

---

# Mathematics Instruction Based on Science Using Didactical Phenomenology Approach in Junior Secondary School in Indonesia

---

Turmudi, Setya Utari, Suprih Widodo and  
Ratnaningsih

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.68437>

---

## Abstract

This chapter presents the result of research that is motivated by a sense of concern where mathematics learning in the class now has been rarely associated with science lesson. We tried to see the existing phenomenon, then we designed science-based teaching materials of mathematics using that phenomenon as an approach to teaching for students. There are two levels when developing instructional design that is at the level of research conducted in the laboratory of physics, by weighing the sugar and water proportionally, then stirred the sugar solution to obtain a wide range concentration of sugar solutions which are stored in the experiment tubes. This experimental tool is then used to facilitate students learning the relationship between two variables such as sugar concentration is expressed in percent on one hand and sedimentation time (in second) of “a clay ball” on each of the sugar solution on the other hand. Pairs of numbers concentration and sedimentation time of “ball” in each solution were plotted in a Cartesian coordinate. The graph reflects a phenomenon of solution viscosity and sedimentation rate of a ball in a solution that can be aligned with the level of consistency of “blood in our body” and that circulation is disturbed when the blood concentration increases. The results of this study indicate that students have an awareness of the importance of the health while maintaining the concentration of the solution for being drunk and eaten. Suggestion from this research is that the readers could consider that sugary drinks with low concentrations, which still be able to maintain a person’s health, are better than the sugary drinks with very high concentration.

**Keywords:** science-based, didactical phenomenology, mathematics instruction

---

## 1. Introduction

Didactic phenomena in our understanding are exploiting the phenomenon as medium or bridge for learning concepts. In learning mathematics, didactical phenomenology interpreted as a means to learn math concepts as [1] stated that the didactical phenomenology is a way to show the teacher place where the learner may step into the learning process of mankind (p. ix).

Starting from the situation that the human body contains thousands or even millions of mysterious phenomena, some of which we can observe through the sheets of the doctor who advised us to go to the lab for having general checkup for our health, having our solutions or body fluids to be tested. It turned out that the concentration of the solutions in the body affects the healthy condition of our body. When the glucose in our body exceeding the normal size, then our health would be affected. When a less glucose (very low concentration) is present in our body then obviously the balance of our body also affected. This situation encourages the research team to take advantage of this phenomenon in mathematics.

Data in the **Figure 1**, represent the result of health lab test of the first author of this chapter (health of Turmudi's lab test) which was conducted in February 25th, 2014 in the Pramita Lab of Bandung. Suppose the number 130 mg/dl for triglycerides showed that as many as 130 mg of triglycerides in 1 dl solution, a healthy person is when she/he has less than 150 mg/dl (<150 mg/dl).

Learning mathematics using mathematician framework usually takes place when introducing the concept of sets and functions and then the "set approach" is used. Therefore, the function is understood without using illustration. Function concept is understood as verbatim. Most mathematics teachers in Indonesia usually introduce relationship or function concepts using arrow diagram. Relating two sets of quantities, such as group of students in one hand, and their shoes number size in the other hand. He/she used arrows to link among two sets of quantities.

**Figure 2** represents the relationships among two quantities such as name of persons in set A and numbers of their shoes in set B. The research team, however, prefers to take advantage of this phenomenon by associating two specific situations. Rather than using data without

PROFIL LEMAK				
Profil Lemak Lengkap				
Cholesterol @	158	Yang diinginkan : < 200 Batas tinggi : 200 - 239 Tinggi : > 239	mg/dL	CHOD-PAP
Trigliserida @	130	Normal : < 150 Batas tinggi : 150 - 199 Tinggi : 200 - 499 Sangat tinggi : >= 500	mg/dL	ENZYMATIC
HDL Cholesterol @	47	Rendah : < 40 Tinggi : >= 60	mg/dL	HEMOTURBIDIMETRI
LDL Cholesterol Direct @	96	Optimal : < 100 Mendekati Optimal: 100-129 Batas tinggi : 130-159 Tinggi : 160-199 Sangat tinggi : >= 190	mg/dL	DIRECT ENZYMATIC
Ratio LDL/HDL	2.0	CARDIO RISK INDEX (CRI) < 3 : Risiko rendah 3 - 5 : Moderat > 5 : Risiko tinggi		CALCULATION

**Figure 1.** Data from a health laboratory.



Figure 2. Example of relation between name and shoes number.

meaning (meaningless), the research team prefers to observe variety of sugar solution concentration, and checking the time sneaking by an object at any percentage of sugar solution. The research team chose some phenomena by conducting experiments for each of these phenomena, and all served in front of students in the classroom. The result is quite amazing because students turned out to have an awareness of the usefulness of the relationship between a quantity and other quantities in the phenomenon.

Sugar solution is prepared with varying concentrations of 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50% by the research team (Figures 3 and 11). Then an object in this case is a small sphere like “ball” made of plasticine dipped in each solution. Here, we have two quantities when a dipping clay ball, i.e., concentrations of the solution expressed in percent and time crept from plasticine ball measured using stopwatch and expressed in seconds. We note the plasticine ball sneaked time in different percentage of sugar solution, so we have ordered pair numbers,

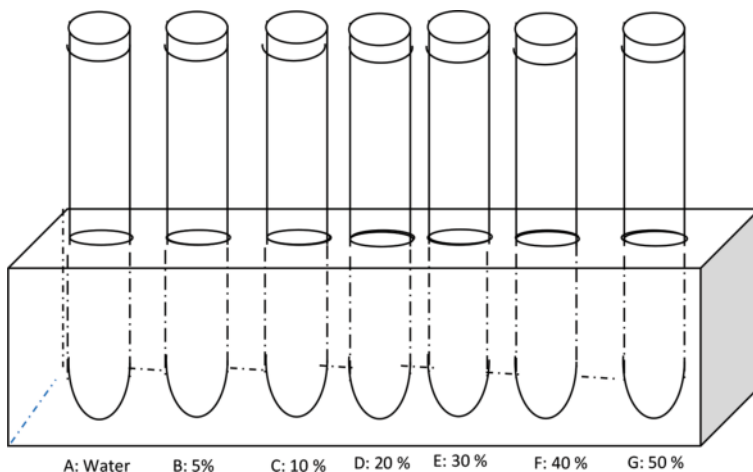


Figure 3. Solutions of sugar cane in percents.

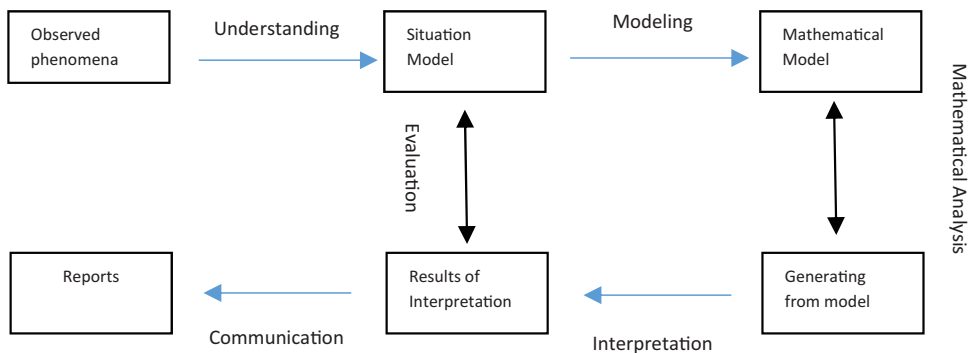
that is the number percentage of sugar solution and sedimentation time of “ball” on each of these solutions.

**Figure 3** represents various concentrations of solutions. Practically, we used more details of sugar solutions by inserting 15, 25, 35, and 45% of sugar solution. Instructional design of mathematics based on the science using didactical phenomenology approaches was presented to the students, so that the students have a sense of the relationship between two quantities. Students have the ability to represent it in a variety of mathematical representations. Furthermore, numbers obtained are processed and then packaged in various forms of mathematical representations. Using this reflective thinking, students can think mathematically and scientifically to instill their awareness to be able to live in a healthy life and harmony.

The chapter presents the results of this study synergically involving scientific activities. Didactical phenomena of instructional design of mathematics were designed by the team in the laboratory activities, recorded using video camera, transferred to the power point presentation. The student tasks in the classroom were to make mathematical model or mathematical graph related to the data as the result of observation was recorded in video camera and power point presentation. These data were presented in the form of table. The pair numbers in the table as coordinates were then be plotted in the Cartesian coordinates. By graphing this phenomenon, the students were asked to interpret the phenomena.

As a design proposed by Verschaffel, Greer, and De Corte (2002, cited by Turmudi et al. [2], **Figure 4**), the process of learning mathematics that involves the modeling process includes the observation of phenomena (reading), understand the situation, modeling, analyzing the model, formulate the results, interpretation, and then make a communication:

The teaching materials of mathematics in this study used the didactical phenomenon in the form of a natural situation or similar situations that conditioned the models created in the science laboratory. The students in the classroom are faced with these instructional materials and the materials were manipulated in the form of power point presentation.



**Figure 4.** Diagram of mathematical modeling process.

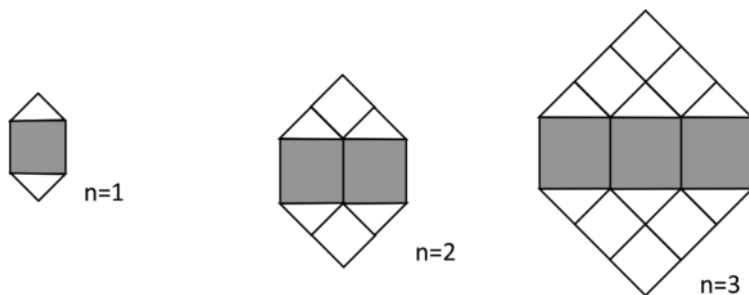
## 2. Background

After doing some research in a clump of innovative teaching, research teams are interested in trying out the learning of mathematics with science-based didactic phenomena within reasonable time. Research studies on realistic mathematics have been conducted with the result that is very promising and could attract the attention of students [3, 4]; encourage teachers in the Bandung city area to realize that they already know their innovative ideas but do not have the ability to present learning with innovative ideas because of the absence of exemplary prototypes [5, 6], the teachers realized that the training through lesson-study has opened their insight to continue working and improving the learning for teaching of mathematics in the classroom [7] but the teachers still require exemplary prototype of mathematics learning using scientific approach that can be implemented in the classroom, so that openly they can watch in open-lesson setting, and in turn the teachers be able to implement it in their respective classes.

Results of research on mathematical modeling by [2, 8] show that the students involved in the study felt there was something new in mathematics instruction. For example in view of the variables that were not visible, but after attending the workshop the participants were able to see the variables in the phenomena, so they were able to make the association among variables that exist to make a mathematical model. Consider the following figures:

**Figure 5** is the pattern model that originally taken from the floor of JICA-FPMIPA building of UPI on the second floor (**Figure 6**, personal collection of photograph), the tile patterns can be seen in the image (**Figure 7**, personal collection of photograph).

At first time, the images were just as the pictures without meaning, the students are not too concerned with tile patterns like that, but with a small call "Let us see and we noticed a pattern (in **Figure 7**), as well as how to process it so we could have an interesting mathematical concept." The invitation make a number of students feel surprised by the mathematical patterns that exist in the JICA Building, in Bandung. When the students were able to see the pattern of the picture and are able to associate with the image number and the area of the geometry shapes, then they obtained a mathematical model that previously did not figure



**Figure 5.** Patterns of floor in JICA-FPMIPA building.



Figure 6. JICA-FPMIPA building in UPI.

out. This association when we continued to the 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup> terms and so on until the n-th term, it will get the functional formula by following form of  $F(n) = n^2 + 2n$  with n is the image number (term) and F(n) is the area of geometrical shapes. By knowing how to build and determine the formula of model, the students can reach through the process of horizontal and vertical mathematization [9] and the process of progressive mathematization [10]). Teacher's capabilities to view such situations nowadays are needed urgently, so that they can try out these competences at the microlevel of the classroom. Such capabilities are seemed urgent, because the implementation of the 2013-curriculum [11], which was characterized scientifically, in fact, was not absorbed completely by the teachers during the 2013-curriculum upgrading. Therefore, the presence of such learning is a positive contribution toward building the current curriculum innovation in Indonesia.

Provisioning capabilities of mathematical concepts, and the pedagogical content knowledge (PCK) of mathematics for teachers would give effects to a teacher as an actor of mathematics learning in action in front of the class. The ability to see the phenomena should be the part of the teachers' as well as the students' competences, so that they always think continuously



Figure 7. Floor in JICA-FPMIPA building.

over time and will always be able to find connections between the existing phenomena by taking into account the specific quantities or the shapes in the flat geometry (2D) or space geometry (3D), therefore it can intertwine the functional relationship between the quantities that appeared in Ref. [54].

### 3. Statement of the problems

Implementation of mathematics instruction based on science using didactical phenomenology approach is asking the question "Does the teaching material of mathematics using science-based didactical phenomenology approach effect positively to the students' cognitive abilities?" This formulation was translated into a number of formulations more specifically as follows: (a) How is the prototype mathematics learning using science-based didactical phenomenology approach? (b) How does the implementation process of learning mathematics using science-based didactical phenomenology approach? (c) What was the students' reaction to the teaching materials? These questions are answered by designing instructional materials, which are prepared in the science-laboratory and implementing them in the real classroom.

### 4. Theoretical framework

Equipped teachers with a number of competencies [12] suggested teachers [mathematics, in addition to the author] to follow the development of professionalism in order to gain new knowledge and skills so as to improve their teaching in the classroom. Nevertheless, we do not deny the condition that the change turned out to be only on the surface, as stated in Ref. [13] "There were not a lot of professional development activities for teachers or other types of innovations implemented as a routine activity for the next stage but there is only the result of the professional development (PD) or innovation is communicated through questionnaires, interviews, or a survey" (p. 77). Symptoms such as those indicating innovation through PD (seminars, training, workshops) face the problem of sustainability, so often teachers are still applying old habits, otherwise known as the "back to basic," even though they have attended a number of times the workshops, seminars, and others. But the situation now is different, although the general teachers feel less comfortable when seen and observed by other teaching [14, 55].

Now gradually the teachers' perception have changed, at least felt by the teachers who attended the lesson-study in Bandung [7]. They have changed their habits according to an anecdote, quoted by [15], "Two jobs that do not like to see by other people. That are work as a teacher and work as a thief," and if this anecdote is true, then for teachers, they are now open to be observed by others either by the teacher (another) or by policy makers (supervisors, department heads, principals). Now, they are open to learn from each other in improving the quality of learning at the microlevel in the classroom. Openness like this makes the chances of a teacher to have the optimal ability to make the classroom productive and allow teachers to apply science-based mathematics instruction, so that the students have an opportunity to be creative in learning mathematics and sciences.

Through the implementation of these learning materials, it was difficult for students to forget it, because it has a very deep impression and also encourages teachers to apply them in their own learning accomplishments.

In a study paper, Ref. [15] recommends to examine deeply whether the teachers' willingness to improve their professionalism in teaching tasks can improve their perform in teaching? Moreover, whether their better perform can improve students' achievement in mathematics? What kind of professionalism improvement could boost their strong willingness to innovate mathematics instruction? To answer the challenge of the recommendation, the author offers a study on the implementation of learning mathematics using science-based of didactical phenomena [1, 54], and empirically tested the implementation of this learning in the classroom.

Mathematics classes with the types of "transmission" as described by Senk and Thompson [16], include the introduction of each topic by declaring a rule which is followed by an example of how to apply the rules (rules, the arguments, the law), and then given a number of exercises, have encouraged developers who are looking for alternatives. Now, the effort to reform the mathematics is to portray the students participation actively, to transform the learning characterized by the "transmission" and to the learning characterized by the "participation."

In studying mathematics and science, the role of the students is constructing knowledge with the teachers. The teacher reveals the problems, asking questions, listening to students' answers, pursuing with follow-up questions (probing questions), and then wait for the responses of the students in the formation of knowledge or mathematical concepts expected. Teachers should be little patience to listen to the arguments, presentation, and reasoning expressed by the students, either in the form of oral or written communication.

Hearing the mathematical ideas of students is an important aspect in learning sound constructivism, i.e., to shift from "telling and describing" to "listening and questioning" and "probing for understanding" [17]. With science-based instruction of mathematics, students are directly retrieving data, processing the data, presenting the data in tables, and describing the data in the table into a chart and then it becomes possible to make a mathematical model of images.

## 5. Didactical phenomenology

The idea of a didactical phenomenology of [42, 1] provided the inspiration to explore the mathematical content through a search phenomenon that is suitable for regions in Indonesia. Suppose how to introduce the concept of linear equations using scales [43], introduce the concept of equation of a straight line or linear function using taxi fares and the cost of photocopying [44], teaches the volume of flat sides of space objects using sand beach [45], teaches the volume of balls and tubes using watermelon [46, 15], and many numbers of phenomena that can be appointed as a "bridge" to understand the concepts of mathematics for students.

An example of how the phenomenon of ball volume is approximated by cleavage of a watermelon is discussed as (**Figure 8** is personal collection of photograph) follows:



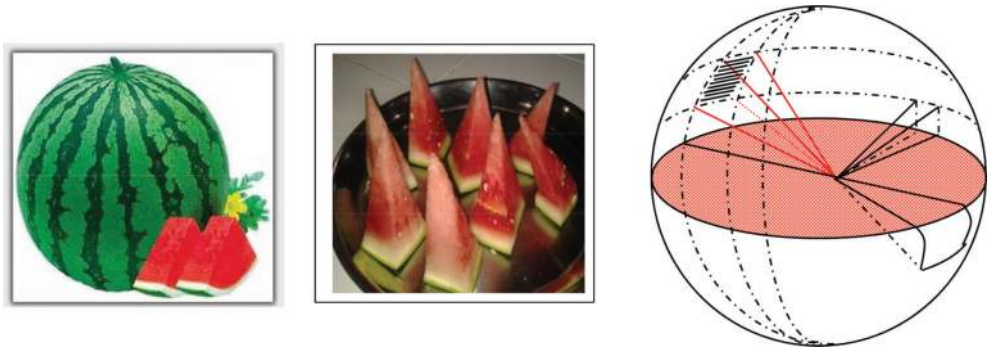


Figure 8. Watermelon ball to show the formula of sphere volume.

By adding the volume of “pyramid models” that are created from a watermelon ball accurately obtained the volume of ball [46], although students are still in doubt because the base of the pyramid-like model was a curved surface. However, this is in line with that proposed by [47].

Using the third figure of **Figure 8**, you can notice the role of “pyramid” in the sphere, that in a sphere we can make many “pyramids-like” models. One can make it easily by using watermelon.

Furthermore, the professional mathematics society which among them are mathematics teachers can help learning how to apply the kind of inquiry studied in the context of exploring didactical phenomenon. Ref. [48] distinguishes between the teachers who are looking for success in their career and teachers who tested their practice in relation to their thoughts. When teachers are tested on the basis of meaning of broad principles, in practice, they are involved in the alteration [48]. Such tests provide support for teachers to learn continuously and make them able to improve their teaching practices continuously anyway.

The existence of such a professional society is very important in supporting experienced teachers to teach in new ways [49, 50]. Professional societies not only provide space and time, but also can provide an environment for teaching practice. Mathematics teachers are the part of the communities involved in the effort to introduce the proceedings of their teaching practices, and can experience this type of learning for students as suggested above. These reforms initiated teachers to strengthen their classrooms with “learning society” in which students explore mathematics in depth [51].

Furthermore, [52] explains that the assumption of “communities of learners” is a form of learning that occurs when people participate actively and discuss with each other. In learning communities, students who are mature or not will share the responsibility to determine, direct, and manage the joint efforts. In view of the innovation, teachers organize students way of thinking, but the role of the teacher is a facilitator not a provider of answers. Mathematics class is seen as a place where students can actively make meaning of themselves and emphasize the process of learning mathematics [50].

The articles on the research and learning in the lesson-study are a matter of joint publications between teachers and lecturers. Students see “the form of linear equations” by comparing it with the “model of scales” a very pleasant experience. Forming a linear function using the “taxi rates and photocopy expenses” is an attribution according to the mathematics teacher that mathematics is so close to the real situation that is faced by the students. Moreover, study the volume of the tube using a “long watermelon” and make it easier for students construct so that they can find the formula for volume of tubes and balls. After mathematics learning students are allowed to consume the watermelon.

## 6. Roadmap of research

Researches that have been carried out by the authors that contribute to this study were presented in the form of road maps (fish backbone), such as research on RME (Realistic Mathematics Education), contextual learning of mathematics [3, 4, 18], mathematical modeling [2], planting consciousness of innovation on mathematics teacher [7], research on ethnomathematics [19, 56], learning with the nuanced phenomenon of didactic in junior secondary student [20], as well as the learning of mathematics using didactical phenomenology in primary school students [54]. The results of the study of RME turned out to encourage students’ enthusiasm for learning mathematics [3, 4, 18], mathematical modeling has opened the horizons of students to be able to see the phenomena that can be modeled [2], it turns ethnomathematics research opens up new horizons of research in the domain of mathematical culture [19, 21]. **Figure 9** is a fish bone of research roadmap within several years which covered realistic mathematic education and contextual teaching of mathematics, mathematical modeling, ethnomathematics, didactical phenomenology in mathematical areas.

Further, Ref. [22] added that for a group of teachers they observed, “the teachers reflection and involvement in professional development opportunities seemed to provide of catalyst and



**Figure 9.** Fish backbone of research roadmap.

change" (p. 130). Professional development of teachers often focuses on helping the teachers to improve learning in the classroom by developing the knowledge and pedagogical skills of the teachers. Professional society engaged in teaching suggests effective ways to provide support to teachers in implementing models of the new learning in their practice [23–27]. But Ref. [27] notes, "... is not so clear how people do or how they create or continue programs and policies" (p. 165).

In conjunction with the program of learning and professional development of teachers, Ref. [28] notes that "one of the two premises report of Glenda (US dept of Education-2000), that better quality learning is at the heart of change, and professional development program cannot be separated from the essence of improving the quality of learning" (p. 331). Our team of researchers, looked at the strength and nature of the professional teacher community, somewhere has significance because (1) the professional community can bridge and translate the efforts of renewal, (2) the professional community can provide support in introducing the kinds of renewal of learning mathematics (e.g., inquiry) required for the practical development of the principles and values are discussed. Empirical evidence and theory suggest that the strength, nature, and focus the professional community in the field of teacher training can bridge the efforts of the school when students learn. Furthermore, the school community can filter the principles, which vary knowledgeably as well as affect the interpretation of the goals of reform (renewal) in mathematics [29–31].

There is a serious criticism of the views of the previous example of the insights that mathematics is a knowledge that is fixed and static [32], as a system, rule, and formal procedure [33], as the rules and right procedures [34], as a set of concepts and skills that must be mastered by students [35]. Suggestions successor is the shift to alternative views, suppose the mathematics as a dynamic subject, as a human activity [10, 32], as the activity of the human senses and problem solving activities [35], or mathematics as humanized and antiabsolutist [36–39]. To facilitate students actively learn mathematics through investigation and exploration, there should be provided a phenomenon that was built by the designer of learning mathematics.

Research on realistic mathematics and their implications on the performance and abilities of students in mathematics further encourages depth curiosity of the research team, how much effect if we add or take properties of learning [56, 40]. From studies conducted on RME, contextual learning, ethnomathematics, modeling, and the phenomenon of didactic raise new questions, "What if the mathematics and science synergize so that students can conduct investigations either individually or together in group in the classroom." Let the students simulated such as how long the water flow from each faucet with various diameter sizes that range from a tub of water.

Suppose a liter of water was expelled through a Faucet A with hole diameter of 2 mm, then we measured how long the pouring time, compared to a Faucet B with hole diameter of 4 mm, we also measured how long the pouring time. Students are required to collect and record the information obtained in the form of a table for which they are asked to describe the graph and determine the mathematical models, equations associating the faucet diameter with the flowing time.

Further to the solution of sugar water with various concentrations of submerged objects that sank in all of the solution, students are asked to interpret the meaning of drowning and are associated with a ratio of the density of objects with the density of each solution. Students are also asked to investigate for how long the objects undergoing the process of sinking from the surface of the solution to the base of the tube solution. The stopwatch is used for recording of each liquid in the tube. Students are also able to model mathematically the magnitude of the solution concentration by the length of the time (in seconds) the object taken to fall from the surface of the solution to the bottom of the bottle.

I wonder what effect it has on the health of the body, if someone drinks a thick liquid of sugar continuously compared with drinking fluids diluting the sugar. Continued impact of what happened to our body turns into increased blood viscosity? How did it effect the blood circulation and the transport of oxygen from the lungs to the brain by the blood? These consequences are expected to sensitize students to maintain their own health.

## 7. Innovative perspectives

The views of this innovative approach affect how the teacher in the classroom and how teachers evaluate students learn mathematics. This is related to the questions of the students related to mathematical ideas, the introduction of mathematical concepts, encourage and promote discussion and group work. The Minister of Education and Culture of Indonesia in the era of 1990s reminds us through his views on mathematics and science "Most schools and teachers treat students as a 'vessel' or something to be filled with knowledge." Another well-known example is the tendency toward right-wrong answer/fact-based learning. School and teachers focus on getting the right answer from the students at the cost of developing the processes that generate the answer [41]. Furthermore, he argued "I would like to challenge you to create greater understanding on how students learn as prerequisite for improving our teaching methods in mathematics and science, and improving the education of teachers for these subjects" (p. 36). These challenges need to be captured and acted wisely, of course. Similar challenges also presented by the President of the National Council of the Teacher of Mathematics (NCTM), Glenda Lappan "Throughout the more recent mathematics education research literature, there have been expressions of growing dissatisfaction with the limitations of the traditionally formal ways of teaching mathematics." Suppose, Lappan (1999, cited by [16]) provides arguments "We have had the longest running experiment in human history about whether rote memorization of facts and skills works." And it does not. Students are coming to universities and to the work place for not understanding mathematics. Why would not I want to try something new?

Challenges like that should be welcomed "Why we do not want to try something new?" After Lappan [16] we had a long trial of the history of humanity, about whether rote memorization of facts and skills can take place either? Challenges of Minister of Education and Culture, to (1) create a better understanding and to create a method of learning in mathematics and science [41], and (2) the growing dissatisfaction with the limited ways of teaching mathematics is

traditionally formal (Glenda Lappan in Ref. [16]), gave rise to the urge to try something new, for example learning by using didactical phenomenon. Lappan (in Refs. [16, 41]) was one of international proponents who are very concerned for better innovative changes.

The underlying issue is how do we support the desire of teachers to improve learning in the classroom and how to provide examples and ideas that can be utilized in a practical way by the teacher in the classroom.

## 8. Action plan

Teaching materials designed in the planning of learning include sugar solution, water fountain with various sized holes, burning fireworks, and opening faucets (various sizes of angle) to record the time of flowing for a certain volume of water. However, because of limited space for reporting in this chapter, learning implementation of sugar solution is only discussed, while others will be described in other chapters.

The research team succeeded designing instructional materials that tried to link the two quantities, namely the percentage of sugar solution and long-time sneaking of a ball of clay.

Sugar solution is formulated by weighing the sugar and water. In **Figure 10**, the researcher team made sugar solutions by balancing the sugar and water proportionally and stirred them, and the results were presented in the tube as in **Figure 11** (personal collection of photographs).

### 8.1. Sugar solution

The instructional design resulted by researcher team produces sugar solution with varying concentrations, as appear in **Figure 12**. With the sugar solution, it is expected that students are able to obtain the numbers as domain and its pair numbers as member of codomain. Suppose that 5% sugar solution is stored in glass tubes. We enter a small ball made of plasticine, then measure the time duration of sneaking the ball when put in a 5% sugar solution. The trial results showed that the sedimentation time of plasticine ball in a 5% sugar solution is 1.22 s;

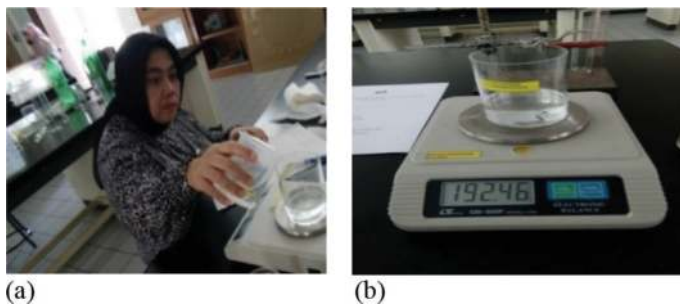


Figure 10. Balancing the sugar and water.

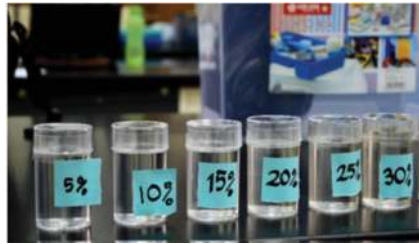


Figure 11. Sugar solutions with various concentrations.

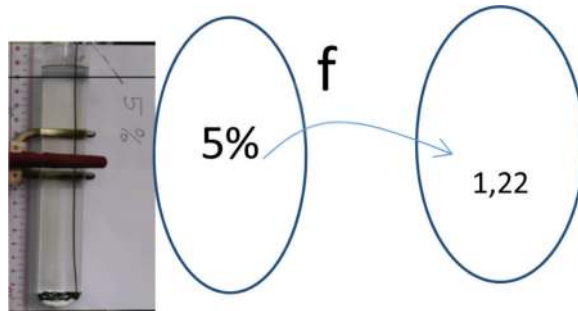


Figure 12. Five percent sugar solution with "time sneaking".

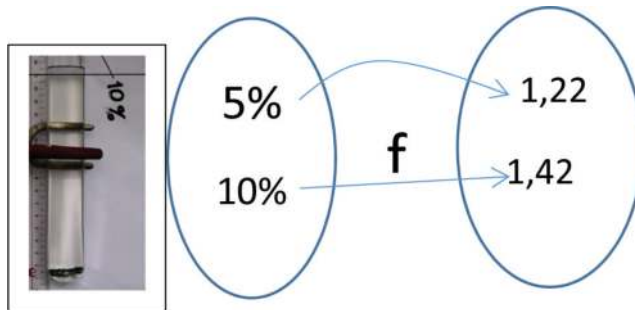


Figure 13. Five percent and 10% sugar solutions with "time sneaking".

and in 10% sugar solution time is 1.42 s, and so on. Therefore we can present the results as in **Figures 12 and 13** (personal collection of photographs).

If we continued this work then we would obtain functional relationship between percentage of sugar solution and time taken by the ball sneaking in the sugar solution. So students can describe



**Figure 14.** (a) Discussion in Sci Lab, (b) working in Sci Lab.

“function that occurs in the form of graphs of functions in Cartesian coordinates”. **Figure 14** indicated the team researcher to obtain the sugar solution accurately (personal collection of the photographs).

## 9. Implementation in the classroom

Learning in the classroom begins with understanding the concept of relations and functions using conventional approaches, but students are asked to make the association between the two functions without involving the phenomenon. Then introduced didactical phenomena of science to the students by presenting a power point presentation as an observation summary of the sneaking of a “clay ball”, asking students for taking note the time in respective bottle of sugar solution (0-50%). Furthermore, students record each of the events and copy them into a table that has been provided. Moreover, the students also observe how the burning of fireworks takes place and record the time duration of fireworks burning from start to finish. Other thing that be learned by the students in the classroom was in making association between the wide hole of diameter of faucet and the debit of flowing water for each faucet. However, due to space limitations to address all the learnings in this chapter, a team of authors only discussed part of the sugar solution.

## 10. Discussion

### 10.1. Design phase

There is one interesting thing that happens when the sugar solution reaches to 50% solution. It turns out that “ball clay” is not sinking, the ball in the solution is not dropped or immersed in the 50% sugar solution, but went up and floating. A member of the researcher who is a junior high school teacher was alarmed and shocked and thus raises the question “Why not

down?" Why and why? She relayed the question over and over again while still in the physics lab, during a process of designing instructional materials that have not been brought into the junior secondary class.

Because the solution that is available only up to 40–50%, while 45% is not yet available, so he had the initiative and desire to deeply make a solution of 45% immediately and she wanted to know how the time crept to the 45% sugar solution. The research team soon made the 45% sugar solution, and measured how long time (how many seconds) a plasticine "clay ball" felt down in the sugar solution. In fact it took 61,22 seconds.

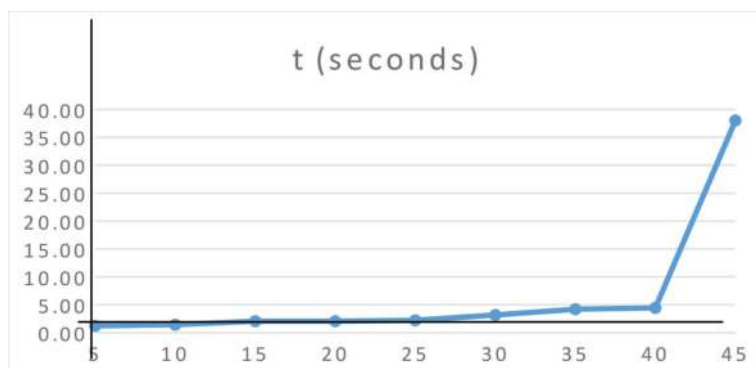
For an ideal situation after discussion with the team (persons of mathematics, physics, computer science, and mathematics teachers), the team suggested we should also know the duration of time sneaking of the "ball" in the solutions of sugar 41, 42, 43, 44, 46, 47, 48, and 49%, but due to time constraints and opportunities, the team finally just gave a prediction of duration time that in graph will look roughly like the image below in **Figure 15** (the graph is made by the researcher team using excel).

For junior secondary students, drawing graph smoothly was not a main target. There was no obligation for students to draw graph smoothly. But the researcher and developer team in this study try to interpret and predict the form of graph look like. It encourage students and recommend the researcher team to investigate further for the numbers around the 45%. It provides an impetus and a recommendation to investigate further around earlier numbers.

Equations or mathematical models in relation to the concentration of the solution with time of "sneaking ball" into a particular function, again for junior high school students, have not been the main target. The junior high school students are required to put or plot dots of various observation results as coordinates (solution, time) or coordinates (time, solution).

## 10.2. Discussion in implementation phase

Before getting into the observation using teaching materials (model) that have been prepared in the laboratory to the students, worksheets were also presented which aim to explore



**Figure 15.** Graph prediction of the sugar solutions.



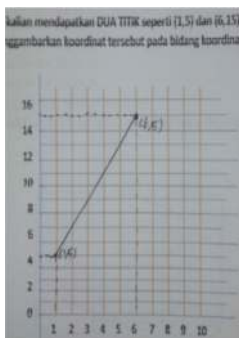


Figure 16. Incorrect graph  $\{(1,5), (6,15)\}$  graph.

knowledge of whether the student has been able to plot the points of known coordinates. In the “worksheet” the known point  $(1,5)$  and point  $(6,15)$ , appears from the existing worksheets, students were asked to plot them in the Cartesian coordinates. There is a group of students who describes a straight line that contains the point  $(1,5)$  and  $(6,15)$ , that is supposed to only two points, namely  $(1,5)$  and  $(6,15)$ . The students plotted correctly, but wrote  $(0,5)$  wrongly, it supposed to be  $(1,5)$  (see **Figure 16**). Students should only plot the coordinates  $(1,5)$  and  $(6,15)$ , not necessary to draw the line from  $(1,5)$  to  $(6,15)$  (the graph were made by students and were photographed by the authors, **Figure 16-18**).

In plotting the coordinate points of  $\{(1,5), (2,7), (3,9), (4,11), (5,13), (6,15)\}$ , generally students worked correctly, but some of students were not correct. The graph of the points are dots as figuring out by students in the **Figure 19**, not as a line segment as figuring out in the **Figure 16** and **20**. (the graphs of **Figure 19** and **20** were made by the students and photographed by researcher team).

Teachers began to deliver lessons after asking a number of questions above and confirmed that the correct point coordinates are the pairs of points  $(x, y)$  such that  $x$  and  $y$  are integers in a couple of points  $\{(1,5), (6,15)\}$  and do not represent a straight line.



Figure 17.  $\{(1,5), (6,15)\}$  correct graph.

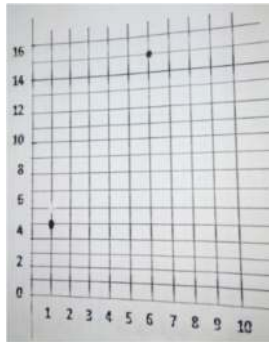


Figure 18.  $\{(1,5), (6, 15)\}$  correct graph.

However, the research team did not worry, because in general the students were able to plot the coordinate points that should be described in the coordinate plane.

### 10.3. Sugar solutions and graph

The next steps, after the students were able to draw coordinate points, they start to learn the part of sugar solutions in relation to viscosity (velocity) of sneaking ball in the various sugar solutions (percent solution of 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50%). They are exposed to the tools that have been recorded in the power point presentation. Furthermore, students use stopwatch (in their mobile phone) to measure the sneaking time of “ball clay” in each of the sugar solution. In this case the student does not measure speed, but measures how long it takes for sneaking “ball clay.”

Some observations of groups of students are outlined in the following table:

Solution	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Time	1.22	1.55	1.93	2.45	2.85	4.76	5.28	6.0	6.95	61.22	~

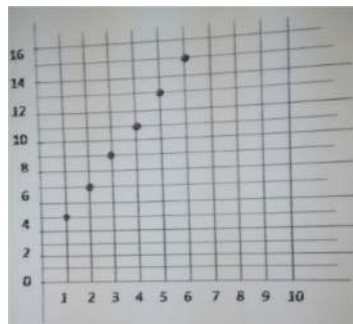


Figure 19.  $\{(1,5), (2,7), (3,9), (4,11), (5,13), (6,15)\}$ .

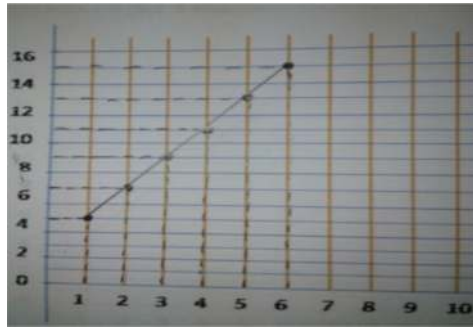


Figure 20.  $\{(x,y) \mid y = 2x + 3, x \text{ dan } y \text{ real dengan } 1 \leq x \leq 6\}$ .

By plotting the points in the above table, there was obtained the following graph (Figure 21).

The graph in Figure 21 indicated the original students' work that can be plotted using excel without any adjustment, except for the data of 35% just predicted. But after entering the data corrected by 35% and the estimate (approximate) data for a solution of 41, 42, 43, and 44%, the graph time-solution was obtained as follows:

Figure 22 represents the relationship between percentage of sugar solution and the sedimentation time of the ball. Each point in the graph represented the number ordered of solution (%) and time.

Students either individually or in groups in the classroom have already understood the concept of the functions and relationships. Modeling of the sugar solution above depicts a phenomenon that in order to precipitate an object into the solution, the more concentrated the solution the longer the time required to precipitate an object. In other words, the more concentrated the solution is, the greater the obstacles encountered objects to penetrate the solution (see Figures 23–25 as students' work after their observation the solution-time. The tables are made by the students, but photographs are made by the researcher team as the personal collections).

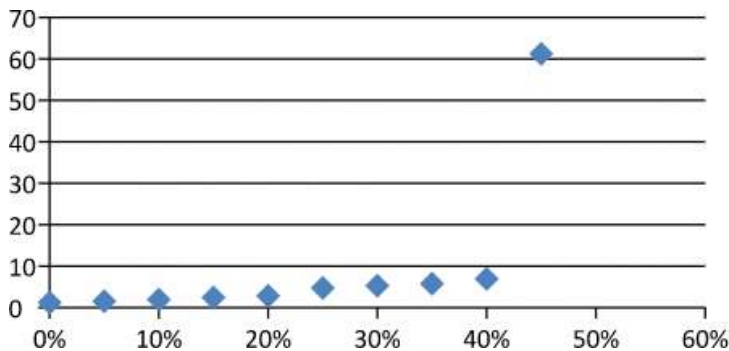


Figure 21. Plotting graph of sugar solution to sedimentation time.

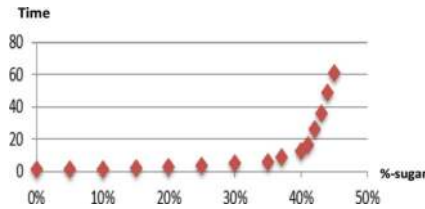


Figure 22. Plotting the percentage of sugar solution versus sedimentation time of plasticine ball.

The conclusion above can be used as a metaphor for our body liquid. If the liquid of our blood in our body more concentrated, the more difficult this liquid transforming objects (e.g., blood carries oxygen” from the heart to the brain). If transport is hindered then the patient will feel pain in his head. **Figure 26** is a circulation system of our body, the blood from the heart transports the oxygen to the brain. When the blood concentration gets high, then the stability of our health will influence.

About the extent to which the student can give reasons why the following arrow diagram is a function and why is not function, descriptions of student work is displayed as follows:

Students above (**Figure 27**) understood the relationship, as they wrote “A Member of A is only be paired with one quantity,” even though, in fact, the relationship is “a very simple relationship among two sets,” as far as the two sets are associated. In a particular association, this relationship was named function, this group gives reason “special relationship in A which paired with exactly one member of C.” Suppose “special relationships that map each member of A with exactly one member of C” (see **Figure 27**).

Students (or other groups) (**Figure 28**) state as to which they answer the following, “because it is the relationship between the set-1 and set-2.”

However, the function according to researcher, the term written by students has not written correctly. It is supposed to be the function “The special relationship that links each element in the set-1 (domain) with exactly one element in the set-2 (codomain)” **Figure 29**.

“(a) is a relation which is a function from A to B,” while

“(c) is the relation which is not a function, because one of the members in A has two images in B.”

Siswa mencatat berapa lama larutan pada masing-masing tabung dijilani oleh kelereng dan siswa membuat tabel:

Larutan	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	46%	47%	48%	50%
Waktu		1,50	1,70	2,22	3,85				6,95	11,34				

Figure 23. Students’ work of Cohort-1.

Location	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	46%	47%	48%	50%
Waktu		1,355	1,682	2,045	2,385	4,76	5,10		5,35	11,01-24				

↓  
61,22

Figure 24. Students' work of Cohort-2 with a correction by researcher.

Location	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	46%	47%	48%	50%
Waktu	122	1,60	1,87	2,17	2,70	4,76	5,10	6,0	6,87	10,70				

Figure 25. Students' work of Cohort-3.

Figure 29 is a problem to be asked for students whether this diagram is a function or not and why? Similarly, Figure 30 is a question for the students, whether this diagram is a function or not? And why? Whereas Figure 31 is student's reasons why the diagram is a relation and function or why the diagram is a relation but not a function. (Figures 29–31 are personal collection of photographs.)

Although the proposed language is less precise, at least the students have an idea that they can distinguish between the functions and relationships. While other students understand the word "function" as a means "to" or "benefit," as the answer to the following students:

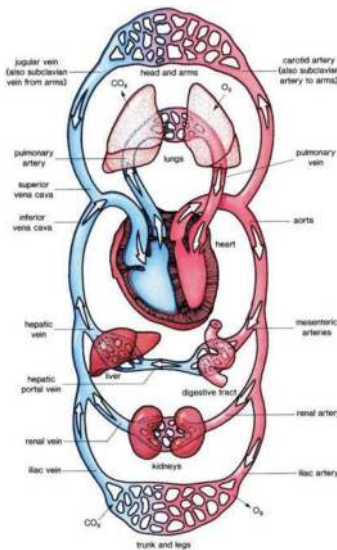


Figure 26. The circulatory system [53].

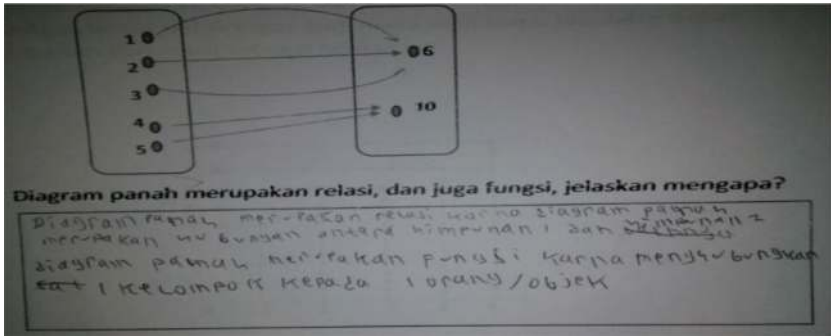


Figure 27. Students' work of function definition.

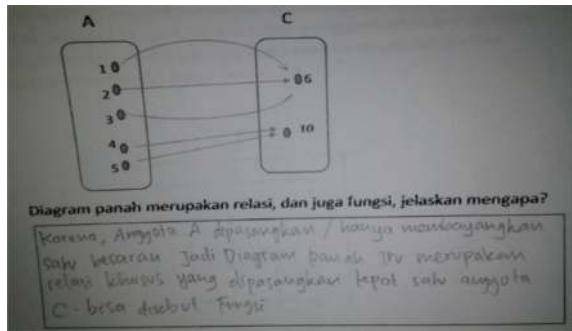


Figure 28. Students' work.

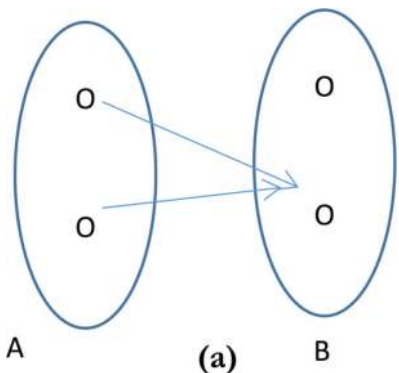


Figure 29. Relation as function.

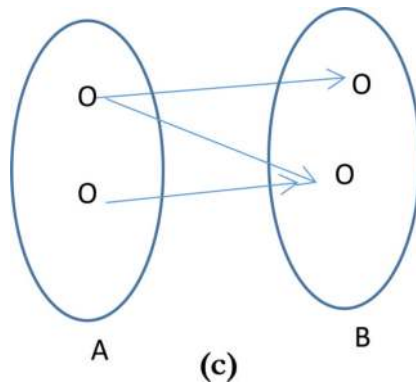
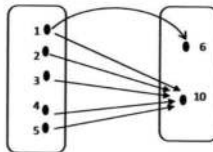


Figure 30. Relation not as function.

1. Gambar a merupakan fungsi karena masing-masing anggota A membayangkan satu dari anggota himpunan B  
 Gambar c bukan fungsi karena salah satu anggota himpunan A membayangkan 2 anggota himpunan B
2. Gambar g bukan fungsi karena masing-masing anggota himpunan A membayangkan satu anggota himpunan B.  
 Gambar c bukan fungsi karena salah satu anggota himpunan A tidak membayangkan siapa-siapa dari himpunan B

Figure 31. Reason for a function and not a function.

Hubungan pada diagram ini relasi namun bukan fungsi. Jelaskan



Karena anggota A dipasangkan dengan Anggota B, jadi dinamakan relasi, tetapi ada satu anggota yang membayangkan dua besaran, jadi tidak berfungsi.

Figure 32. Students' work and reason why this relation is not function.

“There is one member of A that has more than one image in C, then it is not called a function from A to C” (see **Figure 32**, personal collection of photographs).

What do the specific things on the phenomena of “fireworks”, “sugar solution” and “faucet” are in fact a function that each phenomenon can be expressed as diagram of arrow, or with a table, or with the ordered pairs, or with the chart coordinates.

Although the students’ understanding of the concept of relationship is less perfect, the students have already understood the concept of function. They observed a sugar solution and sedimentation time of plasticine ball in each solution through a power point presentation. They detected them using stopwatch provided (or using their own Hand phone or mobile phone (HP)).

## 11. Conclusion

From the study of the implementation of mathematics instruction based on science using didactical phenomenology approach we conclude that (1) the prototype of learning mathematics can be made simply and scientifically in laboratories either by using sophisticated equipment or by using a simple way, as long as all the equipments can produce two groups of quantities, (2) the implementation process of mathematics learning in the classroom does not always use original tools such as original equipment set in the laboratory, but use equipment or software or a power point presentation as a tool or medium for the presentation of the photo or video animation of the laboratory equipment (video water discharge, video sugar solution, and “deposition” of plasticine ball, fireworks video, and video of swivel angle and discharge of water), (3) the reaction of students toward learning materials of mathematics based on science using didactical phenomenology shows positive attitudes and enthusiasm, (4) achievement of mathematical ability even though a group of students who study mathematics based on science using didactical phenomenology approach have a higher average than students who study using conventional approach, statistically both are not significantly different, (5) there are differences in improvement of mathematical ability among students who study mathematics based on science using didactical phenomenology approach (0.48) with students who studied with conventional approach (0.36). But both are in the same categories (middle) and mathematical models were found to show students the results that can be interpreted. Model of fireworks is considered as linear, water discharge models are considered as linear, and model of the sugar solution is considered as a graph arch.

When the researcher team made an experiment in laboratory, there is an interesting finding. When the clay ball was put on the sugar solution, the smaller percentage of the value of sugar solution, the faster the rate of sedimentation of “clay ball” and the higher concentration of sugar solution or the more concentrated of sugar solution, the slower the clay ball penetrates the solution. So the time to reach the base of the bottle is getting long. When looking at the 40% sugar solution, the ball still can be awaited, when a solution to be 45% the ball still could be awaited although it requires longer time. However, when



observing the 50% solution, a teacher who helped designing the study shows surprise and astonishment, "Why is this happening?" In fact she connects the question, "What to do with death?" Then our mutual discussions with the belief held by strengthening the teacher. Approximately what causes happen so? Yes, if the clay ball stops (or floats), it means the same as our blood in our body was stuck because of concentrated so that it can no longer carry oxygen. Interestingly, this teacher seems to associate a sugar solution with the body fluids or blood fluid in our body. The phenomenon of nature (physics) is that a severe type of ball clay is smaller than the density of the sugar solution, so that the "ball clay" floats. If the weight is of the same type, then the ball will be hovering in the 50% sugar solution. For clarifying this situation to the students, then the teachers shares readings about the relationships between viscosity of our blood and maintenance of our health.

Observing this phenomenon, our research team is interested in observing and making a mathematical model of the graph. Apparently, the graph becomes asymptotically at 50% solution. In fact, it still needs to be investigated in a solution with a lower concentration, e.g., 49, 48, 47%, and so on, or fragments are more accurate as 49.5, 49, 48.5, 48, 47.5, 47, 46.5%, 46, 45.5%, and so on. We are talking to mathematicians and they advised to build mathematical models. For students, graphed predictions appear as in **Figure 15**.

Noting the benefits of mathematics instruction based on science using didactical phenomenology approach and its consequences on the students, teachers, and on student achievement, the team delivered the following suggestions.

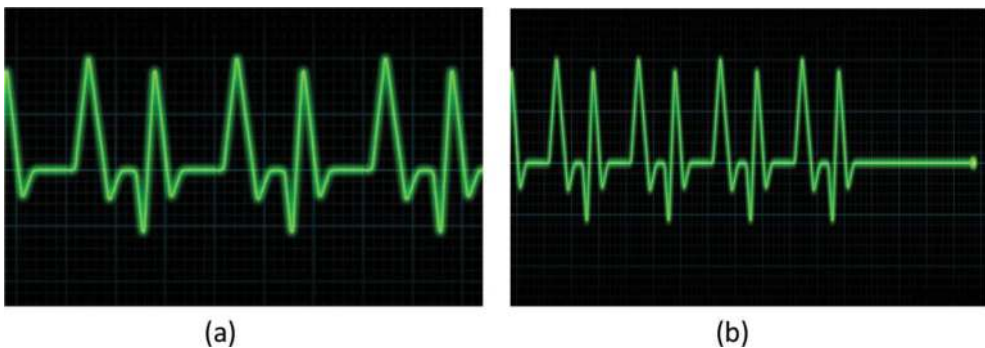
1. That mathematics instruction based on science using didactical phenomenology approach can be an alternative approach in mathematics education, especially for junior high school students who are in a period of transition from concrete thinking to the future abstract thinking.
2. That the issue of didactical phenomenology, both students and teachers become aware of many phenomena in the area of natural and artificial phenomena, and ultimately both have the ability to see phenomena that become real for students. Therefore, the research team suggested to sharpen the sensitivity of seeing the phenomena by repeating ever trained.
3. Although the achievement of mastery of mathematical statistical did not differ significantly, enhancement of mathematical ability in experimental group is higher than the enhancement in control group, this indication further encourages researchers to enhance teaching model like this, so that the sensitivity of the students improved in terms of understanding the didactical phenomenology.
4. The graph of certain phenomena is not always linear, junior high school students can also see the nonlinear phenomenon.
5. It needs further research to a higher level or junior high school so that they can see the other phenomena that can be modeled.

## 12. Recommendation

Learning mathematics based on science using didactical phenomenology approach turned out to inspire teachers and students about the importance of mathematics and science in understanding health. Skill and ability of the students to record and present data in table form become a necessity, much less the ability to characterize graphs modeling capabilities. Ultimately, students are able to interpret the asymptote line as a “death” phenomena, after they are easily interpreting the graph of a straight line in the computer screen of ICCU (Intensive Coronary Care Unit) of a hospital. Similar things can be understood, that when the graph in **Figure 15** turns the curve straight up, then the “ball clay” is difficult to penetrate a sugar solution of 50%, so that the “ball clay” floats, and the time duration for penetrating solution was longer or even never again pierce of 50% sugar solution. The analogy is similar to blood fluid that is no longer able to carry oxygen from the lungs to the brain, so that the mortality occurs as consequences. In classroom, the students were able to give such an interpretation within discussions. As a consequence, students will be cautious when consuming sugar water (such as sweet coffee, or syrup).

In the learning process, when the students are able to observe phenomena, able to represent the data into coordinates or in the ordered pairs, and able to draw its graph, then most students have understanding competencies of mathematics. But the higher competencies such as “mathematical modeling” were still need to be learnt more by students in order to be able to make model of equation of phenomena.

The phenomenon of the sugar solution is a model that is very attractive; students are invited to think about the solution of the blood in the body. When the blood has been thinned, it is still possible to “carry” the oxygen from the lungs to the brain, the condition is very good and smooth (the graph would be as in **Figure 33a**). When blood viscosity increases, the ability of the blood to carry oxygen decreases, so such symptoms affect the health of the human body. At the time when blood is no longer able to carry oxygen to the brain, the oxygen supply to



**Figure 33.** Cardiograph in a hospital [57]. (a) Cardiograph for the normal patient, (b) Cardiograph for a “death” patient.

the brain is stopped, it can be imagined what would happen to our body, and the graph of **Figure 33b** would represent this situation.

## Author details

Turmudi<sup>1\*</sup>, Setya Utari<sup>1</sup>, Suprih Widodo<sup>2</sup> and Ratnaningsih<sup>3</sup>

\*Address all correspondence to: [turmudi@upi.edu](mailto:turmudi@upi.edu)

1 Faculty of Mathematics and Science Education (FPMIPA) of Universitas Pendidikan Indonesia (Indonesia University of Education), Bandung, Indonesia

2 Program of Primary School Teacher Education of Purwakarta Campus of Universitas Pendidikan Indonesia (Indonesia University of Education), Indonesia

3 Mathematics Teacher of Junior Secondary School (SMPN 12), Bandung, Indonesia

## References

- [1] Freudenthal H. *Didactical Phenomenology of Mathematical Structures*. Dordrecht. Kluwer Academic Publishers; 2002
- [2] Turmudi, Hidayat AS, Prabawanto S, Aljupri. Mint: pembelajaran matematika dengan pemodelan (mathematical modeling) berbasis realistik untuk mahasiswa. Dalam *Jurnal Pengajaran Matematika dan Ilmu Pengetahuan Alam (JPMIPA)*. 2014;**19**(1):1-18
- [3] Turmudi, Sabandar J. Kerjasama mahasiswa calon guru dan guru bidang studi dalam mengembangkan desain pembelajaran matematika realistic di SMP Negeri Kota Bandung. Grant Research Report, Bandung: Indonesian University of Education, FPMIPA; 2002
- [4] Turmudi, Dasari D. Peningkatan kemampuan pemahaman konsep matematika bagi siswa SLTP melalui pendekatan realistik. Grant Research Report, Bandung: Indonesian University of Education, FPMIPA; 2001
- [5] Turmudi. *Designing contextual learning strategies for mathematics for Junior Secondary School in Indonesia*. [PhD Thesis]. Australia: La Trobe University; 2006
- [6] Turmudi. Mint: Teachers' perception toward mathematics teaching innovation in Indonesian junior high school: An exploratory factor analysis. *Journal of Mathematics Education*. 2012;**5**(1):97-120
- [7] Turmudi, Juandi D, Harningsih E, Wihatma U. Improving the teacher' awereness to innovaton of mathematics teaching for junior secondary school teachers through lesson study. Research Report. LPPM Universitas Pendidikan Indonesia, unpublished; 2010

- [8] Turmudi, Hidayat AS, Prabawanto S, Aljupri. *Pemodelan matematika dalam pembelajaran Realistik Matematika*. Research Report, FPMIPA-LPPM, un-published; 2009
- [9] Gravemeijer K. *Developmental Research Fostering a Dialect Relationship Between Theory and Practice*. CD-Room of the RME materials produced for the ICME9 Congress in Japan; July 2000
- [10] Freudenthal H. *Revisiting Mathematics Education: China Lectures*. Dordrecht: Kluwer Academic Publishers; 1991
- [11] Nuh M. *Sosialisasi Kurikulum 2013 di Universitas Pendidikan Indonesia*. 16 Maret 2013 di Gedung Gymnasium, UPI, Bandung; 2013
- [12] Djazuli A. *Upaya dan harapan Kanwil Depdikbud Propinsi Jawa Barat dalam meningkatkan kualitas guru MIPA pada Dikdasmen*. In: Rohman I, Imansyah H, Sopandi W, editors. *Seminar on Quality Improvement of Mathematics and Science Education in Indonesia*. Bandung: JICA-IMSTEP-Bandung Institute of Teaching and Educational Sciences; 1999. pp. 15-27
- [13] Coribima D. *Proses dan hasil pembelajaran MIPA di SD, SLTP, dan SMU: Perkembangan penalaran siswa tidak dikelola secara terencana (Studi kasus di Malang, Yogyakarta, dan Bandung)*. In: Rohman I, Imansyah H, Sopandi W, editors. *Seminar on Quality Improvement of Mathematics and Science Education in Indonesia*. Bandung: JICA-IMSTEP- IKIP Bandung; 1999. pp. 73-84
- [14] Shiu C, Hatch G. *Professional development*. In: Johnston-Wilder S, Johnstone-Wilder P, Pimm D, Westwell J, editors. *Learning to Teach Mathematics in the Secondary School: A Companion to School Experiences*. London: Rontledge; 2005. pp. 246-257
- [15] Turmudi, Suherdi U, Kusmana D. *Baby watermelon for Creating the Formula of Cylinder Volume in Junior Secondary Classroom: An Experience in Lesson Study*. MCEIS-2013; 2013
- [16] Senk SL, Thompson DR. *School mathematics curricula: recommendations and issues*. In: Senk SL, Thompson DR, editors. *Standard-Based School Mathematics Curricula: What are They? And What do Students Learn*. New Jersey: Lawrence Erlbaum Associated; 2003. pp. 3-27
- [17] Maher CA, Alston A. *Mint: Teacher development in mathematics in constructivist view on the teaching and learning of mathematics*. *Journal for Reseach in Matematics Education, Monograph*. 1990;4:147-166
- [18] Turmudi. *Implementasi awal pembelajaran geometri dengan pendekatan realistik di sekolah menengah pertama*. Laporan Penelitian mandiri. Bandung: UPI, FPMIPA, un-published; 2001
- [19] Turmudi, Juandi D, Hidayat AS, Puspita E. *Eksplorasi etnomatematik masyarakat kampung Baduy dan kampung Naga*. Laporan hasil penelitian, FPMIPA-LPPM, un-published; 2013

- [20] Turmudi, Hidayat AS, Sispiati R. Pengembangan pembelajaran matematika berbasis fenomena didaktis: (Sebuah Terobosan Inovatif dalam Mengenal, Mendesain, dan Mengimplementasikan serta Memvalidasi Bahan Ajar Matematika di Sekolah Menengah Pertama). Laporan hasil penelitian, FPMIPA-LPPM, un-published; 2014
- [21] Turmudi, Juandi D, Hidayat AS, Puspita E, Ulum AS. Mint: Exploring Ethno-Mathematics: How the baduy of indonesia use traditional mathematics skills in weaving. *International Journal of Control Theory and Applications*. 2016;9(23):324-339. ISSN: 0974-5572
- [22] Anderson J, White P. Problem solving in learning and teaching mathematics. In: Perry B, Anthony G, Diezmann C, editors. *Research in Mathematics Education in Australasia*. Flaxton, Queensland: Post Pressed Flaxton; 2003
- [23] Cochran-Smith M, Lytle S. Relationships of knowledge and practice: Teacher learning in communities. In: Iran-Nejad A, Pearson PD, editors. *Review of Research in Education*. Washington, DC: American Educational Research Association; 1999. pp. 249-306
- [24] Little JW. Mint: Teachers' professional development in a climate of educational reform. *Educational Evaluation and Policy Analysis*. 1993;15(2):129-151
- [25] National Research Council. Adding it up: Helping children learn mathematics. In: Kilpatrick J, Swafford J, Findell B, editors. *Mathematics Learning and Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education*. Washington, DC: National Academy Press; 2001
- [26] Nelson BS. Learning about teacher change in the context of mathematics education reform: Where are we going?. In: Fennema E, Nelson BS, editors. *Mathematics Teachers in Transition*. Mahwah, NJ: Erlbaum; 1997. pp. 403-419
- [27] Smylie MA. Redesigning teachers' work: Connections to the classroom. In *Review of Research in Education*. 1994;20(1):129-177
- [28] Farmer JD, Gerretson H, Lassak M. Mint: What teachers take from professional development: Cases and Implication. *Journal of Mathematics Teacher Education*. 2003;6:331-360. DOI: 10.1023/A:1026318709074
- [29] Knight P. Mint: A systematic approach to professional development: Learning as practice. *Teaching and Teacher Education*. 2002;18:229-241
- [30] Perry RR. The role of teachers' professional communities in the implementation of California mathematics reform. Unpublished doctoral dissertation [thesis]. Palo Alto, CA: Stanford University; 1996
- [31] Talbert JE, Perry R. How Department Ommunities Mediate Mathematics and Science Education Reforms. New Orleans, LA: Paper Presented at the Annual Meeting of the American Educational Research Association; 1994
- [32] Romberg TA, Kaput JJ. Mathematics worth teaching, mathematics worth understanding. In: Fennema E, Romberg TA, editors. *Mathematics Classroom that Promote Understanding*. New Jersey: Lawrence Erlbaum Associates Publishers; 1999. pp. 3-17

- [33] Clarke B, Clarke D, Sullivan P. The mathematics teachers and curriculum development. In: Bishop AJ, et al. editors. *International Handbook of Mathematics Education*. Dordrecht, The Netherlands: Kluwer Academics Publishers; 1996. pp. 1207-1234
- [34] Ernest P. Image of mathematics, values and gender. In: Allen B, Johnston-Wilder S, editors. *Mathematics Education Exploring the Culture of Learning*. London: Routledge Falmer; 2004. pp. 11-25
- [35] Verschaffel L, De Corte E. Number and arithmetic. In: Bishop AJ, Jeremy K, Colette L, Ken C, Kristine K, editors. *International Handbook of Mathematics Education*. Dordrecht, the Netherlands: Kluwer Academics Publishers; 1996. pp. 99-137
- [36] Cockcroft WH. *Mathematics Counts: Report of the Commission of Inquiry into the Teaching of Mathematics in Schools*. UK: Her Majesty's Stationary Office; 1982
- [37] National Council of Teachers of Mathematics. *Principles and standards for school mathematics*. Reston, The National Council of Teachers of Mathematics, Inc.; 2000
- [38] National Council of Teachers of Mathematics. *An agenda for action: Recommendations for school mathematics of the 1980s*. Reston, The National Council of Teachers of Mathematics, Inc.; 1980
- [39] National Council of Teachers of Mathematics. *Curriculum and evaluation standards for school mathematics*. Reston, The National Council of Teachers of Mathematics, Inc.; 1989
- [40] Turmudi. Students' responses to the realistic mathematics teaching approach in junior secondary school in Indonesia. *Proceedings of IICMA -2009*. Yogyakarta: Mathematics Education; 2009. pp. 735-754
- [41] Djojonegoro W. Opening remark: Minister of Education and Culture Republic of Indonesia at the International seminar on science and mathematics education. In the *Proceeding of International Seminar on Science and Mathematics Education (Comparative Study between Indonesia and Japan)*, Bandung, 1995; pp. 32-39
- [42] Freudenthal H. *Didactical Phenomenology of Mathematical Structures*. Dordrecht: Kluwer Academic Publishers; 1984
- [43] Turmudi, Haryanto D. Creating and solving model of linear equation through the balance at junior secondary classroom. In: *Proceeding in International seminar and the 4th National Conference of Mathematics Education, Building the Nation Character Through Humanistic Mathematics Education*; UNY: Yogyakarta. 2011. pp. 619-630
- [44] Turmudi, Ratnaningsih. Tarif Taxi dan Biaya Fotocopy untuk Pengenalan Konsep Fungsi Linear di SMPN 12 Bandung: Lesson Study. *Konfrensi Nasional Pendidikan Matematika ke-16 di UNPAD*, 3-6 Juli 2012; di Jatinangor, Sumedang; 2012
- [45] Turmudi & Erlina E. Constructing formula of Pyramid Volume by Comparing Volume of Prism and Container Media in Junior Secondary School: A Lesson Study. *The 5th*

- International Conference on Research and Education in Mathematics; AIP Conf. Proc. 1450, 2012, pp. 173-182; doi: 10.1063/1.4724136
- [46] Turmudi, Julia. Watermelon for proving the volume of ball in Junior Secondary School. UPI-UPSI conference; Bandung-Indonesia; 2014
- [47] Keirinkan. Mathematics Gateway to the future Math 1 for Junior High School. Hiroshima, Japan; Shinko Shuppansha Keirinkan Co., Ltd. 2013
- [48] Franke ML, Carpenter T, Fennema E, Ansell E, Behrend J. Mint: Understanding teachers' self-sustaining, generative change in the context of professional development. *Teaching and Teacher Education*. 1998;**14**:67-80
- [49] Franke ML, Kazemi E. Teaching as learning within a community of practice: Characterizing generative growth. In: Wood T, Nelson BS, Warfield J, editors. *Beyond Classical Pedagogy*. Mahwah, NJ: Erlbaum; 2001. pp. 47-74
- [50] Stein MK, Silver E, Smith MS. Mathematics reform and teacher development: A community of practice perspective. In: Greeno JG, Goldman S, editors. *Thinking Practices in Mathematics and Science Learning*. Mahwah, NJ: Lawrence Erlbaum Associates; 1998. pp. 17-52
- [51] Thomas G, Wineburg S, Grossman P, Myhre O, Woolworth S. In the company of colleagues: An interim report on the development of a community of teacher learners. *Teaching and Teacher Education*; 1998;**14**:21-32
- [52] Rogoff B. Mint: Developing understanding of the idea of communities of learners. *Mind, Culture, and Activity*. 1994;**1**:209-229
- [53] The circulatory system [Internet] Available in: [http://www.daviddarling.info/encyclopedia/C/circulatory\\_system.html](http://www.daviddarling.info/encyclopedia/C/circulatory_system.html) [Accessed: 2017-10-03]
- [54] Turmudi, Prabawanto S, Nurlaelah E. Pengembangan pembelajaran matematika sekolah dasar berbasis fenomena didaktis. Research report. SPs-UPI. un-published; 2015
- [55] Turmudi. Mint: Professional development for junior secondary school mathematics teacher based on the realistic mathematics framework in Indonesia. *Far East Journal of Mathematical Education*. 2011;**7**(1):11-56. DOI: ISSN: 0973-5631
- [56] Syahrin MA, Turmudi, Puspita E. Study ethnomathematics of aboge (alif, rebo, wage) calendar as determinant of the great days of Islam and traditional ceremony in Cirebon Kasepuhan Palace. In *Conference Proceeding of AIP 2015*. 1708, 060009 (2016). DOI: 10.1063/1.4941172
- [57] EKG (Flatline) - Facebook covers [Internet]. 2017, Available from: <http://covermyfb.com/covers/25434/ekg+%28flatline%29> [Accessed: 2017-09-21]

