Chapter

Bayesian Analysis of Additive Factor Volatility Models with Heavy-Tailed Distributions with Specific Reference to S&P 500 and SSEC Indices

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Abstract

The distribution of the financial return series is unsuitable for normal distribution. The distribution of financial series is heavier than the normal distribution. In addition, parameter estimates obtained in the presence of outliers are unreliable. Therefore, models that allow heavy-tailed distribution should be preferred for modelling high kurtosis. Accordingly, univariate and multivariate stochastic volatility models, which allow heavy-tailed distribution, have been proposed to model time-varying volatility. One of the multivariate stochastic volatility (MSVOL) model structures is factor-MSVOL model. The aim of this study is to investigate the convenience of Bayesian estimation of additive factor-MSVOL (AFactor-MSVOL) models with normal, heavy-tailed Student-t and Slash distributions via financial return series. In this study, AFactor-MSVOL models that allow normal, Student-t, and Slash heavy-tailed distributions were estimated in the analysis of return series of S&P 500 and SSEC indices. The normal, Student-t, and Slash distributions were assigned to the error distributions as the prior distributions and full conditional distributions were obtained by using Gibbs sampling. Model comparisons were made by using DIC. Student-t and Slash distributions were shown as alternatives of normal AFactor-MSVOL model.

Keywords: Bayesian analysis, heavy-tailed, financial markets, stochastic volatility models, MCMC

1. Introduction

In recent years, multivariate time series analysis has become an in important research field due to the positive improvements in both methodological and analytical computations. Based on these developments, it has been possible to assess the estimations of parameters in the models of multidimensional and complex time series. Parallelly with these developments, it has been a necessity to model datasets that have simultaneous and frequently changing together. Besides the increase of dataset

dimensions, multidimensional volatility models have gained importance in respect of both economic and econometric parameter estimations due to the temporal fluctuations and changes. The information provided by the correlation structures of multidimensional volatility models has contributed a lot especially in optimal portfolio management, risk management, asset allocation and financial decisions. Moreover; as the volatility between different assets and markets can move together, multivariate analysis contributes statistical efficiency [1].

GARCH and stochastic volatility (SVOL) models, which are widely used in the estimation of volatility, are developed, analysed, and applied within the frame of multivariate the analysis. While multivariate GARCH (MGARCH) models are widely used, MSVOL models are often used in recent years. In his study [2], juxtaposed the most important studies on analysis and development of these models by comparing univariate and multivariate GARCH and SVOL models.

MSVOL models vary in different structures. These structures can be sorted as alternative specifications such as asymmetric models, factor models, time-varying correlation models and matrix exponential transformation, Cholesky decomposition, Wishart autoregressive models [1]. The reason for the limited use of MSVOL is the problems faced in the method of estimation in these models. The most important one among these problems is the problem of high dimension in multivariate analysis and this problem has been eased by using latent factor structures.

Factor-MSVOL models are divided into two groups according to how the factors involved in the mean equation. The first of these structures is additive Factor-MSVOL (AFactor-MSVOL) in which the factors are added summatively and the second one is multiplicative Factor-MSVOL in which the factors are added multiplicatively [3].

AFactor-MSVOL models are firstly offered by Harvey et al. [4]. Afterward, it was developed by [5–9]. The basic idea is taken from factor multivariate ARCH models; additionally, it is a more general state of factor decomposition of covariance structures in multivariate analysis. Returns are divided into two additive components. The first component involves a limited number of factors. The factors capture the information related to the pricing of the whole assets. The other component is the term of an error on the model and it captures the specific information of the asset [1].

Factor-MSVOL models derive from the field of financial econometrics. These models are often preferred to define the terms uncertainty and risk correctly. Asset allocation and asset pricing can be given as an example here. Additionally, it is also used in the arbitrage pricing theory and financial asset pricing model [10]. In comparison with other multivariate stochastic volatility models, Factor-MSVOL models can be estimated with lesser parameters. In this respect, they are parsimonious models in terms of parameters [11]. Factor models both reduce the number of parameters and allow the changing variance structure, it considerably explains the correlation.

Factor-MSVOL models aim to combine a plain, flexible, and robust structure. Like classical factor models, these models are easier in respect of degrading high-dimensioned observation area into low-dimensioned orthogonal latent factor area [10]. Moreover; in the long term data, it is assessed with lesser deviation thanks to its being robust in case of unusual observations.

This study aims to model parameter estimations concerning AFactor-MSVOL models with normal distribution, Student-t distribution, Slash distribution assigned on the error within based on the Bayesian approach. For this purpose; S&P500 (Standard & Poor's 500) and SSEC (Shanghai Compound Index) index daily return series, involving the period between 10.20.2014 and 10.17.2019, were used. Among

the models, the error was scaled out by normal, Gamma, and Beta distributions; the first one is AFactor-MSVOL-NOR model with normal distribution, the second one is AFactor-MSVOL-St model with Student-t distribution, and the last one is AFactor-MSVOL-Sl robust model with Slash distribution. Estimated AFactor-MSVOL models are bivariate and one-factor structure. Usage of Student-t and Slash distributions, while handling skewness and kurtosis features of returns, enabled a flexible approach as an alternative of normal distribution.

2. Model

Latent factor models prove the notion that high-dimensioned systems are just led by some random resources. Some factors are controlled by these random resources and these factors explain the interaction among the observations. Moreover; latent factor models are an efficient way of estimation of a dynamic covariance matrix. These models enable a decrease in the number of unknown parameters [12].

This model has several attractive features, including parsimony of the parameter space and the ability to capture the common features in asset returns and volatilities. Basic idea of Factor-MSVOL models was taken from multivariate ARCH models. In these models, returns are divided into two additive components. The first component has few factors that capture information about the pricing of all assets, while the other component is the error term that captures asset-specific information.

2.1 Multiplicative Factor-MSVOL model

Stochastic discount Factor-MSVOL, which is also called as multiplicative Factor-MSVOL model, was offered by [13]. He offered Bayesian analysis of structured dynamic factor models. Returns are divided into two multiplicative components in one-factor multiplicative model. As shown below, the first of these components is scalar common factor and the other one is idiosyncratic error vector:

$$y_t = \exp(h_t/2)\varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} N(0, \Sigma_\varepsilon) \tag{1}$$

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t \qquad \eta_t \overset{iid}{\sim} N(0, 1) \tag{2}$$

The first one Σ_ε is accepted as 1 for identification. Compared to the MSVOL model, this model involves lesser parameters and it eases calculation. Different from AFactor-MSVOL model, correlation does not change according to time. Additionally, correlation in log-volatility is always equal to 1. The cross dependence among the returns derives from the dependency in ε_t.

In [14] developed the one-factor model as k-factor. In their studies, [14] researched both the persistence amount of daily stock returns and the factors affecting common persistence components in volatility. In this study, the one-factor multiplicative MSVOL model is expanded as k-factor.

2.2 Additive Factor-MSVOL model

The Factor-MSVOL model is one of the MSVOL approaches allowing the change of implicitly conditioned correlation matrix in time and producing time-varying correlation. Factor models and factors follow a stochastic volatility process. A kind

of Factor-SVOL model that does not allow time-varying correlations was offered by [13]. On the other hand, Harvey et al. [4] introduced a common factor in the linearized state-space version of the basic MSVOL model. In this context, the most basic MSVOL model specification is by:

$$y_{it} = \varepsilon_{it}(\exp\{h_{it}\})^{1/2}\ i = 1, ..., N\ t = 1, ..., T \tag{3}$$

y_{it} refers to the observation values in t period of i serial. $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{Nt})'$ is the error vector which shows normal distribution with Σ_ε covariance matrix and 0 mean. Diagonal elements of Σ_ε covariance matrix are unity and off-diagonal elements are defined as ρ_{ij}. Variance of this model is produced by AR(1) process:

$$h_{it} = \gamma_i + \varphi h_{it-1} + \eta_{it}\, i = 1, ..., N \tag{4}$$

Here, $\eta_t = (\eta_{1t}, ..., \eta_{Nt})'$ with 0 mean and multivariate of $\sum_\eta$ matrix is normal. This model, Eq. (4), $N \times 1\ h_t$, can be generalised as multivariate AR(1) and even ARMA process. If we handle the multivariate random walk model of $\boldsymbol{h_t}$, which is its special case:

$$w_t = -1.27i + h_t + \xi_t \tag{5}$$

$$h_t = h_{t-1} + \eta_t \tag{6}$$

w_t and ξ_t elements are $N \times 1$ vectors in case $w_{it} = \log y_{it}^2$ and $\xi_{\mathbf{t}} = \log \varepsilon_{it}^2 + 1.27\ i = 1, ..., N$. i is $N \times 1$ vector which is composed of unit values.

Common factors can be included in multivariate stochastic variance models; they are unobservable components of time series models. In [4] modelled with a multivariate random walk by considering the persistence in volatility. According to this, Eq. (4) is by:

$$w_t = -1.27i + \theta h_t + \overline{h} + \xi_t, \tag{7}$$

$$\begin{aligned} h_t &= h_{t-1} + \eta_{\boldsymbol{t}} \\ Var(\eta_t) &= \sum\nolimits_\eta \end{aligned} \tag{8}$$

As θ k ≤ N, $N \times k$ parameter matrix, h_t and $\eta_{\mathbf{t}}$ $k \times 1$ vectors, Σ_η $k \times k$ positively defined matrix, $\overline{h}$ is an $N \times 1$ vector in which the first k elements are zeros and the last $N - k$ elements are unbounded, Harvey et al. [4] estimated this model with QML method. Common factors are transformed as $\theta^* = \theta R'$ and $h_t^* = Rh_{\mathbf{t}}$ to evaluate the factor loading [4].

Following the model offered by [15], another kind of MSVOL factor model was handled by [8] as below:

$$y_t = Bf_t + V_t^{1/2}\varepsilon_t \varepsilon_t \sim \mathrm{N_p}(0, \mathrm{I}) \tag{9}$$

$$f_t = D_t^{1/2}\gamma_t \gamma_t \sim N_q(0, I) \tag{10}$$

$$h_{t+1} = \mu + \Phi(h_t - \mu) + \eta_t \eta_t \sim N_{p+q}\left(0, \sum\nolimits_{\eta\eta}\right) \tag{11}$$

$$\mathbf{V_t} = diag\left(\exp(h_{1t}), ..., \exp(h_{pt})\right) \tag{12}$$

$$\mathbf{D_t} = diag\left(\exp(h_{p+1,t}), ..., \exp(h_{p+q,t})\right) \tag{13}$$

$$\Phi = diag\left(\varphi_1, ..., \varphi_{p+q}\right) \tag{14}$$

$$\Sigma_{\eta\eta} = diag\left(\sigma_{1,\eta\eta}, ..., \sigma_{p+q,\eta\eta}\right) \tag{15}$$

$$\Phi = diag\left(\varphi_1, ..., \varphi_{p+q}\right) \tag{16}$$

$$\Sigma_{\eta\eta} = diag\left(\sigma_{1,\eta\eta}, ..., \sigma_{p+q,\eta\eta}\right) \tag{17}$$

and $h_t = \left(h_{1t}, ..., h_{pt}, h_{p+1,t,...,h_{p+q,t}}\right)$.

B is a $p \times q$ matrix of factor loadings. For $i < j, i \leq j\, b_{ij} = 0$, for $i \leq qb_{ii} = 1$, and all remaining elements are unconstrained. Therefore, each of the factors and errors in this model develops according to SVOL models. Similar to this model, except the fact that V_t does not change in time under restriction, another model was handled in [6] and [16]. In [6] estimated their models with Markov Chain Monte Carlo (MCMC) method. On the other hand, [16] showed how to assess MLE with the Efficient Importance Sampling method. Presented by [17], a more generalised version of these models allows spikes in observation equations and the errors are distributed by heavy-tailed-t [18].

In [3] showed that additive factor models are by both time-varying volatility and correlations. In this context, they offered two varieties one-Factor SVOL model and they showed that the correlation between two return series is related to the volatility of the factor. According to this, logarithmic returns observed in t period are expressed as $\mathbf{y}_t = \left(y_{1t}, y_{2t}\right)'$. Additionally, when it is showed as $\boldsymbol{\varepsilon_t} = (\varepsilon_{1t}, \varepsilon_{2t})'$, $\boldsymbol{\eta_t} = (\eta_{1t}, \eta_{2t})'$, $\boldsymbol{\mu_t} = (\mu_{1t}, \mu_{2t})'$ and $\boldsymbol{h}_t = (h_{1,t}, h_{2,t})'$, two varieties one-Factor MSVOL models are such as below:

$$\mathbf{y}_t = Df_t + \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t \overset{iid}{\sim} \mathrm{N}\left(\mathbf{0}, \mathrm{diag}\left(\sigma_{\varepsilon 1}^2, \sigma_{\varepsilon 2}^2\right)\right) \tag{18}$$

$$h_{t+1} = \mu + \varphi(h_t\text{-}\mu) + \sigma_\eta\eta_t, \eta_t \overset{iid}{\sim} \mathrm{N}(0,1) \tag{19}$$

and $h_0 = 0$. This model is offered by [5, 6]. The first component that takes place in return equation involves a small number of factors which includes the information related to the pricing of the whole assets. The second term is error term peculiar to equation; it involves specific information of the asset. A Factor-MSVOL model allows high kurtosis and volatility cluster. It also enables cross dependency in both returns and volatility. h_t represents the log-volatility of the common factor (f_t) which takes place in A Factor-MSVOL model. The conditional correlation between y_{1t} and y_{2t} is as below:

$$\frac{d\, exp\,(h_t)}{\sqrt{\left(exp\,(h_t) + \sigma_{\varepsilon 1}^2\right)\left(d^2\, exp\,(h_t) + \sigma_{\varepsilon 2}^2\right)}} \tag{20}$$

$$= \frac{d}{\sqrt{1 + \sigma_{\varepsilon 1}^2\, exp\,(-h_t)\left(d^2 + \sigma_{\varepsilon 2}^2\, exp\,(-h_t)\right)}} \tag{21}$$

$\sigma_{\varepsilon 1}^2 = \sigma_{\varepsilon 2}^2 = 0$ is not, so correlation coefficient changes in time. Correlation dynamics is dependent on the dynamics of h_t; likewise, the correlation is an increasing function of h_t. It refers that the correlation will be high as much as the common factor volatility is high.

Offered by [3], specification of two varieties one-factor AFactor-MSVOL model, which allows heavy-tailed distribution, is as below:

$$\mathbf{y}_t = Df_t + \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t \overset{iid}{\sim} \mathrm{t}\left(\mathbf{0}, \mathrm{diag}\left(\sigma_{\varepsilon 1}^2, \sigma_{\varepsilon 2}^2\right), \mathbf{v}\right) \tag{22}$$

$$f_t = exp\,(h_t/2)u_t, u_t \overset{iid}{\sim} \mathrm{t}(0, 1, \omega), \tag{23}$$

$$h_{t+1} = \mu + \varphi(h_t\text{-}\mu) + \sigma_\eta \eta_t, \eta_t \overset{iid}{\sim} \mathrm{N}(0, 1) \tag{24}$$

$\mathrm{h}_0 = \mu$, $\mathbf{v} = (\mathrm{v}_1, \mathrm{v}_2)'$. In this model, heavy-tailed Studentt distribution is used for return shocks. The conditional correlation between y_{1t} and y_{2t} is as below:

$$\frac{d}{\sqrt{1 + \frac{\omega}{\upsilon_1}\sigma_{\varepsilon 1}^2 exp\,(-h_t)\left(d^2 + \frac{\omega}{\upsilon_2}\sigma_{\varepsilon 2}^2 exp\,(-h_t)\right)}} \tag{25}$$

In addition to these models, AFactor-MSVOL-Sl model in which error distribution is scaled with Slash distribution is defined as:

The error for the AFactor-MSVOL-Sl model is shown as Slash distribution $(\varepsilon_t|\upsilon_t) \sim slash\left(0, \sum_\varepsilon, \upsilon\right)$ with $\sigma_{\varepsilon_t}^2 \sim beta(\alpha, \beta)$ prior distribution.

$$\mathbf{y}_t = Df_t + \boldsymbol{\varepsilon}_t, \boldsymbol{\varepsilon}_t \sim \mathrm{slash}\left(\mathbf{0}, \mathrm{diag}\left(\sigma_{\varepsilon 1}^2, \sigma_{\varepsilon 2}^2\right), \mathbf{v}\right) \tag{26}$$

$$f_t = exp\,(h_t/2)u_t, u_t \sim \mathrm{slash}(0, 1, \omega) \tag{27}$$

$$h_{t+1} = \mu + \varphi(h_t\text{-}\mu) + \sigma_\eta \eta_t, \eta_t \overset{iid}{\sim} \mathrm{N}(0, 1) \tag{28}$$

Philipov and Glickman [19] offered high-dimensioned additive factor-MSVOL models in their studies. In this study, factor covariance matrix is led by Wishart random process. On the other hand, it is known that daily return series are leptokurtic. In context of stochastic volatility, [20] and [21] presented empirical proofs on the usage of heavy-tailed distribution in conditioned mean equation. Moreover, [22] analysed SVOL models with Student-t distribution and GED. Daily data analysis of JPY/Dollar and TOPIX were carried out by the method of MCMC. Comparison of distributions, in respect of accordance, was calculated with Bayesian factor values. It is determined that SVOL-t model assorts with both of the data compared to SVOL-normal and SVOL-GED models.

In [23] analysed new-class linear factor models. In these models, factors are latent and covariance matrix is followed with MSVOL process. Wu et al. [24] proposed dynamic correlated latent factor SVOL model structure in his studies. According to the results of analysis led by MCMC method, statistically comprehensible results were obtained for financial and economic data.

3. Empirical analysis

3.1 Dataset

This study aims to model parameter estimations concerning AFactor-MSVOL models with Student-t, Slash and normal distributions assigned to the error. For this purpose; S&P500 and SSEC index daily return series, involving the period between 10.20.2014 and 10.17.2019, were used. Among the models the error was scaled by normal, Gamma, and Beta distributions; the first one is AFactor-MSV-NOR model with normal distribution, the second one is AFactor-MSVOL-St model with Student-t distribution, and the last one is AFactor-MSVOL-Sl robust model with Slash distribution. Analyses of data were carried out with R and WinBugs programmes. Daily mean logarithmic return series were determined by:

$$Y_t = 100 \times (\log P_t - \log P_{t-1}) \tag{29}$$

$$y_t = Y_t - \frac{1}{T}\sum_{t=1}^{T} Y_t \tag{30}$$

S&P500 index is composed of stocks of the most valuable 500 companies in USA. On the other hand, SSEC has the most important and the biggest companies of China. Commercial and financial relations between the USA and China not only affect themselves but also global economy. Commercial and financial tensions between them and the anxieties on currency wars can negatively affect Asia and Europe stock markets. Therefore; index values of two grand economies such as China and USA are preferred for analyses. In **Figure 1**, time series plots for S&P500 and SSEC return series are given.

Descriptive statistic values of S&P500 and SSEC series are given in **Table 1**. S&P500 and SSEC series have negative mean returns. It seems that SSEC return series have more volatility. Moreover, both of the series are negatively skew. Kurtosis level is higher for both S&P500 and SSEC. Jarque-Bera normality test results show that series do not have a normal distribution.

In **Table 2**, Ljung-Box and ARCH-LM test results are illustrated in some lags. As Q statistics of Ljung-Box test are examined, null hypothesis that there is not autocorrelation is rejected for both of the series in 20th and 50th lags. It refers that autocorrelation exists in series. According to the ARCH test results, ARCH effect is seen in the whole series. It shows the necessity of preferring the models allowing heteroscedastic structures in the analyses of volatility in return series.

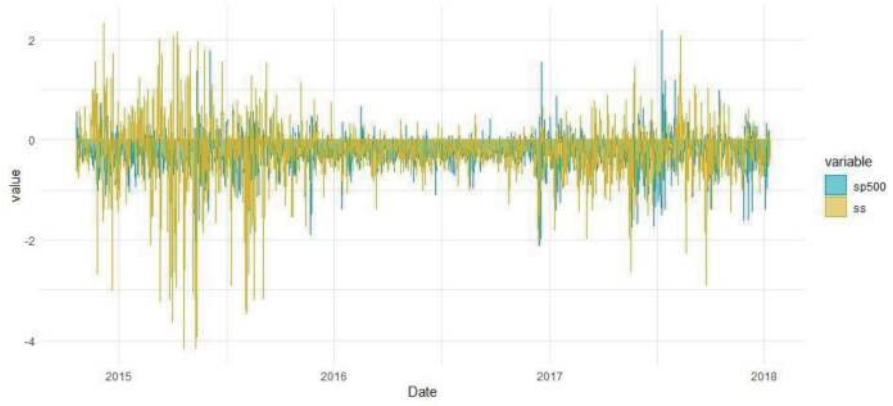

Figure 1.
Time series plots for S&P500 and SSEC returns.

	S&P500	SSEC
Sample size	1177	1177
Mean	−0.2784	−0.2862
Maximum	2.1777	2.3282
Minimum	−2.112	−4.1485
Standard deviation	0.39	0.6802
Skewness	−0.1463	−0.9852
Kurtosis	4.7226	6.0157
Jarque-Bera (possibility)	1098.0 (3.7495e-239)	1965.2 (0.0000)

Table 1.
Descriptive statistics of S&P500 and SSEC return series.

	S&P500	SSEC
Q(5)	7.0117 [0.2197]	12.8221 [0.0251]
Q(10)	16.1735 [0.0947]	20.6526 [0.0236]
Q(20)	29.5883 [0.0768]	59.9778 [0.0000]
Q(50)	71.0158 [0.0269]	115.200 [0.0000]
Q^2(5)	149.806 [0.0000]	262.809 [0.0000]
Q^2(10)	198.313 [0.0000]	323.350 [0.0000]
Q^2(20)	223.106 [0.0000]	575.391 [0.0000]
Q^2(50)	290.457 [0.0000]	937.313 [0.0000]
ARCH-LM(2)	37.195 [0.0000]	52.090 [0.0000]
ARCH-LM(5)	19.487 [0.0000]	31.799 [0.0000]
ARCH-LM(10)	11.359 [0.0000]	16.405 [0.0000]

Table 2.
Ljung-box and ARCH-LM test results.

3.2 Bayesian estimation

The most important factor, which limits the usage of Factor-MSVOL models, is difficulty in estimating the statistics, whereupon some methods were offered for estimation. In these methods, quasi maximum likelihood, simulated maximum likelihood, and Bayesian MCMC are offered as the most efficient methods. Bayesian MCMC method is very efficient against high dimension problems of the dataset [8, 9, 17].

In this study, parameter estimations are obtained by the Bayesian approach. As it is known, in parameter estimation it is supposed that the error term shows the normal distribution, but this assumption is not valid in case unusual points exist, therefore error term has a heterogeneous variance. This case is often faced in longitudinal datasets. In case unusual points exist in datasets, researchers generally prefer some strategies such as keeping the outliers, removing outliers, and recoding outliers. If keeping the outliers is chosen, the heavy-tailed distribution must be preferred rather than normal distribution. Otherwise, it causes statistical inferences.

In recent years, multidimensional analytical operations in computational science have become easier thanks to the advances in computer technology. In parallel with these advances and usage of the Bayesian approach, using more robust models in analyses has increased in the observation of unusual points. In the Bayesian approach, model parameters are random variables and it is supposed that it shows a known distribution. The Bayesian approach relies on the combination of subjective experiences of the researcher, the prior information obtained from the former studies, and the likelihood obtained from data. Posterior information is achieved from the combination with prior information. This information is defined with a known distribution function and parameter estimations are achieved from the posterior distribution.

$$\text{Posterior} \propto \text{Prior X Likelihood}$$

In the Bayesian approach, in obtained of the posterior distribution of parameters requires multidimensional integral computations in multidimensional and longitudinal datasets. This difficulty is overcome by the development of iterative methods such as MCMC. MCMC methods are based on the randomly generate parameter values from posterior distribution; thus, some analytically difficult problems are easily solved by simulation techniques. In this study, parameter estimations are obtained by Gibbs sampling which is also a MCMC method. Gibbs sampling is a

method used in case posterior distribution has a closed-form and it is a kind of iterative method reproducing random values from these values. The full conditional density function is obtained by Gibbs sampling as all the unknown parameters are given and parameters are estimated with this method.

In this study, parameter estimations are obtained by modelling three different prior distributions assigned on the error term. In modelling, error term is scaled with λ variable and normal/independent (or scaled mixture) defined distributions are used. As y variable, which shows normal/independent distribution, is expressed in longitudinal model given below [25];

$$y = \mu + \frac{e}{\sqrt{\lambda}} \tag{31}$$

Here μ is a mean vector, e is error vector and have normal distribution. λ variable that takes place in the model shows different distributions according to the degrees of freedom of υ, and it is defined as random variable with positive valence. As degrees of freedom goes infinite, λ variable is 1 and the error term shows normal distribution. As λ variate shows Gamma $\left(\frac{\upsilon}{2}, \frac{\upsilon}{2}\right)$ distribution, it converges Student-t; and as λ variate shows Beta$(\upsilon, 1)$ distribution in [0,1] closed interval, it converges Slash distribution.

3.3 Findings

As an addition to the AFactor-MSVOL offered by [3] and heavy-tailed AFactor-MSVOL models, bivariate one-factor AFactor-MSVOL model in which the error term is scaled with Slash distribution is estimated in the analysis.

In **Table 3**, posterior mean values of the parameters, standard errors and 95% credible intervals are shown. Using different initial values for each model, two chains are formed. Total iteration number in each chain is determined as 500,000

		AFactor-MSVOL-NOR	AFactor-MSVOL-St	AFactor-MSVOL-Sl
μ	Mean	−1.59	−1.586	−3.930
	Sd	0.2899	0.292	0.614
	%95 CI	[−2.208, −1.054]	[−2.231, −1.072]	[−5.531, −3.102]
Ø	Mean	0.9910	0.991	0.830
	Sd	0,005198	0.006	0.055
	%95 CI	[0.9788,0.9988]	[0.978, 0.999]	[0.708, 0.926]
σ_η^2	Mean	95.68	87.230	0.653
	Sd	36.89	36.339	0.076
	%95 CI	[42.81, 183.0]	[39.950,177.002]	[0.496, 0.793]
d	Mean	0.178	0.158	0.174
	Sd	0.02329	0.018	0.052
	%95 CI	[0.1335,0.2248]	[0.123,0.194]	[0.074, 0.278]
$\sigma^2_{\varepsilon 1}$	Mean	0.003672	0.001	0.345
	Sd	0.006022	0.007	0.042
	%95 CI	[3.984E-8, 0.0216]	[0.000,0.023]	[0.260, 0.426]
$\sigma^2_{\varepsilon 2}$	Mean	0.1781	0.151	0.309
	Sd	0.01054	0.011	0.037
	%95 CI	[0.1583, 0.1996]	[0.131, 0.174]	[0.242, 0.387]

		AFactor-MSVOL-NOR	AFactor-MSVOL-St	AFactor-MSVOL-Sl
v_1	Mean		8.735	8.317
	Sd		5.744	4.665
	%95 CI		[2.396, 24.190]	[3.869, 21.360]
v_2	Mean		4.372	3.652
	Sd		0.599	1.702
	%95 CI		[3.433, 5.780]	[2.418, 8.375]
ω	Mean		7.237	5.299
	Sd		1.670	4.727
	%95 CI		[5.128,11.600]	[2.027, 19.570]

Note: Parameter estimations for AFactor-MSVOL-NOR, AFactor-MSVOL-St and AFactor-MSVOL-Sl models are Rhat = 1.

Table 3.
Posterior mean values of the parameters in the AFactor-MSVOL models.

and the iteration number that must be omitted in the burn-in is 250,000. Thus, when the first burn-in period of 250,000 is omitted, a Gibbs chain of 250,000 is obtained for each parameter by means of saving each iteration value.

It is seen that for AFactor-MSVOL and AFactor-MSVOL-St models Ø parameter of posterior mean value is so close to the unit value. It refers that latent volatility had random walk behaviour. On the other hand, factor process for all the models was highly obtained. It is seen that standard deviation of posterior mean value of Ø parameter is too low. According to this, logarithmic volatility of time-varying latent components shows persistent features. Posterior mean value of Ø parameter is lower in AFactor-MSVOL-Sl model in comparison to the other models, while the posterior means of ϕ are all nearby unity and seem to propose random walk behaviour for h_t. The mean of ϕ is close to unity with a low standard deviation under all specifications, offering persistent time-varying log-volatility for latent components. Factor loading for the estimated models are determined as 0.178, 0.158, and 0.17, respectively. The overall variance-covariance is decomposed into a component which is due to the variation in the common factor and a component reflecting the variation in the idiosyncratic errors. Diebold and Nerlov [26] suggest the common factor reflects the flow of new information relevant to the pricing of all assets, upon which asset-specific shocks represented by the idiosyncratic errors are superimposed (**Figures 2**, **3** and **4**).

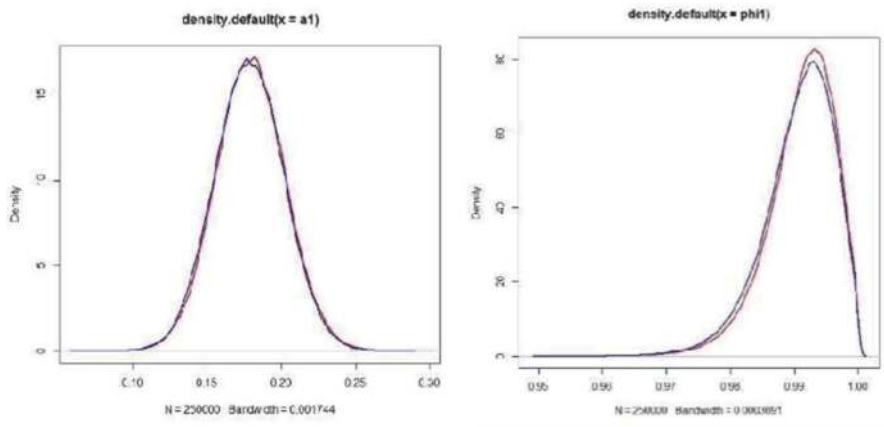

Figure 2.
Kernel density estimation of AFactor-MSVOL-NOR model μ and Ø parameters.

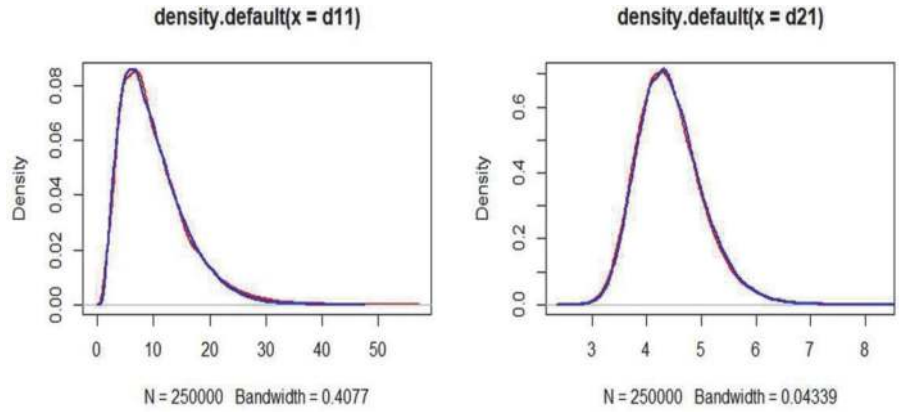

Figure 3.
Kernel density of AFactor-MSVOL-St model v_1 and v_2 parameters.

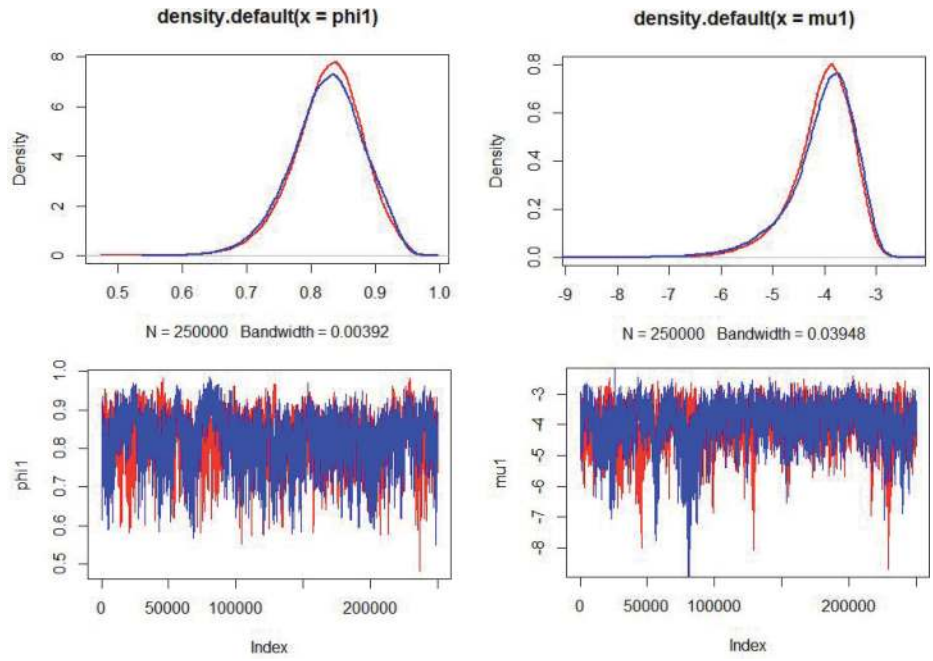

Figure 4.
Kernel density of AFactor-MSVOL-SL model μ and Ø parameters.

Gelman-Rubin statistics is an approach to determining convergence. According to it, convergence takes place in case means of variance within the chain and the variance values between the chains are equal. In this case, Gelman-Rubin statistics is about 1. In **Table 4**, Gelman-Rubin statistics of estimated models for parameter estimation take place. According to this, it is seen that all the parameters take 1 value of Gelman-Rubin statistics and convergence occurs.

	AFactor-MSVOL-NOR	AFactor-MSVOL-St	AFactor-MSVOL-Sl
μ	1.00	1.00	1.01
Ø	1.00	1.00	1.00
σ_η^2	1.02	1.00	1.00
d	1.00	1.00	1.00
$\sigma^2_{\varepsilon 1}$	1.01	1.04	1.00

	AFactor-MSVOL-NOR	AFactor-MSVOL-St	AFactor-MSVOL-Sl
$\sigma^2_{\varepsilon 2}$	1.00	1.00	1.00
v_1		1.00	1.00
v_2		1.00	1.00
ω		1.00	1.00

Table 4.
Gelman-Rubin diagnostic test.

DIC allows comparison between the models by taking into consideration the complexity of the model [27, 28]. p_d is expressed as efficient parameter number. p_d model gives the approximate value of parameter number and measures the complexity of the model. DIC can take both negative and positive values. It causes negative valorisation of both deviation and DIC. In conclusion, the model with the lowest DIC value must be chosen from alternative models [29]. In **Table 5**, DIC values of each three values are given; according to this, the model with the lowest DIC values should be chosen.

	DIC	pd
AFactor-MSVOL-NOR	3705.3	316.1
AFactor-MSVOL-St	3657.7	396.0
AFactor-MSVOL-Sl	3609.3	183.4

Table 5.
DIC values.

4. Conclusion

In financial applications, modelling the correlation structures of the returns is important because empirical analyses show that there is time-varying relation among return-on-assets. In this context, factor-MSVOL models have been preferred. Thanks to these models, volatility dynamics of financial and economic time series can be modelled with few latent factors.

In this study, parameter estimations concerning additive factor-MSVOL models were modelled with normal distribution assigned on the error, Student-t distribution, and Slash distribution in the frame of Bayesian analysis. Normal, Student-t and Slash distributions were assigned as prior distribution to the error distributions and full conditioned posterior distributions were obtained by a kind of MCMC method-Gibbs sampling. Among the criteria of model choosing, DIC is used for comparison and it showed that Student-t and Slash distributions can be used as alternative of normal AFactor-MSVOL models. Provided that the analysis results are evaluated in respect of DIC criteria and model complexity, it is seen that AFactor-MSVOL-Sl model in which the errors are scaled with Slash distribution is better than the other models. In case the error terms are modelled with Slash distribution, analysis of financial return series, which involves deviated and extreme observations, will provide more correct results. Both Student-t and Slash distributions are robust distributions. Both of the distributions better adapted to the data compared to normal distribution. Student-t distribution allows kurtosis in a larger interval for high degrees of freedom but it is possible to say that Slash distribution is more robust as it gives better parameter estimations in case there are more unusual

points. Therefore; it is seen that Student-t and Slash distributions are applicable as an alternative of normal distribution in the analysis of financial return series. Moreover, it is possible to say that heavy-tailed distributions can substitute normal distribution in case deviated observation values are not present.

Acknowledgements

This study was presented at the 20th International Symposium on Econometrics, Operations Research and Statistics, 12–14 February 2016, Ankara, Turkey. (Oral presentation).

Additional classification

JEL codes: C11, C46, G19, C59, C15.

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References

[1] Asai M, McAller M, Yu J. Multivariate stochastic volatility: A review. Econometric Reviews. 2006;**25** (2–3):145-175

[2] McAleer M. Automated inference and learning in modeling financial volatility. Econometric Theory. 2005;**21**: 232-261

[3] Yu J, Meyer R. Multivariate stochastic volatility models: Bayesian estimation and model comparison. Econometric Reviews. 2006;**25**(2/3): 361-384. Research Collection School of Economics

[4] Harvey A, Ruiz E, Shephard N. Multivariate stochastic variance models. The Review of Economic Studies. 1994; **61**(2):247-264

[5] Jacquier E, Polson NG, Rossi PE. Models and Priors for Multivariate Stochastic Volatility. Montreal: CIRANO Working Papers; 1995. pp. 95s-918s

[6] Jacquier E, Polson NG, Rossi PE. Stochastic Volatility: Univariate and Multivariate Extensions. Montreal: CIRANO Working Paper; 1999. pp. 99s-926s

[7] Shephard N. Statistical aspects of ARCH and stochastic volatility. In: Cox DR, Hinkley DV, Barndorff-Neilsen OE, editors. Time Series Models in Econometrics, Finance and Other Fields. London: Chapman&Hall; 1996. pp. 1-67

[8] Pitt MK, Shephard N. Time-varying covariances: A factor stochastic volatility approach. In: Bayesian Statistics. Oxford: Oxford University Press; 1999. pp. 547-570

[9] Aguilar O, West M. Bayesian dynamic factor models and portfolio allocation. Journal of Business & Economic Statistics. 2000;**18**(3):338-357

[10] Kastner G, Frühwirth-Schnatter S, Lopes HF. Efficient Bayesian inference for multivariate factor stochastic volatility models. Journal of Computational and Graphical Statistics. 2017;**26**(4):905-9174

[11] Hepsağ A. Çok Değişkenli Stokastik Oynaklık Modelleri: Petrol Piyasası ile Finansal Piyasalarda İşlem Gören Sanayi Sektörü Endeksi Arasındaki Oynaklık Etkileşimi Üzerine Bir Uygulama. İstanbul: İstanbul University; 2013

[12] Hosszejni D, Kastner G: Modeling Univariate and Multivariate Stochastic Volatility in R with Stochvol and Factor Stochvol. R package vignette. Cornell University Eprint; 2019;**1906**:12123. Available from: https://CRAN.R-project.org/package= factorstochvol/vignettes/paper.pdf

[13] Quintana JM, West M. An analysis of international exchange rates using multivariate DLMs. Stat. 1987;**36**: 275-281

[14] Ray BK, Tsay RS. Long-range dependence in daily stock volatilities. Journal of Business and Economic Statistics. 2000;**18**:254-262

[15] Kim S, Shephard N, Chib S. Stochastic volatility: Likelihood inference and comparison with ARCH models. The Review of Economic Studies. 1998;**65**(3):361-393

[16] Liesenfeld R, Richard J-F. Univaria te and multivariate stochastic volatility models: Estimation and diagnostics. ournal of Empirical Finance. 2003;**4**: 505-531

[17] Chib S, Nardari F, Shephard N. Analysis of high dimensional multivariate stochastic volatility models. Journal of Econometrics. 2006;**134**:341-371

[18] Chib S, Nardari F, Shepard N. Markov chain Monte Carlo methods for

stochastic volatility models. Journal of Econometrics. 2002;**108**:281-316

[19] Philipov A, Glickman M. Factor multivariate stochastic volatility via Wishart processes. Econometric Reviews. 2006;**25**:311-334

[20] Gallant AR, Hsieh D, Tauehen G. Estimation of stochastic volatility models with diagnostics. Journal of Econometrics. 1997;**81**:159-192

[21] Geweke J. Bayesian analysis of stochastic volatility models: Comment. Journal of Business & Economic Statistics. 1994;**12**(4):397-399

[22] Watanabe T, Asai M: Stochastic Volatility Models with Heavy-Tailed Distributions: A Bayesian Analysis. Japan: Technical report, Discussion paper 2001-E-17, Institute for Monetary and Economic Studies; 2001

[23] Nardari F, Scruggs JT. Bayesian analysis of linear factor models with latent factors, multivariate stochastic volatility, and APT pricing restrictions. The Journal of Financial and Quantitative Analysis. 2007;**42**(4): 857-891

[24] Wu S-J, Ghosh SK, Ku Y-C, Bloomfield P. Dynamic correlation multivariate stochastic volatility with latent factors. Statistica Neerlandica. 2018;**72**(1):48-69

[25] Lange KL, Sinsheimer JS. Normal/ independent distributions and their applications in robust regression. Journal of Computational and Graphical Statistics. 1993;**2**:175-198

[26] Diebold FX, Nerlov M. The dynamics of exchange rate volatility: A multivariate latent factor ARCH model. Journal of Applied Econometrics. 1989; **4**:1-21

[27] Spiegelhalter DJ, Best NG, Carlin BP, Linde A. Bayesian measures of model complexity and fit. The Royal Statistical Society. 2002;**64**(4):583-639

[28] Berg A, Meyer R, Yu J. Deviance information criterion for comparing stochastic volatility models. Journal of Business and Economic Statistics. 2004; **22**(1):107-120

[29] Spiegelhalter DJ. Some DIC Slides. Cambridge: MRC Biostatistics Unit; 2006