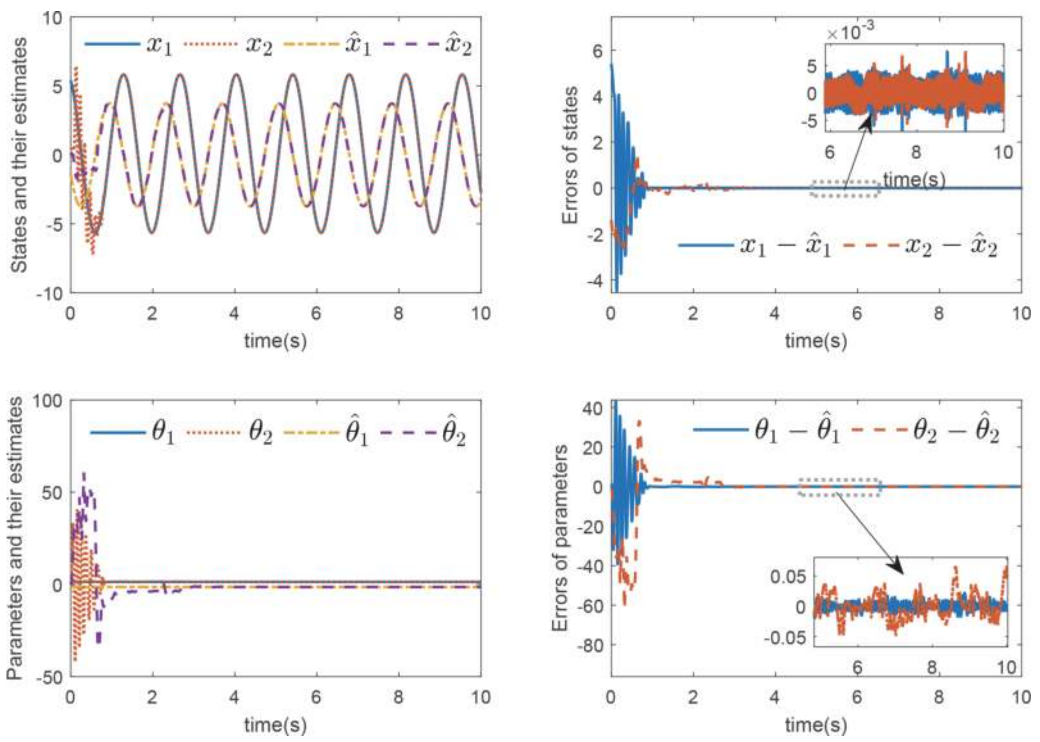

Figure 10.
The observation with noise: The states, time-invariant parameters estimation and estimation errors based on NSP.

For the estimation of time-invariant parameters, $g_1(t) = g_2(t) = 0$, so take $l_{31} = l_{41} = 0$. Take design parameters in simulation analysis $k_1 = k_2 = 0$, $k_{d_1} = k_{d_1} = 0$, $\omega_{11} = \omega_{12} = 0.35$, $\gamma_{11} = \gamma_{12} = 10$, $k_{11} = k_{21} = 10$, $k_{12} = k_{22} = 5$, $k_{31} = k_{32} = 15$, $k_{41} = k_{42} = 10$, $b_{11} = b_{21} = 1$, $b_{12} = b_{22} = 25$, $\alpha_1 = \alpha_2 = 0.1$, $c_1 = c_2 = 6$, $\omega_1 = \omega_2 = 0.35$, $\gamma_1 = 10$, $\gamma_2 = 15$, $\mu_1(0) = \ mu_2(0) = 0$. Suppose the initial state of the state observer is $(0, 0)$. The state and parameter estimation, and estimation error obtained by simulation are shown in **Figure 9**. The simulation result in **Figure 9** shows that the above parameter method is still applicable to the estimation of time-invariant parameters. We can see from the simulation results that the estimation error of the state is controlled within 5×10^{-3}, and the estimation error of the parameter estimation is controlled within 2×10^{-2}, or even better (see **Figure 9** $\theta_1 - \hat{\theta}_1$). If we tune the design parameters properly, we can get a more accurate estimate.

When the observation of the system (29) contains noise, for example, there is noise that obeys uniformly distribute in $[-0.001, 0.001]$, that is, $y(t) = x(t) + \epsilon(t)$, where $\epsilon(t) \sim U[-0.001, 0.001]$. In this case, the design parameters are the same as above, and the estimated errors of the states and parameters are shown in **Figure 10**, where the estimated error of the states are controlled within 5×10^{-3}. The parameter estimation error is larger than the parameter estimation error without noise, but the parameter estimation error can still be controlled within 5×10^{-2}.

In summary, this section analyzes the estimation problem of multiple time-varying parameters in nonlinear systems based on the parameter estimation method combined the observer with the new stripping principle. Simulation research shows that the parameter estimation method proposed this chapter can estimate multiple time-varying parameters (this section only considers the estimation of two parameters), and the time-invariant and time-varying conditions of the parameters in the analysis both illustrate the applicability of the parameter estimation method. In addition, the simulation research on whether there is observation noise in the observations verifies the robustness and feasibility of the parameter estimation method proposed in this section.

4. Conclusions

This chapter studies the state observer method of nonlinear system parameter estimation. When the unknown parameters have explicit expressions, we can use the nonlinear tracking-differentiator-based method to estimate the parameters. The unknown parameters which is relatively non-linear system in nonlinear form or is not easy to express by explicit are main considered in this chapter. According to the different characteristics of the parameters contained in the dynamic process, based on the research of the existing literatures, this chapter proposes a new parameter estimation method based on the state observer and NSP. The parameter estimation method based on the combination of state observer with new stripping principle for dynamic systems containing multiple time-varying parameters. This chapter not only proves the feasibility of the method in theory, but also do the simulations. The simulation results show that the design method can approximate the true value of the parameter within a certain error range. The simulations also consider the presence or absence of observation noise. The simulation results not only show that the parameter estimation method introduced in this chapter is robust to noise, but also show the adaptability of the design parameters. Because it is found in the design parameter adjustment that: adjusting the design parameters within a certain range has little effect on the accuracy of parameter estimation, so in the adjustment of design parameters, according to the characteristics of the error system, the thought and method of control system design can be used to give an approximate value to make the state and the parameter converge, and it can also make fine adjustments to make the estimated error meet the actual demand.

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