

Theory of Tribo-Systems

Xie You-Bai

*Shanghai Jiaotong University and Xi'an Jiaotong University
China*

1. Introduction

What is tribology? Why people need tribology?

Some people say that tribology is friction, wear or lubrication. Others say that it is friction plus wear and plus lubrication. However both of them are not accurate enough. Tribology takes all theoretical and applied results from friction, wear and lubrication obtained in the past, inputs into them with much more new senses and contents based on the development of science and technology. It is striving to constitute a theoretical and technical platform to meet the future requirement. Tribology cannot be looked simply as equal only to friction, wear, lubrication or any other technique related.

The early stage of applying knowledge of friction, wear and lubrication in human productive and living practice can be traced back to 3000 BC or earlier (Dowson, 1979). This multi-disciplinary branch of science and technology and its application in comprehensive areas were studied in many different sub-subjects independently from very different points of view over a long period. A suggestion from H Peter Jost gave this old field a powerful impact and poured into it youthful vigor (Her Majesty's Stationery Office, 1966). It developed quicker and quicker thereafter. Tribology is a both old and young discipline.

In the first phase of development of tribology since 1965, due to its universal existence in nature according to the definition given by Jost on one side and the belief of tribologists in many countries that they could make huge benefit for industry on another side, the influence of tribology increased dramatically fast in the seventies and eighties of the last century. Promise of saving 5 billion pounds per year in UK in the Jost Report pushed forward tribologists working on applying existed knowledge of friction, wear and lubrication to solve engineering problems. New techniques related to friction, wear and lubrication developed then rapidly in the following phase even though some people they did not like the name "tribology". Many books published in this stage with the title "Tribology" but no one discussed on the questions that what was tribology and why they used the word "tribology" except Jost did in his famous Report.

The later situation has shown that to achieve the potential benefit is not so easy (Xie, 1986; Xie & Zhang, 2009). A name, a definition and simply putting all knowledge components together subjectively are not enough. A concept system, theory system and method system, which can match the name, definition and nature of tribology and then can promote an independent development and application of tribology, are expected.

It is valuable to mention that "Tribology" was defined as one of the four major disciplines of Mechanical Systems by a Committee of NSF of US in 1983 (The Panel Steering Committee for the Mechanical Engineering and Applied Mechanics Division of the NSF, 1984) and then

the "Journal of Lubrication Technology" was renamed as "Journal of Tribology" of Transaction of ASME. Only ten years later, a gentleman from US indicated in an informal speech in Beijing that a change under way was the gradual disappearance of the term "tribology" from programs and projects of NSF in US. In this period fewer papers which dealt with the relation between tribology and mechanical systems could be found in the journal. It implies that no enough effort has been made to carry out the original intention of the committee. Many famous tribologists prophesied that tribology would become or be replaced by surface engineering.

It shows some undesired situation in the development of tribology. There are at least three problems with it. Firstly tribology was born on the foundation of known appearances of friction, wear and lubrication but the difference between tribology and friction, wear and lubrication has not been paid attention to investigate into. Naturally the traditional way of studying friction, wear and lubrication independently is still having its visible influence on tribology. Secondly as tribology is so universal and so important to engineering and industry, much attention has been paid to the tribology-based applied techniques and a very fast development of the techniques has been achieved. Due to the nature of tribology, which will be discussed later, most of the techniques can be applied only to a specific branch of field for a specific target. Many people they work in the field of tribology but they don't think they are tribologists. Some of them think they are chemists, material scientists, biologists or mechanical engineers. Therefore the theoretical study of tribology, especially the efforts on finding a systematic framework for tribology cannot benefit further from such a fast development of technique. Thirdly people don't know how to use the results obtained under one condition to another condition and how to compare the results from one kind of test machines with what of another kind of test machines, in other words, there is no general model for tribology and almost no modern mathematic tool can be used in tribology. Engineers have to make each decision individually in design depending on experience or experiment. They cannot construct a tribological design in true sense for their products because no model can be found in simulating the behaviors of tribology other than friction, wear or lubrication individual. Therefore there is no strong enough attraction to take tribology as an independent discipline in industry further.

Tribology has been defined in 1965 as "the science and technology of interacting surfaces in relative motion and of the practices related thereto" (Her Majesty's Stationery Office, 1966) and modified later as "the science of behaviors of interaction surfaces in relative motion together with the active medium concerned (each of them is a tribo-element) in natural systems, their results and the technology related thereto" (Xie, 1996). A question arises then that why people need *the interacting surfaces in relative motion*? Both definitions deal with appearance aspects rather than functional aspects of tribology and cannot answer the question. Obviously any surface cannot exist independently and must be a part of a component. The relative motion of surfaces is defined by the relative motion of components and where the surfaces reside on. The interactions transmitted between surfaces are from the components in contact on the surfaces as well. In most (not all) cases two interacting surfaces in relative motion function as a *joint* which permits only some kinds of relative motion and prevents other kinds of relative motion between two components in contact. Such joints are named kinematic pairs in mechanisms. The interacting surfaces in relative motion must function with other elements in a system or function with other elements for a system. Therefore the problems with tribology are problems of systems science and systems engineering. In a sense, without system there would be no tribology.

In the very early stage of tribology people have begun to think about system problems (Fleischer, 1970; Czichos, 1974; Salomon, 1974). A comprehensive study on applying system concepts to friction, wear and lubrication was given by Czichos which described how to use general systems theory and engineering system analysis in treating tribological problems (Czichos, 1978). Without an effective way for mathematic computation limited its application. Dai and Xue (Dai & Xue, 2003) tried to evaluate tribological behaviors with an entropy calculation in tribo-systems while Ge and Zhu (Ge & Zhu, 2005) worked out through a fractal analysis for a similar attempt. Either entropy calculation or fractal analysis cannot describe explicitly and quantitatively the character of movement of interacting surfaces in relative motion. Since they deal only with entropy or fractal parameters, transforming all other physical and geometric behaviors into an entropy or fractal change in calculation is unavoidable. It involves the transformation of a large amount of knowledge concerning with tribology getting together in the past into the thermodynamic or fractal knowledge and is almost impossible in practice.

Considering that relative motion is the first important character of tribology and is also a basic behavior studied in mechanisms, some concepts in mechanisms should be discussed before going to construct a *function based* systems theory for tribology.

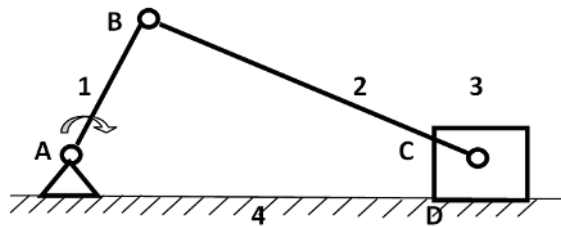


Fig. 1. A crank-slider mechanism

When several components are joined together by kinematic pairs it constructs a kinematic chain. The necessary condition that a kinematic chain becomes a mechanism, in other words a mechanical system is all components in the chain having definitive relative motions. This condition can be rewritten as that the number of motion conditions given (input) outside of the system equals to the number of residual degrees of freedom of the chain. For example, as shown in Figure 1 there is a plane kinematic chain of four components (1 - 4) with one fixed component (4, chassis), three revolute pairs (A - C) and one prismatic pair (D). Each movable component has three degrees of freedom while each revolute pair or each prismatic pair cancels two degrees of freedom. Revolute pairs and prismatic pairs all formed with surface contact are known as lower pairs and pairs formed with point contact or line contact are known as higher pairs in mechanisms. Each higher pair cancels one degree of freedom in plane analysis. Then the residual degrees of freedom of the plane chain can be calculated as

$$RDOF = 3MC - 2L - H = 3(4 - 1) - 2 \cdot 4 = 1 \quad (1)$$

In formula (1) MC , L and H are the number of movable components, the number of lower pairs and the number of higher pairs respectively. The result shows 1 motion condition input is in need of becoming the chain to a crank-slider mechanism. In this example when a rotating speed of the crank (1) is given the relative motions of all other movable components

are defined and can be derived from the crank rotating speed. In the derivation each pair (interacting surfaces in relative motion) functions to permit some kinds of relative motion and prevent the others between two components joined by the pair, and each component functions to keep the surfaces of pairs having fixed positions on the component. Then the mechanism can be looked as consisting of two sub-systems: a component system and a pair system. The component system and the pair system work together to guarantee the mechanism with a definite motion when the number of motion conditions input is enough. Such function is a motion guarantee function.

Back to tribology, a tribo-pair is the physical realization of a kinematic pair and the tribo-pairs in total in a machine system constitutes the main part of a tribo-system.

Tribo-systems can be understood in another way. A system of higher rank can be divided into several systems of lower rank or sub-systems and vice versa. The division can be implemented in different ways. For example, a machine as a system can be divided into assembles, such as a rotor assemble, a chassis assemble, or can be divided according to the function of the sub-systems as well, such as a coolant circulation system, a brake system etc. A machine system can also be divided according to the character of behaviors of the sub-systems, for example, dividing the machine into a mechanical system, a thermodynamic system, an electric system and what will be discussed in detail in tribology, a tribo-system, etc.

Such a division is making an abstraction of machine systems. It does not limit the division to groups of elements or kinds of functions but keep the investigation into a given category of behaviors and their results. For example, in general there is an electric circuit diagram for a machine and the diagram is just a description of the electric system abstracted from the machine.

When a machine system or another natural system is abstracted into a system consisting of tribo-elements and some supporting auxiliary sub-systems for studying behaviors on or between the interacting surfaces in relative motion, results of the behaviors and technology related to, a tribo-system is then constructed.

In most cases there will be a liquid, a gas or a fat lubricant film kept between the interacting surfaces in relative motion to reduce friction and wear. Solid lubricant films will not be included in this discussion and looked as parts of the surfaces with a motion similar to the surfaces. The auxiliary sub-system for fluid lubrication including at first a cycling sub-system, a cooling sub-system and a filtering sub-system operates to keep the fluid film staying between the interacting surfaces in relative motion and to make the surfaces work efficiently, reliably and friendly to human and environment.

Due to there is a time variable character with the tribo-systems which will be discussed later, the change in structure, behavior and function or the change of working condition in short should be monitored (necessary) and be adaptively controlled (suggested) to avoid low efficiency work, abnormal wear, environment pollution or catastrophic damage. Therefore the condition monitoring sub-system or the condition controlling sub-system usually takes place in the auxiliary sub-system of tribo-systems. Figure 2 gives a general construction diagram for tribo-systems (Xie, 1996).

It can be concluded that a machine system is consisted of a component system and a tribo-system from the view point of motion. The tribo-system together with the component system plays a motion guarantee function which keeps each part of the machine system with a definite motion when the number of motion conditions input outside of the system is enough.

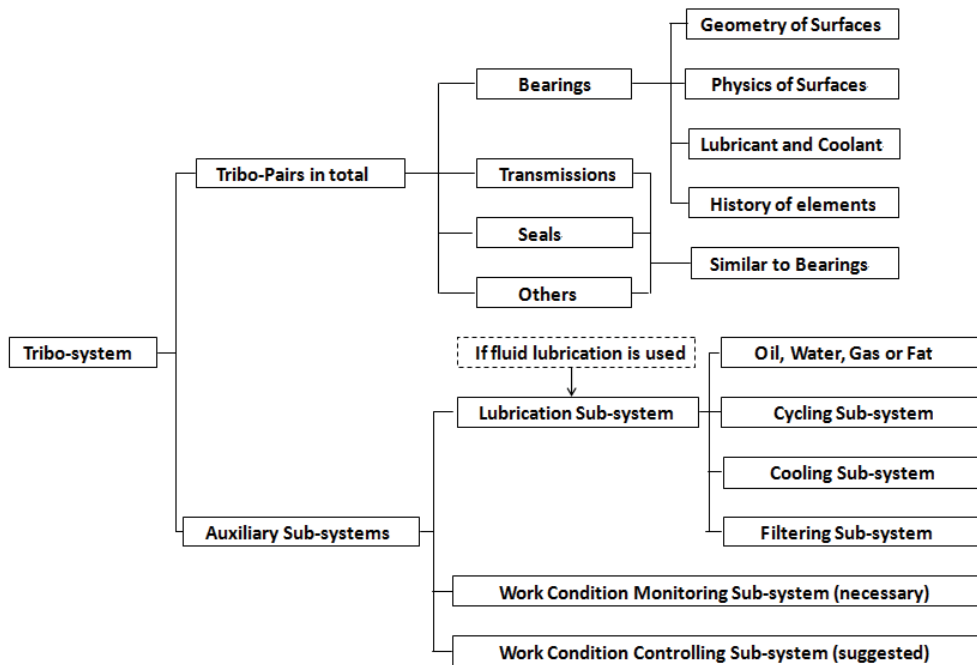


Fig. 2. The block diagram of a typical tribo-system

Tribology science and technology is very important in obtaining *the best way* (theory and application) to complete the motion guarantee function of tribo-systems. Tribology exists universally. Where there is relative motion there is tribology. Tribo-systems play sometimes very critical roles in machine systems and work sometimes under extreme severe condition. It implies that the motion guarantee function must be implemented with high reliability, low energy consumption, low cost, low pollution, human and environment friendship etc.

The study of friction, wear, lubrication and other tribological techniques are part of the efforts in finding the best way. Ignoring the fundamental function of tribo-systems and appreciating an *appearance based* study of friction, wear, lubrication or techniques from surface engineering, nanotechnology or biology etc, even though some physical or geometric results can be obtained, the study cannot give a clear overview on the relation between the results. Putting the results together in practice usually throws engineers into confusion. It will also increase the difficulty in tribo-system modeling and in looking for mathematic tools for an overall system and life cycle behavior simulation. Lack of overall system model and mathematic tool for simulation makes that the results from one working condition cannot be used in another condition, from one period of the life cannot be used in another period and from the study of friction cannot be used in the study of wear or the study of lubrication etc. Furthermore such a situation makes almost impossible to implement a tribological design since the tribo-design is the design of tribo-systems. It is well known that tribological design is a main channel for embedding tribology knowledge into products.

Therefore different from the independently study of friction, wear, lubrication or any technique related, tribology should be studied with a system viewpoint and cannot overlook the basic function of tribo-systems, the motion guarantee function for a machine system. Tribology study is to find the best way (theory and application) to complete the motion guarantee function. It is no doubt that all results from the study of friction, wear, lubrication and other technique related are indispensable in reaching the goal.

2. How tribo-systems behave?

After a detailed discussion on the basic function of tribo-systems the question arises that how a tribo-system behaves to complete the function?

Some basic knowledge about the character and pattern of change of the systems is necessary in describing their behaviors. For example, one character of a mechanical system or a mechanism is that all components in the system have definitive relative motion and it can be checked with formula (1). The pattern of change of the mechanism is governed by principles in kinematics from which the behaviors can be derived. For the crank-slider mechanism shown in Figure 1 the relative motions of all movable components can be derived with a given rotating speed of the crank. Such a requirement will be similar for thermodynamic systems, electric systems and all other systems abstracted from a system of higher rank according to a given category of behaviors. Tribology is a multi-disciplinary area. Any principle in other disciplines cannot describe and govern the character and pattern of change of tribo-systems accurately. A task of top priority is to organize the basic knowledge which matches the name, the definition and the nature of tribology rather than matches what concerning with only friction, wear, lubrication or any individual technique related.

After the investigation in many years the author suggests that there are three axioms in tribology and they can be used as a base or a start point to study into the character and pattern of change of tribo-systems. So-called an axiom means what people cannot find any opposite example with the axiom even though they cannot prove it theoretically (Suh, 1990). The three axioms in tribology (Xie, 2001) are: (1) The first Axiom: Tribological behaviors are system dependent. (2) The second Axiom: The property of tribo-elements and then the systems containing tribo-elements are time dependent. (3) The third Axiom: The results of tribological behaviors are the results of mutual action and strong coupling of many behaviors of other disciplines under a tribological condition consisted of interacting surfaces in relating motion. The three axioms will be discussed in more detail in the following.

2.1 The first axiom: Tribological behaviors are system dependent

Changes taking place on or between the interacting surfaces in relative motion are what to be investigated into in tribology and called tribological behaviors. Interactions and relative motion are causes of the behaviors. Results of the behaviors include a recoverable and irrecoverable change of intrinsic property of elements, a change of the state of the system consisted of the elements and a material, energy and information exchange with the environment in the forms of input and output. The intrinsic property includes geometric, physical and historic aspects and will be discussed later. A single surface or medium substance cannot implement any tribological behavior. In Fig. 3 there is a simplest tribo-system including three elements. The system is enveloped with a system block and exchanges material, energy and information via input and output with environment. The system dependent character governs not only the behaviors of simplest systems but also any

more complex system consisted of simplest systems and their supporting auxiliary sub-systems (Xie, 2010). The first axiom focuses on the relationship of structures.

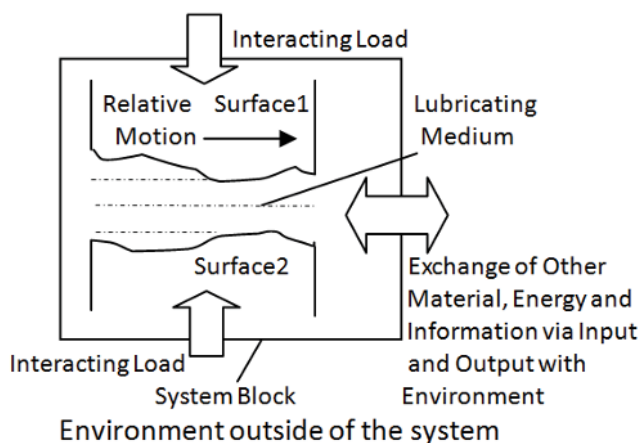


Fig. 3. A simplest tribo-system

2.2 The second axiom: The property of tribo-elements and then the systems containing tribo-elements are time dependent

In comparison with the material in the body of a component, the material of any element in a tribo-pair bears much more intensive load and works under a much more severe condition. As shown in Figure 4 when the roughness of surfaces in contact is considered, the real contact area is much smaller than the nominative contact area. Through the much smaller contact area it transmits a load equal to what transmitted by the body of the component with an area of the body section. The load density is then very high at the real contact area. On the other hand the transmission is implemented between different materials in a pair while it is through the same material in the case of the body of a component. Additional physical or chemical reaction between different materials may occur under such a condition. Furthermore there is a relative motion. It accelerates the change of their physical property, chemical composition and geometric configuration, especially due to the relative motion the change is continuously repeated, sometimes with very high frequency. The relative motion produces heat and then high temperature and other kinds of active energy. They will no doubt promote the change in physical aspects and chemical aspects. All of them make the change of the property of each element in the tribo-pair much more fast in comparison with what in the body of a component. Due to the severity in most cases the change is irrecoverable. Therefore as in performance analysis or design people usually consider what they deal with is a time-invariable system they must consider it as a time-variable system in tribological analysis and tribological design. As shown in Fig. 5 the speed of change of performance is variable in a life cycle. In the earlier stage of work of a new system the speed of change is high and this is a running in stage. Afterwards the speed of change will be slow in a stable operation stage. At last the speed of change increases faster and it predicts the end of life. The variation of speed of change is very complex in many systems.

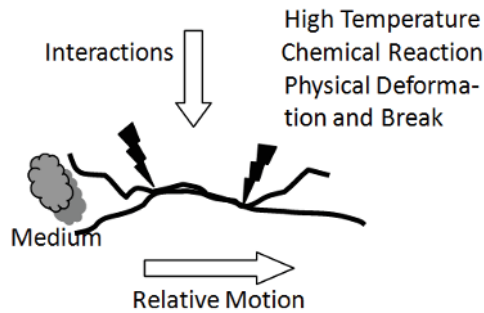


Fig. 4. The severity of work condition of a tribo-pair

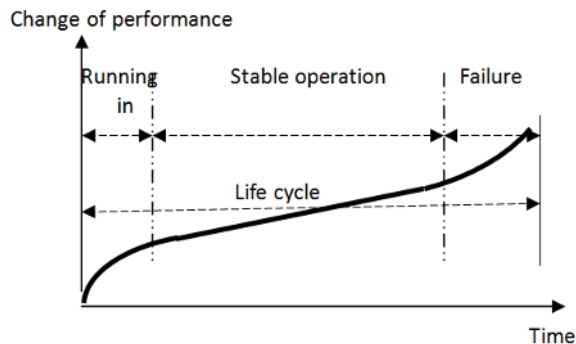


Fig. 5. Change of performance of a tribo-system in a life cycle

2.3 The third axiom in tribology: The results of tribological behaviors are the results of mutual action and strong coupling of many behaviors of other disciplines under a tribological condition consisted of interacting surfaces in relating motion

Obviously from the simplest tribo-system shown in Fig. 1 the force interaction, relative motion of the surfaces and the medium substance between the surfaces are a mechanical behavior. Transformation of the mechanical energy consumed in motion into heat energy and the diffusion of heat in surrounding, which makes a stable or unstable temperature field, are thermodynamic behaviors and heat transfer behaviors. The molecular interaction (including transferring) between surfaces and surfaces with medium is a physical or physical-chemical behavior. The reactions in ion level and atomic level are chemical behaviors. If there is any electric or magnetic field, which produces attractive or repelling interaction, or changes the arrangement of molecules in materials, or induces eddy current and heat, all of them are electric behaviors or magnetic behaviors, and so on. Most of them behavior simultaneously and inevitably change the structure of the system recoverably and irrecoverably. Then in turn they bring about the final results different from the results when they behavior singly. The results are different also from a simple addition of the results of individual behaviors. There is strong coupling between such behaviors. Tribology is the science and technology, which provides theories and techniques to describe and control the pattern of coupling between behaviors under tribological condition. No other discipline can

meet such a need other than tribology, even though for any individual behavior there are principles from the discipline related which can predict its results. The difficulty for tribologists is that they have to know all the relative disciplines together with tribology simultaneously. Such a character of inter-disciplines and multi-disciplines requires a new methodology for tribology different from what for friction, wear or lubrication. In distinguish with the first axiom the third axiom focuses on the relationship of behaviors.

3. How to model a tribo-system and simulate its behaviors?

The structure is a description of intrinsic facts of a tribo-system while *the behaviors* are a description of change of the tribo-system. The structure of a system exists regardless whether there is an input. The behaviors must follow an input and can be derived from the input for a given structure in principle.

3.1 The structure of a tribo-system

Czichos (Czichos, 1978) described the structure of a tribo-system with a parameter set as

$$S = \{E, P, R\} \quad (2)$$

where $E = \{e_1, e_2, \dots, e_N\}$ is a sub-set showing that there are N tribo-elements in total in the system, $P = \{p_{e1}, p_{e2}, \dots, p_{eN}\}$ is a sub-set describing the property of each element in the system and $R = \{r_{e1e2}, r_{e1e3}, \dots, r_{e1eN}, r_{e2e3}, \dots\}$ is a sub-set collecting all relations between elements in the system.

Such description carries out a problem. Since the relative motion is the first important character of tribology and then the relative displacements between elements changing with the motion condition input, therefore the relative displacements between elements cannot be treated as an intrinsic fact. Several other examples can be listed as well.

To avoid the problem the author modified the description of a structure as (Xie, 2010)

$$S = \{E, P, H\} \quad (3)$$

where $H = \{h_w, h_{e1}, h_{e2}, \dots, h_{eN}\}$ is a sub-set including the history of the system as a whole and of each element.

Each element e_i , $i = 1 \dots N$, in the sub-set E represents a surface or a medium substance, for example a journal surface, a bearing surface, a cylinder bore surface, a piston skirt surface or the lubricant film between the surfaces.

Each element p_{ei} , $i = 1 \dots N$, in the sub-set P describes the property of element e_i . In more detail the contents of property of each element can be divided into two groups, i.e. $p_{ei} = \{pg, pp\}_i$, $i = 1 \dots N$, where pg is the geometric parameter group of property of the element, for example the diameter and width of the bearing surface in macro scale and the roughness in micro scale, and pp is the physical parameter group of property of the element. pp should be understood in a generalized sense including all physical, chemical and biological features besides geometric. It usually can be described by a group of physical, chemical and biological parameters, such as hardness, viscosity, acidity, activity etc. but there are some exceptions. Such features are affected by material composition, manufacturing process, service history, surrounding temperature, atmosphere, etc.

According to the second axiom in tribology, the property of elements and systems is time depended. The structure is a description of intrinsic facts but it is not invariable for a tribo-system. There are recoverable changes and irrecoverable changes in the structure due to the interaction and relative motion of surfaces. As described in formula (3), E is obvious invariable, the only variable things in S are P and H . Each element $h_w, h_{ei}, i = 1 \dots N$, in the sub-set H are too complex to be described with parameters, usually they are a series of records in natural language. Using H rather than using a time parameter t here is because of that t notes only a time scale but what happened at t is more important for understanding the change of the structure. The elements of H do not act directly upon the structure but affect the values of parameters in pg and pp . For each effect some principles which govern the progress of effect can be found in related discipline. For example an elastic deformation of the surfaces is a recoverable change of pg which follows the change of interacting load on the surfaces governed by principles in the theory of elasticity, while a plastic deformation or wear of the surfaces is an irrecoverable change of pg , it is defined by what happened in the history and governed by principles in the theory of plasticity and tribology.

3.2 The behavior simulation of a tribo-system

Different from what used in references (Dai & Xue, 2003; Ge & Zhu, 2005), a state space method is applied here to simulate the behaviors. The state space method is a combination of general systems theory with engineering systems analysis and has wide application in dynamic system analysis, control engineering and many non-engineering analysis (Ogata, 1970, 1987). It takes a vector quantity called *state* as a scale to coordinate and evaluate the results of behaviors. When an input is applied upon a system, the system behaves from one state to another state and gives an output. For a time-invariable linear system a state equation (4) and an output equation (5) can be used to describe the results of behaviors:

$$\dot{X} = AX + BU \quad (4)$$

$$Y = CX + DU \quad (5)$$

where X, U, Y are the state vector, input vector and output vector of the system respectively. A and B are the system matrix, input matrix for equation (4) while C and D the output matrix for equation (5) respectively. All of them consist of the elements of structure of the system. A, B, C and D are constant for a time-invariable linear system.

In general the elements in a state vector are what concerned with the results of behaviors. As discussed previously, the first important behavior to be studied in tribo-systems is the relative motion. Any surface cannot exist independently and must be a part of a component of the machine system from which the tribo-system abstracted. The relative motion of surfaces is defined by the relative motion of components and where the surfaces reside on. Therefore for tribo-systems in the state vectors there are usually the parameters of displacements and time derivatives of displacements of components. For example the state of a single mass moving horizontally can be written as

$$X = [x, \dot{x}]^T$$

in which, x is the coordinate of the mass in x direction. When there are behaviors besides mechanics to be studied, parameters of related disciplines may emerge in the state vector,

for example the electric current i in the coil of the electric magnet of an adaptive magnetic bearing.

For tribo-systems the situation will be complex. There are three possible ways to be selected.

1. If in behavior simulation the change of structure is not considered there will be a time-invariable linear system, i.e.

$$S = const \quad (6)$$

and simultaneously

$$A = const, B = const, C = const, D = const \quad (6a)$$

2. If in behavior simulation the recoverable change of structure is considered only there will be a time-invariable non-linear system, i.e.

$$S = S(X), A = A(X), B = B(X), C = C(X), D = D(X) \quad (7)$$

Simultaneously there will be also

$$P = \{pg, pp\} = P(X) = \{pg(X), pp(X)\} \quad (7a)$$

For any artifact system a requirement of behavior repeatability in an observation of short period is obviously necessary for reuse. Therefore the state X is repeatable. The recoverable change of structure implies that the structure is a function of the state and independent to time. Whenever a similar input applied on a system with a similar state the system will have a similar state change and similar output. In other words the system behaves similarly. In an observation of short period the irrecoverable change due to very small in value in comparison with the recoverable change is negligible.

In an observation of short period, pg or pp changes with X due to many causes under the tribological condition, i.e. on or between the interacting surfaces in relative motion. Because X is repeatable and pg or pp is a function of X only, the patterns of change of pg or pp are relative simple. For each cause there will be some principles dealing with how the cause affects the change of parameters of pg or pp . These principles are in general relative to a discipline independent to tribology. Meanwhile a governing equation system, which may be a theoretical, experimental or statistical one, can be found in the discipline to describe the patterns of change of parameters of pg or pp under the tribological condition. As discussed before, for an elastic deformation the governing equation system can be found in the theory of elasticity and dynamics for a temperature distribution change the governing equation system can be found in the thermodynamics and heat transfer, for a change of viscosity of lubricants in terms of relative motion the governing equation system can be found in rheology, etc.

3. Irrecoverable changes are performed in entire processes of manufacturing, assembling, packaging, storing and transporting and will accumulate with service time and reach a comparable extent at last. It is history depended. In behavior simulation a time-variable non-linear system have to be treated, i.e.

$$S = S(X, t) \text{ or more accurate that } S = S(X, H) \quad (8)$$

$$\text{and } A = A(X, H), B = B(X, H), C = C(X, H), D = D(X, H)$$

Since formula (3) and that the elements of H do not act directly upon the structure but affect the values of parameters in pg and pp , the following formula can be established

$$P = \{pg, pp\} = P(X, H) = \{pg(X, H), pp(X, H)\} \quad (8a)$$

It shows that the property of a tribo-system changes with the system state and the history of the system.

In an observation of long period, pg or pp changes not only with X but also with H . There are many issues concerning with irrecoverable changes of the structure of machine systems. Wear, fatigue, plastic flow, creep, aging and corrosion are the most important irrecoverable changes. It is no doubt that wear is one of the issues studied in tribology. Fatigue takes place on the surfaces bringing forth a kind of fatigue wear. Plastic flow or creep carries out a permanent deformation of surfaces in macro scale which harms the motion guarantee function. Plastic flow in micro scale makes a change of elastic contact to plastic contact and will generate origins of surface fatigue after a number of cycles of repeat. Aging changes parameters in pp for solid surface materials and makes them inclining to failure. Aging spoils the performance of lubricants, increases corrosiveness and decreases the capability of lubrication. Corrosion of interacting surfaces in relative motion is also a kind of wear due to the chemical reaction of some compositions in lubricant or atmosphere with the materials of surfaces or due to the mechanical effect of break of air bubbles in the lubricant film. Obviously most issues concerning with irrecoverable changes are taken place in tribo-systems and studied in tribology.

According to the third axiom in tribology, the results of tribological behaviors are the results of mutual action and strong coupling of behaviors of many disciplines under a tribological condition constituted by interacting surfaces in relative motion. Because of that history or time is unrepeatable, the irrecoverable change is more complex in description than the recoverable change and almost no simple equation system can be found in any discipline. The different causes occurred singly or jointly at different moment in the history and their results were accumulated or coupled each other and result an irrecoverable change of the structure at a given time. In other words the structure is a carrier of mutual action and strong coupling of behaviors of many disciplines and gives a structure change in total at last as the results.

3.3 How to solve the state equations and output equations

In the behavior simulation of tribo-systems a time-variable non-linear system must be faced. The state equations and output equations will be as

$$\dot{X} = A(X, H) \cdot X + B(X, H) \cdot U(t) \quad (9)$$

$$Y = C(X, H) \cdot X + D(X, H) \cdot U(t) \quad (10)$$

Solving state equations is an initial value problem.

For a time-invariable linear system formula (4) can be integrated analytically when in formula (6a) A and B are constant. At any instant t_1 an input $U(t)$ is applied to a system in an initial state X_1 , then the system behaves to a state X_2 at an instant $t_2 = t_1 + \Delta t$ and give an output Y based on formula (5). It implies that similar initial state and similar input result similar change of state and similar output after a similar time interval Δt . After obtaining a new X_2 the new output Y_2 can be computed accordingly with formula (5) and constant matrixes C and D .

For time-variable non-linear systems the situation will be a little complex. Since matrix A , B , C or D is a function of the state and time (history related), integrating formula (9) and (10) analytically is in general impossible. The problem is similar with time-invariable non-linear systems when the matrixes A , B , C and D are functions of state X as shown in formula (7) and (7a) and will not be discussed separately in the following.

Numerical method is used for solving formula (9) and (10) for a time-variable non-linear system. The equations are discretized and integrated in a small time increment Δt step by step. When the Δt is small enough one can suppose that matrix A , B , C or D is independent to X and t and is constant in the time interval Δt , i.e. the system becomes a time-invariable linear system. In the integration, matrix A , B , C or D as a constant matrix and the values of their elements are calculated base on the results of last step with state X_1 and time t_1 . After integration, there will be a change for both state and time, i.e. $X_2 = X_1 + \Delta X$ and $t_2 = t_1 + \Delta t$. Afterwards the elements in matrixes A , B , C and D should be recalculated according to X_2 and t_2 for the next step of integration if any of them is state and time related. Similar to the time-invariable and linear assumption made in the integration, a decoupling assumption is made also that the effect of any behavior on the values of elements in matrixes A , B , C and D can be calculated independently with the governing equations of related discipline or obtained from an experiment under a condition considering only the change of X and t ignoring other coupling effects. For example, in the simulation of the lubrication behavior in a piston skirt – cylinder bore pair, the lubricant film between the skirt surface and the bore surface undergoes a viscosity change when the piston changes its position along the bore due to a non-uniform distribution of temperature. The viscosity is a parameter in pp and its change may affect some elements in matrix A , B , C or D . A viscosity η_1 corresponding to temperature T_1 at y_1 , the coordinate of skirt in the bore, is used for obtaining matrix A , B , C or D . After integrating over a Δt , y_1 becomes to y_2 , T_1 becomes to T_2 , η_1 becomes to η_2 and the matrix A , B , C or D will be recalculated with η_2 for the next integration. For recoverable change in an observation of short period the function $\eta(T)$ can be obtained by fitting experiment data and accurate enough. For irrecoverable change in an observation of long period a function in the form of $\eta(T, H)$ is necessary. In the history, many causes of very different kinds can affect the relation between η and T and make the lubricant aging. The causes before service include the kind of base oil, the technology and process of refining, the additive used etc. while the causes after service include the service temperature, service atmosphere, pollution condition and filtration efficiency in service etc. Knowledge of $\eta(T, H)$ have to be acquired for each application. Aging is a long period change and progresses very slowly. In numerical integration one can use a relative long time interval for such kinds of irrecoverable change other than recoverable change while a small time interval has to be used to keep the accuracy of simulation for recoverable change in time-invariable non-linear system.

There are many mathematic tools which make such an application available, for example, the Runge-Kutta Procedure (Chen, 1982). The difficulty in solving the problem is to find a balance between time consuming while a smaller time step (Δt) is used and low precision while a larger time step is used in integration (Xu, 2007).

4. Examples of modeling and simulation

4.1 Example 1

The cylinder – piston – conrod – crank system of a single cylinder internal combustion engine is shown in Fig. 6. The system can be abstracted into a tribo-system with following

tribo-pairs: piston skirt – cylinder bore, wrist pin – small end bearing, journal of crank – big end bearing and journal of crankshaft and main bearing, i.e. one prismatic pair and three revolutive pairs in the system totally.

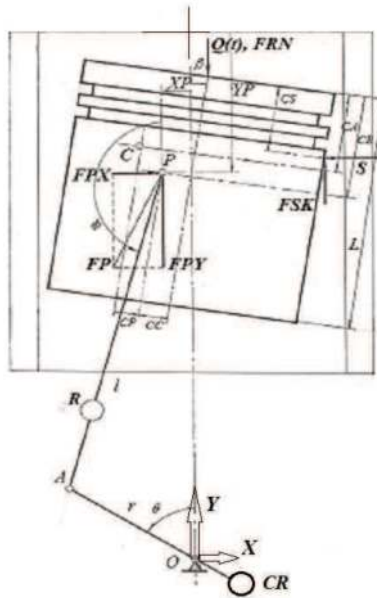


Fig. 6. Cylinder-piston-conrod-crank mechanism

A system block diagram for the piston skirt – cylinder bore pair is shown in Fig. 7 which gives a survey on the relationship between the skirt-bore tribo-pair and environment.

In the simulation the secondary motion of the piston and the change of inertia of the conrod are considered. The influence of the offset of the wrist pin can be considered as well and is taken as zero, positive or negative for comparison. The behaviors of lubricant film between the skirt surface and bore surface are treated according to the theory of thermal-hydrodynamic lubrication. The configuration of the skirt and bore can be given in simulation and a thermal distortion, force deformation and wear process can be calculated separately with the theory of heat transfer, theory of elasticity and regression of measured wear data. Their effects will couple between each other and with what of other behaviors in the iteration but a rigid skirt and a rigid bore without wear are supposed in the example. All of the eccentricities of journals in bearings and all of the elastic deformation of other components in the system are neglect. Their behaviors can be simulated separately and is decoupling with other behaviors in the global simulation.

The tribological behavior concerned in the system is a hydrodynamic lubrication behavior between the skirt surface and the bore surface. It results a thrust force S which balances the interacting load on the lubricant film and a shear resistant (friction) force FSK against the relative motion. In general there are two ways to treat the hydrodynamic behavior. One is looking the forces produced in the film like the inputs of the system. The other is taking the lubricant film as a structure element between surfaces. In this example the first way of treatment is applied.

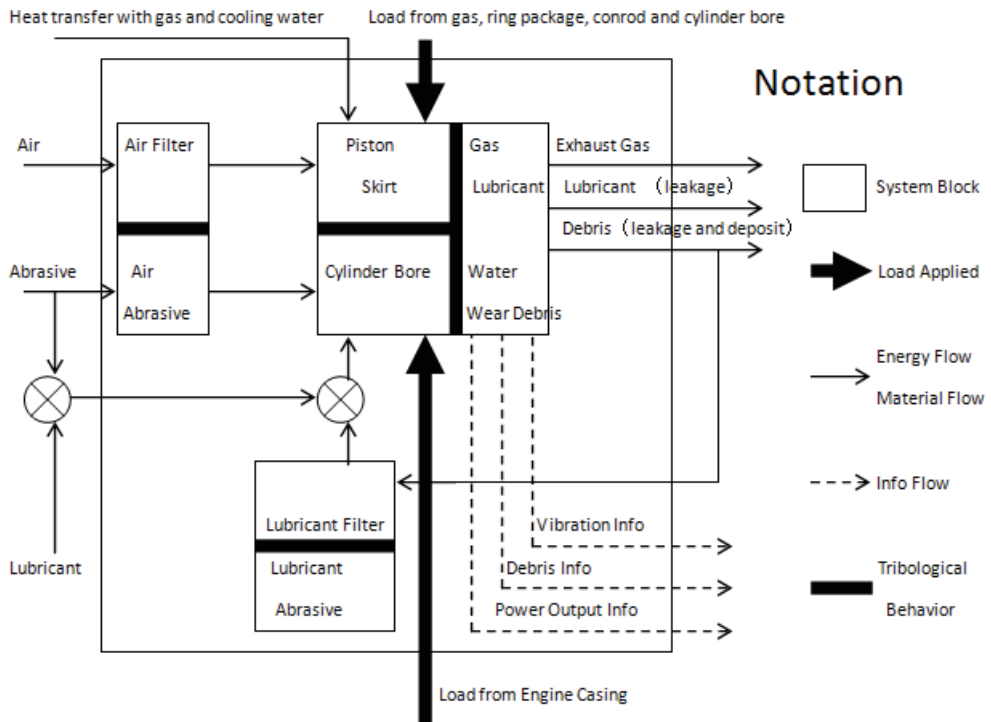


Fig. 7. The system block diagram of a cylinder bore-piston skirt

Piston ring package is considered separately also and the friction force between ring surfaces and cylinder bore is treated as an input (FRN in Fig. 6) applied on the piston. Other inputs are the gas pressure $Q(t)$ on the top of the piston, the thrust force from the cylinder bore surface on the piston skirt surface S , the force on the wrist pin FP . All of them are balanced by a resistant torque moment (load) on the crankshaft.

The output can be selected according to what one wants to know in the simulation.

The state matrix equation of the system and the output matrix equation can be written as follows.

$$\begin{bmatrix} X_p \\ \dot{X}_p \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix}' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{26} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{46} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} X_p \\ \dot{X}_p \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ U_2 \\ 0 \\ U_4 \\ 0 \\ U_6 \end{bmatrix} \tag{11}$$

When the hydrodynamic behavior between the skirt surface and bore surface is looked as an input applied on the system (via skirt surface), the resultant force of the hydrodynamic film pressure S and the resultant force of the resistant shear stress FSK will be the elements in U_2

and U_4 . The hydrodynamic behavior depends on the gap geometry, the relative motion of surfaces and the lubricant viscosity. The gap geometry is changed with the wrist pin center displacement X_p and the piston tilting angle β in this case. The relative motion includes a tangential and normal component. The lubricant viscosity changes with temperature which has a distribution along the cylinder wall in y direction. The temperature distribution changes with the engine working condition but keeps unchanged in the example. All of them will be calculated in a separate program based on Reynolds Equation (Pinkus & Sternlicht, 1961).

$$\begin{bmatrix} \dot{\theta} \\ P_{LOSS} \\ X_p \\ \beta \\ F_{RHT} \\ F_{LFT} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & C_{16} \\ 0 & 0 & 0 & 0 & 0 & C_{26} \\ 0 & 0 & 0 & 0 & 0 & C_{36} \\ 0 & 0 & 0 & 0 & 0 & C_{46} \\ 0 & 0 & 0 & 0 & 0 & C_{56} \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} X_p \\ \dot{X}_p \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad (12)$$

Fig. 8 gives the change of output in 720° crankshaft rotating angle by formula (12), where (a), (b), (c), (d), (e) and (f) are the deviation of crankshaft speed $\dot{\theta}$, change of friction power loss P_{LOSS} in the skirt-bore pair, displacement X_p of the wrist pin center in X direction, tilting angle β around the wrist pin center, thrust force F_{RHT} on the right side of the skirt and thrust force $FLFT$ on the left side of the skirt from the hydrodynamic lubrication film respectively.

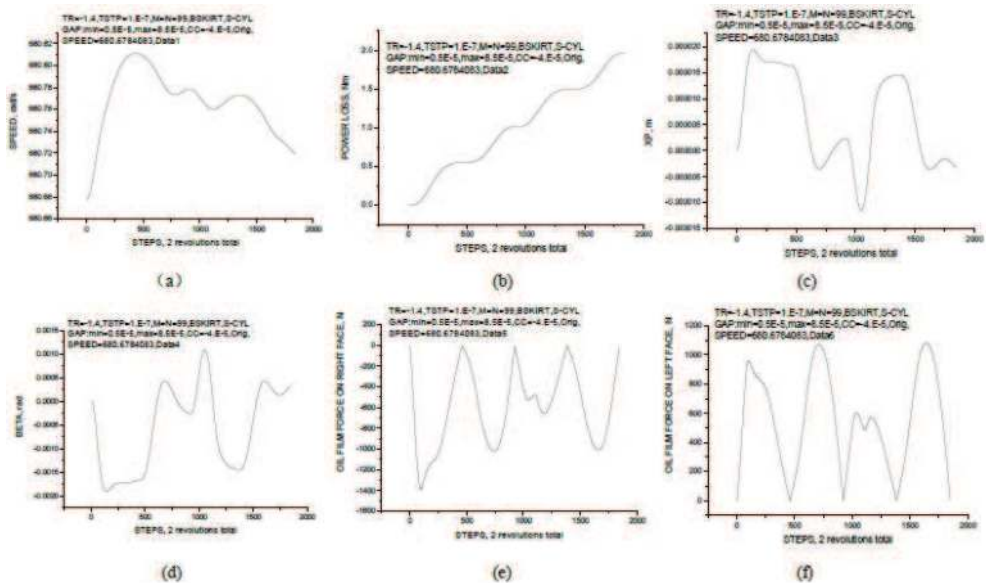


Fig. 8. Output of the system in 720° rotating angle of crankshaft

Fig. 9 gives a comparison on the friction power loss when different skirt configurations are used. The geometry of skirt influences the gap between surfaces and then changes the

hydrodynamic film pressure in values and distribution and changes the shear stress. It shows that the barrel skirt has a smaller friction loss.

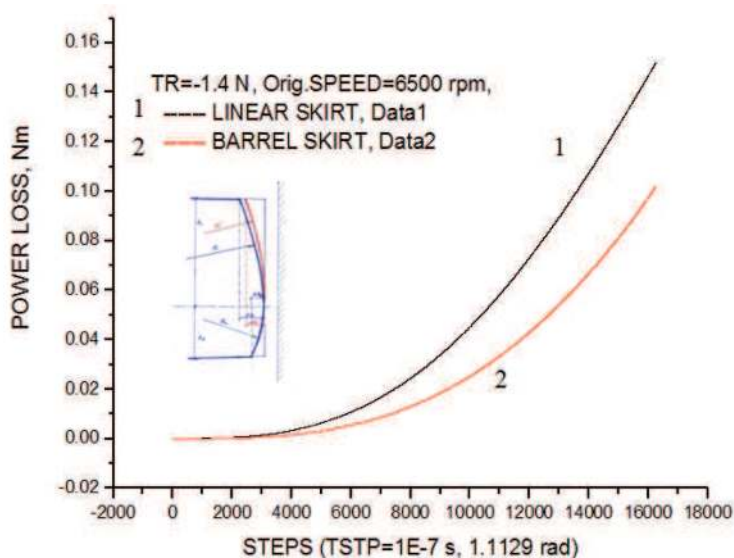


Fig. 9. Influence of skirt configuration on the friction power loss

Table 1 shows a comparison on the friction power loss between different values of wrist pin offset. The linear skirt is more sensitive to the offset than the barrel skirt is.

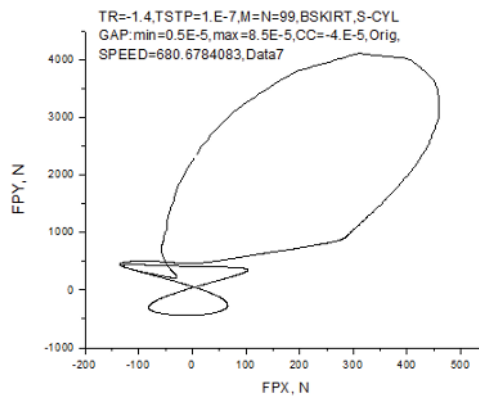
Computation number	Wrist Pin Offset	Friction Power Loss in 720°
Linear Skirt LS99-2-C-1	Left Offset CC=+4.E-5 m	2.32121 Nm
Linear Skirt LS99-2-C-0	Zero Offset CC=0.m	2.31236 Nm
Linear Skirt LS99-2-C-2	Right Offset CC=-4.E-5 m	2.30477 Nm
Barrel Skirt BS99-2-C-1	Left Offset CC=+4.E-5 m	1.97164 Nm
Barrel Skirt BS99-2-C-0	Zero Offset CC=0.m	1.97038 Nm
Barrel Skirt BS99-2-C-2	Right Offset CC=-4.E-5 m	1.96907 Nm

Table 1. Effects of wrist pin offset and skirt profile on piston skirt friction power loss

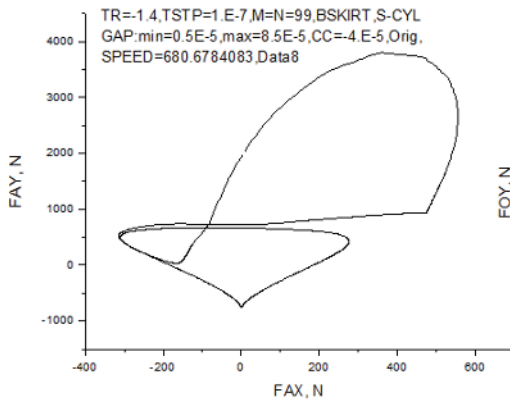
If the forces transmitted in the pairs P , A and O are interesting there will be another output matrix equation as

$$\begin{bmatrix} F_{PX} \\ F_{PY} \\ F_{AX} \\ F_{AY} \\ F_{OX} \\ F_{OY} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & C'_{16} \\ 0 & 0 & 0 & 0 & 0 & C'_{26} \\ 0 & 0 & 0 & 0 & 0 & C'_{36} \\ 0 & 0 & 0 & 0 & 0 & C'_{46} \\ 0 & 0 & 0 & 0 & 0 & C'_{56} \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix} \begin{bmatrix} X_P \\ \dot{X}_P \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix} \quad (13)$$

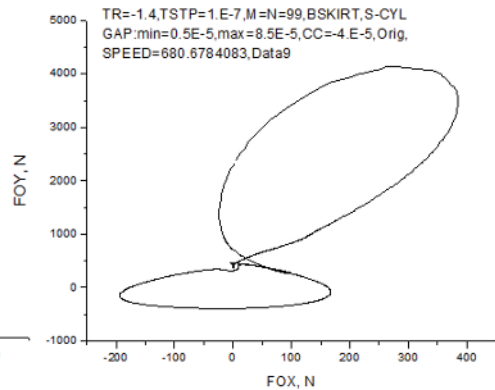
Where F_{PX} , F_{PY} , F_{AX} , F_{AY} , F_{OX} and F_{OY} are the force components transmitted in the small end bearing of conrod, in the big end bearing of conrod and in the main bearings (in total) of crankshaft respectively of the IC engine in discussion. The change of such forces in 720° crankshaft rotating angle is shown in Fig. 10.



(a)



(b)



(c)

Fig. 10. Forces transmitted in the bearing of an IC engine. (a) Small end bearing of conrod. (b) Big end bearing of conrod. (c) Main bearing of crankshaft

The derivation of elements A_{16} to A_{66} , C_{16} to C_{66} , U_2 to U_6 and C'_{16} to C'_{66} in formulas (11), (12) and (13) can be found in Appendix.

4.2 Example 2

As shown in Fig. 11 there is a rotor-bearing system of a 300MW turbo-generator set consisted of the rotor of a high pressure cylinder (HP), an intermediate pressure cylinder (IP), a low pressure cylinder (LP), a generator, an exciter and eight hydrodynamic bearings (1# - 8#) on pedestals. A simplification is made in the example that the eight bearings are all plane bearings to reduce the amount of computation. The rotor in total is an elastic component supported by the bearings and can vibrate laterally. Obviously it is a statically indeterminate problem. The load on each bearing is determined by the relationship between the elevations of journal centers which are controlled by a camber curve checked at last in installation. There are many reasons which can change the relationship, for example the journals may float with different eccentricity e (Fig. 17) on the hydrodynamic film and the pedestals may change their heights due to the changes of working temperatures during different turbine output and then change the bearing loads under a statically indeterminate condition.

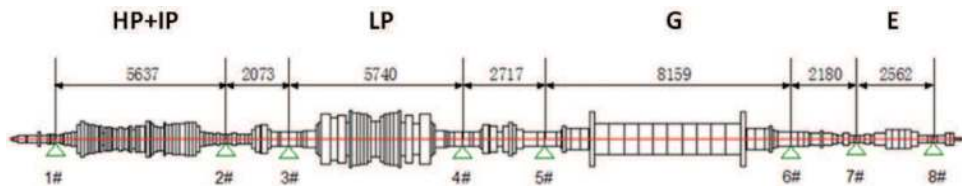


Fig. 11. The rotor-bearing system of a 300MW turbo-generator set

The tribological behaviors considered in the example are the hydrodynamic behaviors in bearings. There are three points to be considered.

1. For a hydrodynamic bearing the rotating journal is floating on the hydrodynamic film and there is an eccentricity between the journal center and the bearing center. During installation the journal is dropped upon the bottom surface of the bearing bore. The eccentricity changes with the load on the bearing.
2. The change of the load or eccentricity changes the geometric property and physical property (pg , pp - see section 3.1) of the film when taking it as a structure element between surfaces.
3. If the change of pp approaching to some extent the film will excite a kind of severe vibration of the system called oil whirl or oil resonance (Hori, 2002) and may result a catastrophic damage of the turbo-generator set.

In general it is recognized that the oil whirl begins at the threshold of instability of the rotor-bearing system and usually has a frequency half the rotor speed. It is a tribological behavior induced vibration and indicates a decrease or loss of motion guarantee function.

The treatment of the hydrodynamic behavior in the film looks like inserting a structure element between surfaces and is different from what has done in example 1 (see section 4.1). In this case the film is a linearized spring-damper in time interval Δt and its pp can be represented by four constant stiffness coefficients k_{xx} , k_{xy} , k_{yx} , k_{yy} and four constant damping coefficients d_{xx} , d_{xy} , d_{yx} , d_{yy} . It implies an assumption of using $pp=const$ instead of $pp=pp(X)$

during integration in time interval Δt . The eight coefficients can be calculated before integration with a separate program for a given film configuration (bearing bore geometry, eccentricity and attitude angle) and relative motion (tangential and normal) between journal surface and bearing surface (Pinkus & Sternlicht, 1961). The eight spring-dampers together with the distributed mass-stiffness-damping of the rotor defines the threshold of instability. To constitute the state space equation the rotor is discretized into 194 sections (Fig. 12) according to a concentrated mass treatment which can be found in rotor-bearing system dynamics (Glienicke, 1972) and its detail is omitted in the example.



Fig. 12. A discretized model of the rotor

Each section (Fig. 13) consists of a field of length l with stiffness but without mass and a station with mass, inertia moment but without length.

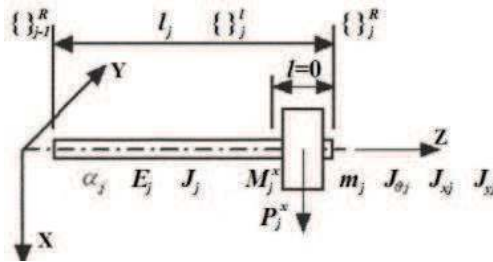


Fig. 13. A section of the rotor with a field and a station

The forces and moments applied on both side of a field and the related deformations are shown in Fig. 14 and Fig. 15.

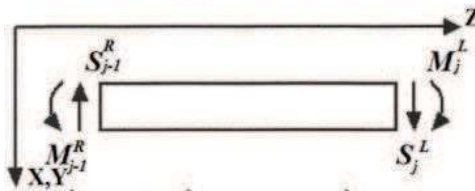


Fig. 14. The forces and moments on a field

The angular displacements and inertia moments of a station are described in Fig. 16. All of the inputs (forces and moments) apply only on the station. They make a balance between the forces and moments applying by the fields (right and left) and the inertia forces and moments. If there is a bearing attached to a section then the station is looked like supported by a linearized spring-damper with four direct stiffness and damping coefficients k_{xx} , k_{yy} , d_{xx} , d_{yy} and four cross stiffness and damping coefficients k_{xy} , k_{yx} , d_{xy} , d_{yx} as shown in Fig. 17. The cross stiffness and damping coefficients show an important difference between the

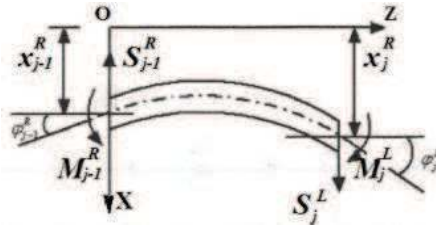


Fig. 15. The lateral deformation of a field

hydrodynamic film and isotropic solid material. The hydrodynamic film then plays the role of a component of the system. It should be emphasized that the height of the journal center is determined by the sum of the height of bearing center controlled by pedestal and the project of eccentricity e of the journal center on ordinate axis while the load on each bearing is determined by the journal height under a static indeterminate condition.

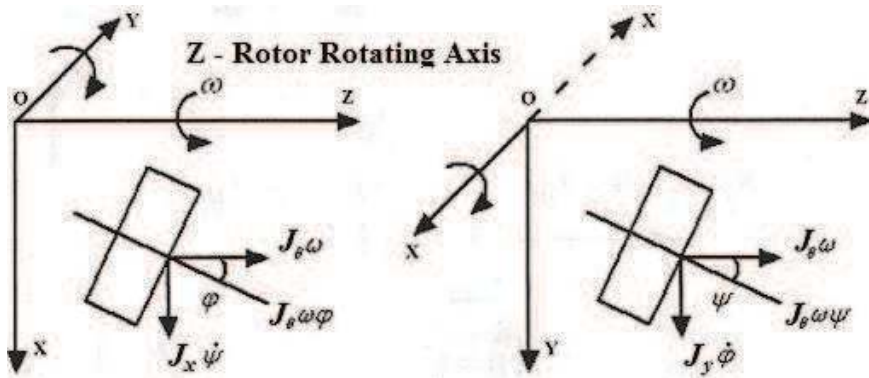


Fig. 16. Angular displacements and inertia moments of a station in X-Z and Y-Z plane

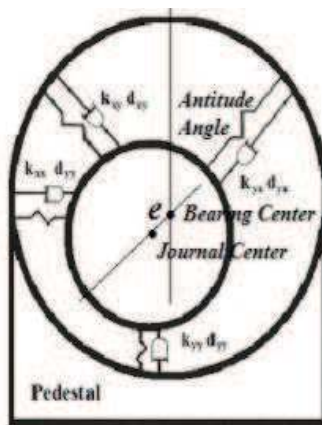


Fig. 17. A linearized model of the hydrodynamic film

Another form of formula (4) for one section, for example for section j , can be written as

$$\begin{aligned}
 & \begin{bmatrix} m\ddot{x} \\ m\ddot{y} \\ J_x\ddot{\phi} \\ J_y\ddot{\psi} \end{bmatrix}_j + \begin{bmatrix} d_{xx} & d_{xy} & 0 & 0 \\ d_{yx} & d_{yy} & 0 & 0 \\ 0 & 0 & 0 & -J_\theta\omega \\ 0 & 0 & J_\theta\omega & 0 \end{bmatrix}_j \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix}_j + \begin{bmatrix} k_{xx} & k_{xy} & 0 & 0 \\ k_{yx} & k_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}_j \begin{bmatrix} x \\ y \\ \phi \\ \psi \end{bmatrix}_j \\
 & + \begin{bmatrix} \frac{12EJ}{l^3} & 0 & \frac{6EJ}{l^2} & 0 \\ 0 & -\frac{12EJ}{l^3} & 0 & \frac{6EJ}{l^2} \\ -\frac{6EJ}{l^2} & 0 & \frac{2EJ}{l} & 0 \\ 0 & -\frac{6EJ}{l^2} & 0 & \frac{2EJ}{l} \end{bmatrix}_{j+1} \begin{bmatrix} x \\ y \\ \phi \\ \psi \end{bmatrix}_{j+1} + \begin{bmatrix} \frac{12EJ}{l^3} & 0 & \frac{6EJ}{l^2} & 0 \\ 0 & \frac{12EJ}{l^3} & 0 & \frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & 0 & \frac{4EJ}{l} & 0 \\ 0 & \frac{6EJ}{l^2} & 0 & \frac{4EJ}{l} \end{bmatrix}_{j+1} \begin{bmatrix} x \\ y \\ \phi \\ \psi \end{bmatrix}_{j+1} \\
 & + \begin{bmatrix} \frac{12EJ}{l^3} & 0 & -\frac{6EJ}{l^2} & 0 \\ 0 & \frac{12EJ}{l^3} & 0 & -\frac{6EJ}{l^2} \\ -\frac{6EJ}{l^2} & 0 & \frac{4EJ}{l} & 0 \\ 0 & -\frac{6EJ}{l^2} & 0 & \frac{4EJ}{l} \end{bmatrix}_j \begin{bmatrix} x \\ y \\ \phi \\ \psi \end{bmatrix}_j + \begin{bmatrix} -\frac{12EJ}{l^3} & 0 & -\frac{6EJ}{l^2} & 0 \\ 0 & -\frac{12EJ}{l^3} & 0 & -\frac{6EJ}{l^2} \\ \frac{6EJ}{l^2} & 0 & \frac{2EJ}{l} & 0 \\ 0 & \frac{6EJ}{l^2} & 0 & \frac{2EJ}{l} \end{bmatrix}_j \begin{bmatrix} x \\ y \\ \phi \\ \psi \end{bmatrix}_{j-1} = \begin{bmatrix} P^x \\ P^y \\ M_k \\ N_k \end{bmatrix}_j
 \end{aligned} \tag{14}$$

where E is the Young's module of the rotor material and J is the area moment inertia, other parameters can be found in Fig. 12 to 17. The state space equation for the rotor bearing system can be obtained by assembling formula (14) for $j=1$ to $j=n$ with free boundary condition at the two terminal ends. The assembled result formula will not be presented in the example.

A question arises that how the change of elevation distribution influences the threshold of instability of the system? It can be transformed into an eigenvalue problem. In general the solutions of equation are as follows

$$\begin{aligned}
 x_i &= x_{0i}e^{v_i t} = x_{0i}e^{-a_i t}e^{ib_i t}, \\
 y_i &= y_{0i}e^{-a_i t}e^{ib_i t}, \\
 \phi_i &= \phi_{0i}e^{-a_i t}e^{ib_i t}, \\
 \psi_i &= \psi_{0i}e^{-a_i t}e^{ib_i t}, i = 1 \sim N.
 \end{aligned} \tag{15}$$

N is defined by the practical requirement and the computational facility. Only some interesting solutions should be paid attention to, for example the solution i in this discussion to explain the tribological behavior. In formula (15) the item $e^{ib_i t}$, the virtual part of the solution where $j = \sqrt{-1}$, gives b_i which is the frequency of vibration (oil whirl). Meanwhile the item $e^{-a_i t}$, the real part of the solution, gives a_i which is the system damping of the system and predicts a speed of changing the amplitude of vibration concerning with the

solution. When a_i takes a negative value the amplitude of vibration will increase with time and the solution is then unstable. Only when it is positive the solution can be stable. Therefore $a_i = 0$ is a condition of threshold of instability of the system.

Back to formula (14), if the input vector $[p^x, p^y, M_x, M_k, N_k]^T$ is constant, most structure parameters are constant in a short period of observation except the eight stiffness and damping coefficients which are defined by the relative motion (the rotating speed of the rotor) and the load on the bearing. Under a given elevation distribution the change of system damping can be expressed in another form, the logarithmic decrement

$$\Delta = 2\pi a_i / b_i$$

Figure 18 gives two logarithmic decrement curves versus rotor rotating speed. The intersection point of each curve and abscissa ($\Delta = 0$) gives a margin of threshold of instability with related elevation distribution. The turbo-generator set in power plant must work under a speed of 3000 rpm. In Fig. 18 one can find that at a speed of 3000 rpm, before and after the change of elevation of 4# bearing (decreasing a value of 0.15 mm) and 7# bearing (increasing a value of 0.7 mm) the logarithmic decrement changes from 0.95 to -0.05. It implies that the change makes the system becoming not stable. Some turbo-generator set works normally in full output but during low output in middle night a half frequency vibration component emerges. Elevation distribution change might be an important cause of such phenomena. Many efforts have been given to understand it (Li, 2001).

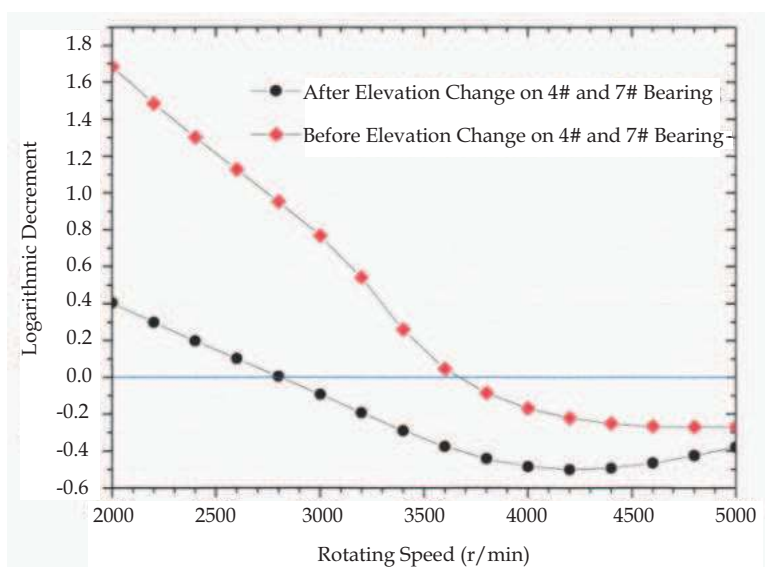


Fig. 18. Logarithmic decrement versus rotating speed for two different elevation distributions

5. Conclusion

The problems with tribology are problems of systems science and systems engineering. In a sense, without system there would be no tribology. A machine system is consisted of a

component system and a tribo-system from the view point of motion. The tribo-system is consisted of tribo-elements and some supporting auxiliary sub-systems abstracted from a machine system for studying behaviors on or between the interacting surfaces in relative motion, results of the behaviors and technology related to. The tribo-system together with the component system plays a motion guarantee function which keeps each part of the machine system with a definite motion. Tribology science and technology is very important in obtaining the best way (theory and application) to complete the motion guarantee function of tribo-systems.

Tribological behaviors are system dependent. The property of tribo-elements and then the systems containing tribo-elements are time dependent. The results of tribological behaviors are the results of mutual action and strong coupling of many behaviors of other disciplines under a tribological condition consisted of interacting surfaces in relating motion.

A state space method which is a combination of general systems theory with engineering systems analysis can be successfully applied to simulate the behaviors. Two examples are given to show how the system structure can be connected with the system behaviors via the state space method. With the state space method the structure is a carrier in realizing the mutual action and coupling. The structure can have a recoverable change and an irrecoverable change while the behaviors can be repeatable and unrepeatable in the simulation.

6. Acknowledgment

This study is supported by the National Science Foundation of China in a long period especially the key item 50935004/E05067. The author wishes to thank Professor H. Xiao for his kind help on proofreading the whole chapter, Dr. Z. S. Zhang on having the calculation results of the example 2, Dr. Z. N. Zhang on preparing the manuscript and Professor J. Mao, she read the first draft and pointed out some mistakes.

Appendix: Derivation of elements in the state space and output equations in example 1

In this example, the study will focus mainly on the skirt - bore tribo-pair of a cylinder - piston - conrod - crank system of an internal combustion engine.

As shown in Fig. 6 and in the following formulas, symbols Q - gas pressure on the top of piston, F - force or friction force, S - thrust force in total on piston skirt, T - torque moment load on the crankshaft, t - time, m - mass of a component, I - inertial moment of a component, P - center of small end pair of conrod, A - center of big end pair of conrod, O - center of crankshaft pair on casing, C - center of mass of piston assembly, R - center of mass of conrod, CR - center of mass of crankshaft, X, Y - coordinate directions, PIS - piston, PIN - wrist pin, SK - skirt, RN - piston ring package, R - conrod respectively and l - length of conrod, r - length of crank, jl - distance from A to R , hr - distance from O to CR .

Suppose that the influence of secondary motion of piston on the motion and equilibrium of conrod and crankshaft can be neglected. The following formulas yield the geometry and motion relationship between the conrod and crankshaft:

$$l \sin \phi = r \sin \theta, \quad \sin \phi = \frac{r}{l} \sin \theta, \quad \dot{\phi} = \frac{\dot{\theta} r \cos \theta}{l \cos \phi}, \quad \ddot{\phi} = \dot{\theta}^2 \left[\left(\frac{r \cos \theta}{l \cos \phi} \right)^2 \tan \phi - \frac{r \sin \theta}{l \cos \phi} \right] + \ddot{\theta} \frac{r \cos \theta}{l \cos \phi}$$

Following parameters are used for short in further discussion

$$m_p = m_{PIS} + m_{PIN}$$

$$W1 = I_{PIS} + m_{PIS} \left((C_B - C_A)^2 + C_P^2 \right) \quad (1A)$$

$$W1' = m_{PIS} (C_B - C_A) \quad (2A)$$

$$W2 = \left[\frac{(r \cos \theta)^2}{l \cos^3 \varphi} - r \cos \theta - r \sin \theta \tan \varphi \right] \quad (3A)$$

$$W2' = \left[\frac{j(r \cos \theta)^2}{l \cos^3 \varphi} - r \cos \theta - jr \sin \theta \tan \varphi \right] \quad (4A)$$

$$W2'' = \left[\left(\frac{r \cos \theta}{l \cos \varphi} \right)^2 \tan \varphi - \frac{r \sin \theta}{l \cos \varphi} \right] \quad (5A)$$

$$W3 = (r \cos \theta \tan \varphi - r \sin \theta) \quad (6A)$$

$$W3' = jr \cos \theta \tan \varphi - r \sin \theta \quad (7A)$$

$$W4 = \frac{I_R}{m_p} \left(\frac{W2''}{l \cos \varphi} \right) + \frac{m_R}{m_p} W2' j \tan \varphi + \frac{m_R}{m_p} r (1-j) j \sin \theta + W2 \tan \varphi \quad (8A)$$

$$W4' = \frac{I_R}{m_p} \left(\frac{r \cos \theta}{l \cos \varphi} \right) + \frac{m_R}{m_p} W3' j \tan \varphi - \frac{m_R}{m_p} r (1-j) j \cos \theta + W3 \tan \varphi \quad (9A)$$

$$I(\theta) = I_C + m_C h^2 r^2 + I_R \left(\frac{r \cos \theta}{l \cos \varphi} \right)^2 + m_p W3^2 + m_R \left[r^2 (1-j)^2 \cos^2 \theta + W3'^2 \right] \quad (10A)$$

$$I'(\theta) = 2I_R \left(\frac{r \cos \theta}{l \cos \varphi} \right) \left(\left(\frac{r \cos \theta}{l \cos \varphi} \right) \tan \varphi - \tan \theta \right) \\ + 2m_p W3 W2 - 2m_R r^2 (1-j)^2 \sin \theta \cos \theta + 2m_R W3' W2' \quad (11A)$$

$$g(\theta) = gr \left[m_p (\cos \theta \tan \varphi - \sin \theta) + m_R (j \cos \theta \tan \varphi - \sin \theta) + m_C h \sin \theta \right] \quad (12A)$$

$$Q(t, \theta) = (Q(t) - F_{SK} - F_{RN}) r (\cos \theta \tan \varphi - \sin \theta) \quad (13A)$$

$$W5 = -\frac{I'(\theta)}{2I(\theta)} \quad (14A)$$

$$W5' = -\frac{g(\theta) + Q(t, \theta) - T}{I(\theta)} \quad (15A)$$

$$W6 = -m_C \cdot h \cdot r \sin \theta + m_R \cdot r(1-j) \sin \theta - m_P \cdot W4 \quad (16A)$$

$$W6' = m_C \cdot h \cdot r \cos \theta - m_R \cdot r(1-j) \cos \theta - m_P \cdot W4' \quad (17A)$$

$$W7 = m_C \cdot h \cdot r \cos \theta + m_R \cdot W2' + m_P \cdot W2 \quad (18A)$$

$$W7' = m_C \cdot h \cdot r \sin \theta + m_R \cdot W3' + m_P \cdot W3 \quad (19A)$$

The displacements, the first derivatives and second derivatives of displacements of points P , R and O are given as follows

$$Y_p = r \cos \theta - l \cos \phi$$

$$\dot{Y}_p = -\dot{\theta} \cdot r (\sin \theta - \cos \theta \tan \phi)$$

$$\ddot{Y}_p = \dot{\theta}^2 \cdot W2 + \ddot{\theta} \cdot W3$$

X_p, \dot{X}_p and \ddot{X}_p will be given later because they need the values of secondary motion of piston which have to be obtained from the equations of equilibrium.

$$X_R = -r \sin \theta \cdot (1-j)$$

$$Y_R = r \cos \theta - l \cdot j \cos \phi$$

$$\dot{X}_R = -\dot{\theta} \cdot r(1-j) \cos \phi$$

$$\dot{Y}_R = -\dot{\theta}(r \sin \theta - j \cdot r \cos \theta \tan \phi)$$

$$\ddot{X}_R = \dot{\theta}^2 \cdot r(1-j) \sin \theta - \ddot{\theta} \cdot r(1-j) \cos \theta$$

$$\ddot{Y}_R = \dot{\theta}^2 \cdot W2' + \ddot{\theta} \cdot W3'$$

For

$$X_C = h \cdot r \sin \theta$$

$$Y_C = -h \cdot r \cos \theta$$

Then

$$\dot{X}_C = \dot{\theta} \cdot h \cdot r \cos \theta$$

$$\dot{Y}_C = \dot{\theta} \cdot h \cdot r \sin \theta$$

$$\ddot{X}_C = -\dot{\theta}^2 \cdot h \cdot r \sin \theta + \ddot{\theta} \cdot h \cdot r \cos \theta$$

$$\ddot{Y}_C = \dot{\theta}^2 \cdot h \cdot r \cos \theta + \ddot{\theta} \cdot h \cdot r \sin \theta$$

In the equilibrium analysis of the piston, conrod and crankshaft two other parameters are used for short also

$$FY = \frac{S}{m_p} - \frac{Q(t) - (F_{SK} + F_{RN}) + gm_p + gm_R j}{m_p} \tan \varphi \quad (20A)$$

$$FY' = Q(t) - (F_{SK} + F_{RN}) \quad (21A)$$

The equilibrium equations for the piston assembly, conrod and crankshaft can be written as follows

$$\Sigma F_{PX} = 0, F_{PX} + S - \ddot{X}_p m_p = 0 \quad (22A)$$

$$\Sigma F_{PY} = 0, F_{PY} + F_{SK} + F_{RN} - Q(t) - gm_p - \ddot{Y}_p m_p = 0 \quad (23A)$$

$$\Sigma M_p = 0, M + \ddot{X}_p W1' + \ddot{Y}_p m_{PIS} C_p - \ddot{\beta} W1 = 0 \quad (24A)$$

$$\Sigma F_{RX} = 0, -\ddot{X}_R m_R - F_{PX} + F_{AX} = 0 \quad (25A)$$

$$\Sigma F_{RY} = 0, -\ddot{Y}_R m_R - gm_R - F_{PY} + F_{AY} = 0 \quad (26A)$$

$$\Sigma M_R = 0, -\ddot{\theta} I_R - F_{BX} (1-j) l \cos \varphi - F_{BY} (1-j) l \sin \varphi - F_{AX} j l \cos \varphi - F_{AY} j l \sin \varphi = 0 \quad (27A)$$

$$\Sigma F_{CX} = 0, F_{OX} - F_{AX} - \ddot{X}_C m_C = 0 \quad (28A)$$

$$\Sigma F_{CY} = 0, F_{OY} - F_{AY} - gm_C - \ddot{Y}_C m_C = 0 \quad (29A)$$

$$\Sigma M_C = 0, -\ddot{\theta} I_C + T + F_{AX} (1+h) r \cos \theta + F_{AY} (1+h) r \sin \theta - F_{OX} h r \cos \theta - F_{OY} h r \sin \theta = 0 \quad (30A)$$

Considering that the study focuses mainly on the piston skirt - cylinder bore tribo-pair, parameters relative to the motion of the piston and the parameters concerning with motion condition input will be selected in the state vector $X = [X_p, \beta, \theta, \dot{X}_p, \dot{\beta}, \dot{\theta}]^T$, i.e.. Inputting (1A) - (21A) and equilibrium conditions (22A) - (30A) into formula (22A) yield

$$\ddot{X}_p = -\dot{\theta}^2 W4 - \ddot{\theta} W4' + FY \quad (31A)$$

Similarly yield

$$\ddot{\beta} = -\dot{\theta}^2 \frac{W4 \cdot W1' - m_{PIS} W2 \cdot C_p}{W1} - \ddot{\theta} \frac{W4' \cdot W1' - m_{PIS} W3 \cdot C_p}{W1} + \frac{FY \cdot W1' + M}{W1} \quad (32A)$$

$$\ddot{\theta} = \dot{\theta}^2 \cdot W5 + W5' \quad (33A)$$

Inputting formula (33A) into formulas (31A) and (32A) yield

$$\ddot{X}_p = -\dot{\theta}^2 (W4 + W5 W4') - W5' W4' + FY \quad (34A)$$

$$\ddot{\beta} = -\dot{\theta}^2 \left[\frac{W4W1' - W2m_{pIS}C_P + W5(W4'W1' - W3m_{pIS}C_P)}{W1} \right] - \frac{W5'(W4'W1' - W3m_{pIS}C_P)}{W1} + \frac{FYW1' + M}{W1} \tag{35A}$$

After reorganizing the state equation for the cylinder - piston - conrod - crank system can be derived as follows

$$\begin{bmatrix} X_p \\ \dot{X}_p \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{26} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{46} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} X_p \\ \dot{X}_p \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ U_2 \\ 0 \\ U_4 \\ 0 \\ U_6 \end{bmatrix}$$

where

$$\begin{aligned} A_{26} &= -\dot{\theta}(W4 + W5 \cdot W4') \\ A_{46} &= -\dot{\theta} \left[\frac{W4 \cdot W1' - m_{pIS}W2 \cdot C_P + W5(W4' \cdot W1' - m_{pIS}W3 \cdot C_P)}{W1} \right] \\ A_{66} &= W5 \\ U_2 &= -W5' \cdot W4' + FY \\ U_4 &= -\frac{W5'(W4' \cdot W1' - m_{pIS}W3 \cdot C_P)}{W1} + \frac{FY \cdot W1' + M}{W1} \\ U_6 &= W5' \end{aligned}$$

When the behaviors of piston are interesting in study, the output equations can be written as

$$\begin{bmatrix} \dot{\theta} \\ P_{LOSS} \\ X_p \\ \beta \\ F_{RHT} \\ F_{LFT} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & C_{16} \\ 0 & 0 & 0 & 0 & 0 & C_{26} \\ 0 & 0 & 0 & 0 & 0 & C_{36} \\ 0 & 0 & 0 & 0 & 0 & C_{46} \\ 0 & 0 & 0 & 0 & 0 & C_{56} \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} X_p \\ \dot{X}_p \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

where, $C_{16} = 1$ and $C_{26}, C_{36}, C_{46}, C_{56}, C_{66}$ concern the solution of Reynolds equation which governs the hydrodynamic lubrication behaviors between skirt and bore surfaces and cannot be presented explicitly. They will be computed numerically with a separate procedure before every integrating step from the value of elements in state vector obtained in last integrating.

If the forces transmitting in the pairs P , A and O are interesting the forces can be obtained with an equilibrium condition analysis for the piston on P , for the conrod on A and for the crankshaft on O . Replacing the first and second derivatives of displacements in formulas (22A) to (30A) and reordering yields

$$\begin{aligned}
F_{PX} &= -\dot{\theta}^2 m_P (W4 + W4' \cdot W5) - m_P (W4' \cdot W5' - FY) - S \\
F_{PY} &= \dot{\theta}^2 m_P (W2 + W3 \cdot W5) + m_P (W3 \cdot W5' + g) + FY' \\
F_{AX} &= \dot{\theta}^2 [-m_P (W4 + W4' \cdot W5) + m_R r (1-j) (\sin \theta - W5 \cos \theta)] \\
&\quad + [-m_P (W4' \cdot W5' - FY) - m_R r (1-j) W5' \cos \theta - S] \\
F_{AY} &= \dot{\theta}^2 [m_P (W2 + W3 \cdot W5) + m_R (W2' + W3 \cdot W5)] \\
&\quad + [m_P (W3 \cdot W5' + g) + m_R (W3' \cdot W5' + g) + FY'] \\
F_{OX} &= \dot{\theta}^2 (W6 + W5 \cdot W6') + W5' \cdot W6' + m_P \cdot FY - S \\
F_{OY} &= \dot{\theta}^2 (W7 + W5 \cdot W7') + W5' \cdot W7' + FY' + (m_P + m_R + m_C) \cdot g
\end{aligned}$$

Then the output matrix equation becomes

$$\begin{bmatrix} F_{PX} \\ F_{PY} \\ F_{AX} \\ F_{AY} \\ F_{OX} \\ F_{OY} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & C'_{16} \\ 0 & 0 & 0 & 0 & 0 & C'_{26} \\ 0 & 0 & 0 & 0 & 0 & C'_{36} \\ 0 & 0 & 0 & 0 & 0 & C'_{46} \\ 0 & 0 & 0 & 0 & 0 & C'_{56} \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix} \begin{bmatrix} X_P \\ \dot{X}_P \\ \beta \\ \dot{\beta} \\ \theta \\ \dot{\theta} \end{bmatrix}$$

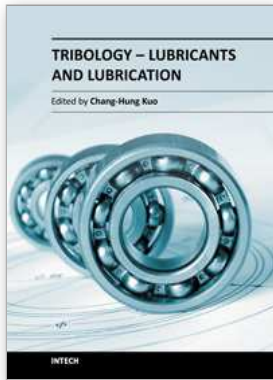
where

$$\begin{aligned}
C'_{16} &= -\dot{\theta} m_P (W4 + W4' \cdot W5) - [m_P (W4' \cdot W5' - FY) - S] / \dot{\theta} \\
C'_{26} &= \dot{\theta} m_P (W2 + W3 \cdot W5) + [m_P (W3 \cdot W5' + g) + FY'] / \dot{\theta} \\
C'_{36} &= \dot{\theta} [-m_P (W4 + W4' \cdot W5) + m_R r (1-j) (\sin \theta - W5 \cdot \cos \theta)] + \\
&\quad [-m_P (W4' \cdot W5' - FY) - m_R r (1-j) \cdot W5' \cdot \cos \theta - S] / \dot{\theta} \\
C'_{46} &= \dot{\theta} [m_P (W2 + W3 \cdot W5) + m_R (W2' + W3' \cdot W5)] + \\
&\quad [m_P (W3 \cdot W5' + g) + m_R (W3' \cdot W5' + g) + FY'] / \dot{\theta} \\
C'_{56} &= \dot{\theta} (W6 + W5 \cdot W6') + [W5' \cdot W6' + m_P \cdot FY - S] / \dot{\theta} \\
C'_{66} &= \dot{\theta} (W7 + W5 \cdot W7') + [W5' \cdot W7' + FY' + g(m_P + m_R + m_C)] / \dot{\theta}
\end{aligned}$$

7. References

- Chen, P. (1982). *State Space Methods and Application*. Publishing House of Electronics Industry, Beijing, China (In Chinese)
- Czichos H. (1978). *Tribology: A Systems Approach to the Science and Technology of Friction, Lubrication and Wear*, ISBN 978-0444416766, Elsevier
- Czichos H. The Principle of System Analysis and their Application to Tribology. *ASLE Trans*, Vol. 17, No. 4, (1974), pp. 300-306, ISSN 0569-8197

- Dai, Z.; Xue, Q. Exploration Systematical Analysis and Quantitative Modeling of Tribo-System Based on Entropy Concept. *Journal of Nanjing University of Aeronautics & Astronautics*, Vol. 25, No.6, (2003), pp.585 ~ 589 ISSN 1005-2615 (In Chinese)
- Dowson, D. (1979). *History of Tribology*. Wiley, ISBN 978-1860580703, London
- Fleischer G. *Systembetrachtungen zur Tribologie*. Wiss. Z. TH Magdeburg, Vol. 14, (1970), pp.415-420
- Ge, S.; Zhu, H. (2005). *Fractal in Tribology*. China Machine Press, ISBN 7-111-16014-2, Beijing, China (In Chinese)
- Glienicke, J. (1972). Theoretische und experimentelle Ermittlung der Systemdaempfung gleitgelagerter Rotoren und ihre Erhoehung durch eine aeussere Lagerdaempfung. Fortschritt Berichte der VDI Zeitschriften, Reihe 11, Nr. 13, VDI-Verlag GmbH, Duesseldorf
- Her Majesty's Stationery Office. (1966). *Lubrication (Tribology) Education and Research: A Report on the Present Position and Industry's Needs*. London
- Hori, Y. (2005). *Hydrodynamic Lubrication*. Springer, ISBN 978-4431278986
- Li, J. Analysis and Calculation of Influence of Steam Turbo-Generator Bearing Elevation Variation on Load. *North China Electric Power*, No. 11, (2001), pp.5 ~ 7, ISSN 1007-2691 (In Chinese)
- Ogata, K. (1970). *Modern Control Engineering*. Prentice-Hall, ISBN 9780135902325, New Jersey, USA
- Ogata, K. (1987). *Discrete-time Control Systems*. Prentice-Hall, ISBN 9780132161022, New Jersey, USA
- Pinkus, O.; Sternlicht, B. (1961). *Theory of Hydrodynamic Lubrication*. McGraw-Hill, New York, USA
- Salomon G. Application of Systems Thinking to Tribology. *ASLE Trans*, Vol.17, No.4, (1974), pp.295-299, ISSN 0569-8197
- Suh, N. (1990). *The Principle of Design*, Oxford University Press, ISBN 978-0195043457, USA
- The Panel Steering Committee for the Mechanical Engineering and Applied Mechanics Division of the NSF. Research Needs in Mechanical Systems-Report of the Select Panel on Research Goals and Priorities in Mechanical Systems. *Trans ASME, Journal of Tribology*, Vol. 1, (1984), pp. 2~25, ISSN 0022-2305
- Xie, Y. On the Systems Engineering of Tribo-Systems. *Chinese Journal of Mechanical Engineering* (English Edition), No 2, (1996), pp. 89-99, ISSN 1000-9345
- Xie, Y. On the System Theory and Modeling of Tribo-Systems. *Tribology*, Vol.30, No.1, (2010), pp.1-8, ISSN 1004-0595 (In Chinese)
- Xie, Y. *On the Tribological Database*. *Lubrication Engineering*, Vol.5, (1986), pp. 1-7, ISSN 0254-0150 (In Chinese)
- Xie, Y. Three Axioms in Tribology. *Tribology*, Vol.21, No.3, (2001), pp.161-166, ISSN 1004-0595 (In Chinese)
- Xie, Y.; Zhang, S. (Eds.). (2009). *Status and Developing Strategy Investigation on Tribology Science and Engineering Application: A Consulting Report of the Chinese Academy of Engineering (CAE)*. Higher Education Press, ISBN 978-7-04-026378-7, Beijing, China (In Chinese)
- Xu, S. (2007). *Digital Analysis and Methods*. China Machine Press, ISBN 978-7-111-20668-2, Beijing, China (In Chinese)



Tribology - Lubricants and Lubrication

Edited by Dr. Chang-Hung Kuo

ISBN 978-953-307-371-2

Hard cover, 320 pages

Publisher InTech

Published online 12, October, 2011

Published in print edition October, 2011

In the past decades, significant advances in tribology have been made as engineers strive to develop more reliable and high performance products. The advancements are mainly driven by the evolution of computational techniques and experimental characterization that leads to a thorough understanding of tribological process on both macro- and microscales. The purpose of this book is to present recent progress of researchers on the hydrodynamic lubrication analysis and the lubrication tests for biodegradable lubricants.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Xie You-Bai (2011). Theory of Tribo-Systems, Tribology - Lubricants and Lubrication, Dr. Chang-Hung Kuo (Ed.), ISBN: 978-953-307-371-2, InTech, Available from: <http://www.intechopen.com/books/tribology-lubricants-and-lubrication/theory-of-tribo-systems>

INTECH

open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821

© 2011 The Author(s). Licensee IntechOpen. This is an open access article distributed under the terms of the [Creative Commons Attribution 3.0 License](#), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.