

# Response Surface Designs Robust against Nuisance Factors

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## Abstract

This paper discusses an algorithmic approach to constructing trend-free and orthogonally-blocked response surface designs. The constructed designs have the main effects, 2-factor interactions and second-order effects being orthogonal or near-orthogonal to the nuisance factors such as the time-trend or the blocking factors. The paper also provides a catalogue of (near-) trend-free Box–Behnkens designs and orthogonally blocked Box–Behnkens designs arranged in rows and columns.

**Keywords:** Box–Behnken designs, D-optimality, Interchange algorithm, Orthogonal blocking, Trend-free designs

## 1. Introduction

Consider an experiment to study the effect of processing time, temperature and shear stress caused by pumping on the quality of skim milk powder. The milk for this experiment is blended and stored at 4°C and is used over a week for a series of experimental runs. As the milk quality deteriorates over the week, it is desirable for the scientist to have a design whose runs are in a particular order such that the main effects (MEs), 2-factor interactions (2FIs) and second-order effects (SOEs) are orthogonal or near-orthogonal to the time trend.

Box et al. [1], p. 486 discussed an experiment using the Box–Behnken design or BBD [2] to study the influence of four factors on nylon flake blender efficiency. The four factors are (1) particle size, (2) screw lead length, (3) screw rotation and (4) blending time. The design has 27 runs in three blocks of nine runs each. Let us assume there is a need to add an additional factor, i.e. supplier in addition to the existing blocking factor, say operator and find a design which can accommodate the new blocking factor.

The two mentioned experiments emphasise the need for a special class of response surface designs (RSDs) and experiment designs in general which are robust against nuisance factors such as the time trend and the heterogeneous environment. Particular attention will be given to the BBDs since BBDs are the most widely used 3-level designs.

## 2. Measure of goodness of a (near-) trend-free design or orthogonally blocked design

Consider the following second-order model for an  $n$ -run design with  $v$  time trend columns (or blocking factor columns)  $z_1, \dots, z_v$  and  $m$  factors  $x_1, \dots, x_m$ , of which  $m_3$  factors are at 3-level and the rest at 2-level:

$$y_u = \delta_1 z_{1u} + \dots + \delta_v z_{vu} + \beta_0 + \sum_{i=1}^{m_3} \beta_{ii} x_{iu}^2 + \sum_{i=1}^m \beta_i x_{iu} + \sum_{i=1}^{m-1} \sum_{j=i+1}^m \beta_{ij} x_{iu} x_{ju} + \epsilon_u \quad (1)$$

where  $y_u$  ( $u = 1, \dots, n$ ) is the response value of the  $u$ th run,  $z_w$ 's are the nuisance columns/variables and  $\epsilon_u$  is a random error associated with the  $u$ th run. If the nuisance columns  $z_w$ 's are the time trend ones, there will be two columns, one for linear trend and one quadratic. The linear trend column (first column) is created by scaling a column of numbers  $(1, \dots, n)'$  by subtracting each value from the column mean and then dividing the resulting number by the largest one. The quadratic trend column (second column) is created by scaling a column obtained by squaring each element of the linear trend column.

If the nuisance columns  $z_w$ 's are associated with the blocking factors, there will be  $v$  ( $= \sum_{i=1}^r (b_i - 1)$ ) columns where  $r$  is the number of blocking factors and  $b_i$  is the number of blocks/categories for each blocking factor. Before standardisation by subtracting the values of each column by the column mean, these columns are dummy variables, which take value 1 if the  $u$ th run is in the  $w$ th block and zero otherwise (see [3], Section 8 and [4], Chapter 8).

Eq. (1) can also be written in matrix form as:

$$\mathbf{y} = \mathbf{Z}\boldsymbol{\delta} + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \quad (2)$$

where  $\mathbf{y}$  is an  $n \times 1$  response vector,  $\mathbf{Z}$  a matrix of size  $n \times v$  containing  $v$   $z_w$  columns in (1),  $\boldsymbol{\delta}$  a  $v \times 1$  column vector representing nuisance effects,  $\mathbf{X}$  is the expanded design matrix of size  $n \times p$ ,  $\boldsymbol{\beta}$  a  $p \times 1$  column vector of parameters to be estimated, and  $\boldsymbol{\epsilon}$  an  $n \times 1$  column vector of random errors. The least square solution for the unknown parameters  $\boldsymbol{\delta}$  and  $\boldsymbol{\beta}$  in (2) is thus the solution of the following equation:

$$\begin{bmatrix} \mathbf{Z}' \\ \mathbf{X}' \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{Z}'\mathbf{Z} & \mathbf{Z}'\mathbf{X} \\ \mathbf{X}'\mathbf{Z} & \mathbf{X}'\mathbf{X} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\delta}} \\ \hat{\boldsymbol{\beta}} \end{bmatrix} \quad (3)$$

When the condition

$$\mathbf{Z}'\mathbf{X} = \mathbf{0} \quad (4)$$

is satisfied, it can be seen that the solution for  $\boldsymbol{\beta}$  from (3) will be the same as the one from the equation  $\mathbf{X}'\mathbf{y} = \mathbf{X}'\mathbf{X}\hat{\boldsymbol{\beta}}$ , i.e. the equation for a model without the nuisance columns. The condition in (4) is called the time trend-free condition or orthogonal block condition.

To measure how good of a (near-) trend-free design or orthogonally blocked design is, we use the following fraction:

$$(|\mathbf{X}'\mathbf{X}| / (|\mathbf{Z}'\mathbf{Z}| |\mathbf{X}'\mathbf{X}|))^{1/p} \quad (5)$$

where  $\mathbf{X} = [\mathbf{Z} \ \mathbf{X}]$  includes the nuisance factors and  $p$  is the number of parameters of the model to be estimated (i.e. the number of columns of  $\mathbf{X}$ ). A well known result of Fischer [5] states that given a positive definite hermitian matrix  $\mathbf{G}$  of the form:

$$\mathbf{G} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D} \end{bmatrix} \quad (6)$$

then  $|\mathbf{G}| \leq |\mathbf{A}||\mathbf{D}|$ , and the equality holds if and only if  $\mathbf{B} = \mathbf{0}$ . It follows from this inequality that:

$$|\mathbf{X}'\mathbf{X}| \leq |\mathbf{Z}'\mathbf{Z}||\mathbf{X}'\mathbf{X}| \quad (7)$$

which implies the measure in (5) is less than or equal to 1, and it becomes 1 if and only if  $\mathbf{Z}'\mathbf{X} = \mathbf{0}$ .

### 3. An algorithm to attain the orthogonality condition $\mathbf{Z}'\mathbf{X} = \mathbf{0}$

Let  $\mathbf{x}'_i$  and  $\mathbf{x}'_u$  be two rows of  $\mathbf{X}$  and  $\mathbf{z}'_i$  and  $\mathbf{z}'_u$  be the corresponding rows of  $\mathbf{Z}$ . Swapping the  $i$ th and  $u$ th rows of  $\mathbf{X}$  is the same as adding the following matrix to  $\mathbf{Z}'\mathbf{X}$ :

$$-(\mathbf{z}_i - \mathbf{z}_u)(\mathbf{x}_i - \mathbf{x}_u)'. \quad (8)$$

We use this matrix result to develop an algorithm to achieve the condition  $\mathbf{Z}'\mathbf{X} = \mathbf{0}$ , i.e. for constructing (near-) trend-free designs and orthogonally blocked designs. This algorithm has two main steps:

1. Construct the nuisance matrix  $\mathbf{Z}$  and the expanded design matrix  $\mathbf{X}$ . Randomly assign each of the  $n$  rows of  $\mathbf{X}$  to each of the  $n$  rows of  $\mathbf{Z}$ . Calculate  $f$ , the sum of squares of the elements of  $\mathbf{Z}'\mathbf{X}$ .
2. Repeat searching for a pair of rows of  $\mathbf{X}$  such that the swap of the positions of these two rows results in the biggest reduction in  $f$ . If the search is successful, swap their positions, update  $f$  and  $\mathbf{Z}'\mathbf{X}$ . This step is repeated until  $f = 0$  or until  $f$  cannot be reduced further.

**Table 1 (a)** and **1 (b)** display the  $\mathbf{X}(= (\mathbf{Z} \ \mathbf{X}))$  matrices of a BBD for three factors in 15 runs: (a) ordered in the presence of both linear and quadratic trends; (b) arranged in three blocks of five runs each. This example shows how the  $\mathbf{Z}$  matrices are constructed. The  $\mathbf{X}$  matrix contains a column of 1's representing the intercept, followed by three columns representing SOEs, three columns representing the MEs and three columns representing 2FIs.

#### Remarks

1. The above two steps make up one computer try. Thousands of tries are required for each design parameters and the one with the smallest  $f$  will be chosen. Among designs with the same  $f$ , the one with the smallest fraction calculated in (5) will be chosen.
2. For a factorial or fractional factorial design (FFD), the orthogonality between MEs and nuisance variables is considered more important than the orthogonality between 2FIs and nuisance variables. For an RSD, the orthogonality between the MEs (and 2FIs) and nuisance variables are considered more important than the orthogonality between SOEs and nuisance variables. In these situations, partition  $\mathbf{X}$  as  $(\mathbf{X}_1 \ \mathbf{X}_2 \ \dots)$  where  $\mathbf{X}_2$  is associated with the more important effects. Similarly, partition  $\mathbf{Z}'\mathbf{X}$  as  $(\mathbf{Z}'\mathbf{X}_1 \ \mathbf{Z}'\mathbf{X}_2 \ \dots)$ . Let  $g$  be the sum of squares of the elements of  $\mathbf{Z}'\mathbf{X}_2$  and  $f$  the sum of squares of the elements of  $\mathbf{Z}'\mathbf{X}$  as defined previously. Step 2 of our algorithm now becomes

repeating searching for a pair of runs such that swapping their run positions results in the biggest reduction in  $g$  (or  $f$  if  $g$  cannot be reduced further). If the search is successful, swap their positions, update  $f$  and  $Z'X$ . This step is repeated until  $f = 0$  or until both  $g$  and  $f$  cannot be reduced further.

$z_1$	$z_2$	$x_0$	$x_1^2$	$x_2^2$	$x_3^2$	$x_1$	$x_2$	$x_3$	$x_1x_2$	$x_1x_3$	$x_2x_3$
-1	1	1	0	0	0	0	0	0	0	0	0
-0.86	0.57	1	0	1	1	0	-1	1	0	0	-1
-0.71	0.21	1	1	0	1	1	0	-1	0	-1	0
-0.57	-0.09	1	1	0	1	-1	0	-1	0	1	0
-0.43	-0.32	1	0	1	1	0	1	1	0	0	1
-0.29	-0.48	1	1	1	0	-1	1	0	-1	0	0
-0.14	-0.58	1	1	1	0	1	1	0	1	0	0
0	-0.61	1	0	0	0	0	0	0	0	0	0
0.14	-0.58	1	1	1	0	-1	-1	0	1	0	0
0.29	-0.48	1	1	1	0	1	-1	0	-1	0	0
0.43	-0.32	1	0	1	1	0	-1	-1	0	0	1
0.57	-0.09	1	1	0	1	1	0	1	0	1	0
0.71	0.21	1	1	0	1	-1	0	1	0	-1	0
0.86	0.57	1	0	1	1	0	1	-1	0	0	-1
1	1	1	0	0	0	0	0	0	0	0	0

(a)

$z_1$	$z_2$	$x_0$	$x_1^2$	$x_2^2$	$x_3^2$	$x_1$	$x_2$	$x_3$	$x_1x_2$	$x_1x_3$	$x_2x_3$
0.67	-0.33	1	1	0	1	-1	0	1	0	-1	0
0.67	-0.33	1	1	0	1	1	0	1	0	1	0
0.67	-0.33	1	0	0	0	0	0	0	0	0	0
0.67	-0.33	1	0	1	1	0	1	-1	0	0	-1
0.67	-0.33	1	0	1	1	0	-1	-1	0	0	1
-0.33	0.67	1	0	0	0	0	0	0	0	0	0
-0.33	0.67	1	1	0	1	-1	0	-1	0	1	0
-0.33	0.67	1	1	0	1	1	0	-1	0	-1	0
-0.33	0.67	1	0	1	1	0	-1	1	0	0	-1
-0.33	0.67	1	0	1	1	0	1	1	0	0	1
-0.33	-0.33	1	1	1	0	1	-1	0	-1	0	0
-0.33	-0.33	1	1	1	0	-1	1	0	-1	0	0
-0.33	-0.33	1	1	1	0	-1	-1	0	1	0	0
-0.33	-0.33	1	1	1	0	1	1	0	1	0	0
-0.33	-0.33	1	0	0	0	0	0	0	0	0	0

(b)

**Table 1.** The  $X (= Z'X)$  matrices of a BBD for three factors in 15 runs (a) ordered in the presence of both linear and quadratic trends; (b) arranged in three blocks of five runs each.

The designs whose  $X$  matrices are in **Table 1 (a)** and **1 (b)** were constructed by this approach. In **Table 1 (a)**, the  $X_2$  matrix is associated with the MEs and in **Table 1 (b)**, the  $X_2$  matrix is associated with the MEs and 2FIs. Readers can verify that  $Z'X_2 = \mathbf{0}$ .

The algorithm we describe in this Section is closely aliased to the ones of Nguyen [6] and Nguyen [7]. As such, it does not require matrix inversions and therefore is much faster than the ones by other authors (See e.g. [8–11]).

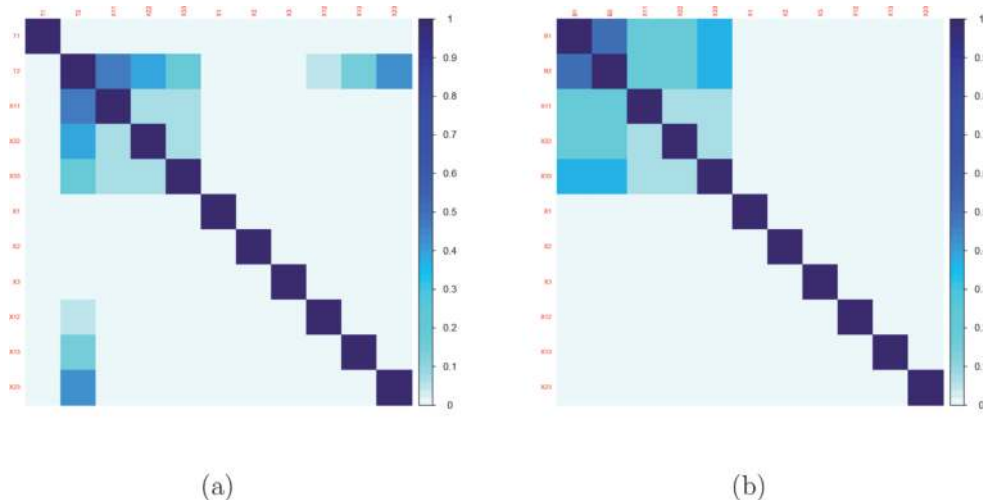
#### 4. BBDs robust against time trend and blocking factors

The algorithm in the previous Section has been used to construct BBDs for 3–7 factors which are (near-) trend-free and orthogonally blocked with two blocking factors (i.e. rows and columns).

We use the correlation cell plots (CCPs) proposed by Jones & Nachtsheim [12] to display the magnitude of the correlation between the SOEs, MEs and 2FIs with the columns/variables representing nuisance factors. The colour of each cell in these plots ranges from white (no correlation) to dark (correlation of 1 or close to 1). **Figures 1 (a)** and **1 (b)** show the CCPs of the BBDs whose  $X$  matrices are **Table 1 (a)** and **1 (b)**. In **Figure 1 (a)**, it can be seen that the MEs are orthogonal to both the linear and quadratic trends. In **Figure 1 (b)**, it can be seen that both the MEs and 2FIs are orthogonal to the block effects. For both Figures, the SOEs are slightly correlated with the nuisance effects.

Appendix 1 tabulates the BBDs which are near- trend-free. These BBDs are for three factors in 15 runs, four factors in 27 runs, five factors in 46 runs and six factors in 54 runs. The BBD for seven factors in 62 runs can be found in the link given in the next Section. These BBDs have the MEs being orthogonal to the time trends and have CCPs very similar to the one in **Figure 1 (a)**. Let TF (time factor) be the fraction computed in (5) for these designs. The TF values of these five designs are 0.91, 0.959, 0.986, 0.974 and 0.976 respectively.

Appendix 2 tabulates the BBDs which are near-orthogonally blocked. These BBDs are for three factors in 16 runs arranged in two rows and two columns, four factors in 28 runs arranged in two rows and two columns, five factors in 48 runs arranged in two rows and three columns, and six factors in 54 runs arranged in two rows in three columns. The BBD for seven factors in 60 runs arranged in two rows



**Figure 1.** CCPs of the BBDs whose  $X$  matrices are **Table 1 (a)** and **1 (b)**.

and three columns can be found in the link given in the next Section. With the exception of the first BBD, these BBDs have both the MEs and 2FIs being orthogonal to the block effects and have CCPs very similar to the one in **Figure 1 (b)**. Let BF (blocking factor) be the fraction computed in (5) for these designs. The BF values of these five designs are 0.944, 1, 0.992, 0.927 and 0.962 respectively. Note that for the 4-factor BBD, all effects are orthogonally blocked.

### 5. Conclusion

This paper describes an algorithmic approach to arrange the runs of an experimental design in general and an RSD in particular so that it is robust against nuisance factors such as time trend and blocking factors. Designs constructed by this approach can supplement the existing catalogue of designs in the literature. Although 3-level designs are used in this paper to illustrate our blocking approach, ours can also be used with 2-level designs (the factorial and fractional factorial designs) or mixed-level designs [13–15] or mixture designs [16].

The link of the supplemental material which contains the Java implementation of the algorithm in Section 3 and additional examples is: <https://drive.google.com/drive/folders/14g7E3I4F8KIL2rcZMovlvJmsaM7iC7pJ?usp=sharing>.

### Appendix 1: BBDs for 3–6 factors with the trend-free main effects

-1	1	0	0	0	-1	1	-1	0	-1	0	0	-1	1	0	0	0	0	0	0	
-0.857	0.571	-1	0	-1	-0.956	0.867	1	0	-1	0	0	-0.962	0.887	-1	1	0	0	0	-1	
-0.714	0.209	0	1	1	-0.911	0.739	0	0	0	1	1	-0.925	0.778	1	1	0	0	0	1	
-0.571	-0.088	0	-1	1	-0.867	0.618	0	0	0	0	0	-0.887	0.673	0	0	0	1	-1	1	
-0.429	-0.319	1	0	-1	-0.822	0.503	0	1	0	0	-1	-0.849	0.573	0	-1	0	0	1	-1	
-0.286	-0.484	1	-1	0	-0.778	0.394	0	-1	0	-1	0	-0.811	0.478	1	0	0	-1	1	0	
-0.143	-0.582	1	1	0	-0.733	0.291	-1	0	1	0	0	-0.774	0.386	0	-1	-1	-1	0	0	
0	-0.615	0	0	0	-0.689	0.194	1	0	0	0	-1	-0.736	0.299	-1	0	0	-1	1	0	
0.143	-0.582	-1	-1	0	-0.644	0.103	0	0	0	0	0	-0.698	0.216	0	0	0	1	-1	-1	
0.286	-0.484	-1	1	0	-0.6	0.018	0	0	1	1	0	-0.66	0.138	1	0	-1	0	-1	0	
0.429	-0.319	-1	0	1	-0.556	-0.061	0	0	1	0	1	-0.623	0.064	-1	0	1	0	-1	0	
0.571	-0.088	0	1	-1	-0.511	-0.133	1	-1	0	0	0	-0.585	-0.006	0	0	0	0	0	0	
0.714	0.209	0	-1	-1	-0.467	-0.2	0	0	0	-1	1	-0.547	-0.071	0	-1	0	0	1	1	
0.857	0.571	1	0	1	-0.422	-0.261	0	0	0	0	0	-0.509	-0.132	1	-1	0	1	0	0	
1	1	0	0	0	-0.378	-0.315	0	-1	0	0	-1	-0.472	-0.189	0	1	-1	0	-1	0	
					-0.333	-0.364	0	1	1	0	0	-0.434	-0.241	1	-1	1	0	0	0	
					-0.289	-0.406	-1	0	0	-1	0	-0.396	-0.289	-1	0	0	-1	0	1	
-1	1	0	-1	-1	0	-0.244	-0.442	1	1	0	0	0	-0.358	-0.332	-1	0	0	1	1	0
-0.923	0.769	1	1	0	0	-0.2	-0.473	-1	1	0	0	0	-0.321	-0.372	0	1	1	0	1	0
-0.846	0.557	1	0	0	-1	-0.156	-0.497	0	0	1	-1	0	-0.283	-0.406	0	-1	0	0	-1	-1
-0.769	0.363	0	0	1	1	-0.111	-0.515	0	0	0	0	0	-0.245	-0.437	-1	1	0	0	0	1
-0.692	0.188	-1	1	0	0	-0.067	-0.527	0	-1	0	1	0	-0.208	-0.463	0	1	1	-1	0	0
-0.615	0.031	-1	0	1	0	-0.022	-0.533	0	0	-1	1	0	-0.17	-0.485	0	0	1	1	0	-1
-0.538	-0.108	0	0	-1	1	0.022	-0.533	0	0	-1	0	-1	-0.132	-0.502	1	1	0	0	0	-1

-0.462	-0.228	0	0	1	-1	0.067	-0.527	0	1	-1	0	0	-0.094	-0.515	-1	-1	0	1	0	0
-0.385	-0.329	0	0	0	0	0.111	-0.515	0	0	0	0	0	-0.057	-0.524	0	0	0	0	0	0
-0.308	-0.412	-1	-1	0	0	0.156	-0.497	1	0	0	0	1	-0.019	-0.528	0	0	-1	0	1	1
-0.231	-0.477	-1	0	-1	0	0.2	-0.473	-1	0	0	0	1	0.019	-0.528	-1	0	-1	0	-1	0
-0.154	-0.523	0	0	0	0	0.244	-0.442	0	0	1	0	-1	0.057	-0.524	0	0	1	1	0	1
-0.077	-0.551	0	-1	0	-1	0.289	-0.406	-1	-1	0	0	0	0.094	-0.515	1	1	0	1	0	0
0	-0.56	0	-1	0	1	0.333	-0.364	0	0	-1	-1	0	0.132	-0.502	1	0	0	-1	0	1
0.077	-0.551	1	0	0	1	0.378	-0.315	0	0	-1	0	1	0.17	-0.485	0	0	0	0	0	0
0.154	-0.523	0	1	0	-1	0.422	-0.261	0	0	0	-1	-1	0.208	-0.463	0	0	0	0	0	0
0.231	-0.477	0	1	1	0	0.467	-0.2	0	1	0	1	0	0.245	-0.437	1	0	0	-1	0	-1
0.308	-0.412	1	-1	0	0	0.511	-0.133	-1	0	0	1	0	0.283	-0.406	-1	0	0	-1	0	-1
0.385	-0.329	1	0	-1	0	0.556	-0.061	0	-1	-1	0	0	0.321	-0.372	0	1	1	0	-1	0
0.462	-0.228	0	1	-1	0	0.6	0.018	1	0	0	1	0	0.358	-0.332	0	-1	1	-1	0	0
0.538	-0.108	0	1	0	1	0.644	0.103	1	0	1	0	0	0.396	-0.289	0	0	-1	0	1	-1
0.615	0.031	-1	0	0	-1	0.689	0.194	0	0	0	1	-1	0.434	-0.241	0	-1	0	0	-1	1
0.692	0.188	0	0	0	0	0.733	0.291	1	0	0	-1	0	0.472	-0.189	0	0	1	0	1	-1
0.769	0.363	1	0	1	0	0.778	0.394	0	-1	0	0	1	0.509	-0.132	0	0	-1	1	0	-1
0.846	0.557	0	-1	1	0	0.822	0.503	0	-1	1	0	0	0.547	-0.071	0	0	-1	1	0	1
0.923	0.769	0	0	-1	-1	0.867	0.618	0	1	0	-1	0	0.585	-0.006	1	-1	-1	0	0	0
1	1	-1	0	0	1	0.911	0.739	0	1	0	0	1	0.623	0.064	-1	-1	-1	0	0	0
						0.956	0.867	0	0	0	0	0	0.66	0.138	0	0	0	-1	-1	1
						1	1	-1	0	0	0	-1	0.698	0.216	0	1	-1	0	1	0
													0.736	0.299	0	1	-1	-1	0	0
													0.774	0.386	0	0	1	0	1	1
													0.811	0.478	0	0	0	-1	-1	-1
													0.849	0.573	0	0	0	0	0	0
													0.887	0.673	1	0	0	1	1	0
													0.925	0.778	-1	1	0	1	0	0
													0.962	0.887	1	0	1	0	-1	0
													1	1	-1	-1	1	0	0	0

## Appendix 2: BBDs for 3–6 factors arranged in rows and columns

1	1	-1	-1	0	1	1	1	-1	0	0	1	0	1	1	0	0	1	0	1	1
1	1	1	0	-1	1	1	0	0	1	0	1	1	1	1	-1	-1	0	0	0	0
1	1	0	0	0	1	1	0	-1	0	0	-1	1	1	1	1	0	0	0	0	1
1	1	0	1	1	1	1	0	0	0	0	0	1	1	1	1	0	0	0	0	-1
1	2	-1	0	1	1	1	0	0	1	0	-1	1	1	0	0	1	0	1	0	-1
1	2	0	1	-1	1	1	0	0	0	0	0	1	1	1	0	1	0	-1	0	0
1	2	1	-1	0	1	1	0	1	0	-1	0	1	1	0	0	0	0	0	0	0
1	2	0	0	0	1	1	1	0	-1	0	0	1	1	1	-1	1	0	0	0	0

2	1	0	-1	1	1	2	0	1	0	0	-1	1	1	-1	0	1	0	-1	0	
2	1	-1	0	-1	1	2	0	0	0	0	0	1	2	0	1	1	0	-1	0	
2	1	0	0	0	1	2	0	0	-1	-1	0	1	2	0	-1	0	0	-1	-1	
2	1	1	1	0	1	2	1	0	0	0	-1	1	2	0	1	-1	-1	0	0	
2	2	1	0	1	1	2	0	-1	0	-1	0	1	2	0	-1	-1	-1	0	0	
2	2	0	0	0	1	2	0	0	-1	1	0	1	2	0	-1	0	0	-1	1	
2	2	0	-1	-1	1	2	0	0	0	0	0	1	2	-1	-1	0	1	0	0	
2	2	-1	1	0	1	2	1	0	0	0	1	1	2	0	0	0	0	0	0	
					1	3	-1	0	-1	0	0	1	2	0	1	-1	0	-1	0	
					1	3	0	-1	1	0	0	1	2	-1	1	0	1	0	0	
1	1	0	0	-1	-1	1	3	0	0	0	1	1	1	3	-1	0	0	-1	0	1
1	1	-1	0	0	-1	1	3	-1	-1	0	0	0	1	3	0	1	-1	0	1	0
1	1	1	1	0	0	1	3	0	0	0	-1	1	1	3	0	0	-1	1	0	1
1	1	-1	0	0	1	1	3	1	0	0	1	0	1	3	-1	0	0	-1	0	-1
1	1	0	-1	1	0	1	3	0	1	1	0	0	1	3	0	-1	0	0	1	-1
1	1	1	-1	0	0	1	3	-1	1	0	0	0	1	3	0	0	-1	1	0	-1
1	1	0	1	0	1	2	1	1	0	1	0	0	1	3	0	1	1	0	1	0
1	2	0	1	1	0	2	1	0	1	-1	0	0	1	3	0	0	0	0	0	0
1	2	1	0	-1	0	2	1	0	-1	0	0	1	1	3	0	-1	0	0	1	1
1	2	0	0	0	0	2	1	0	0	0	0	0	2	1	-1	-1	-1	0	0	0
1	2	0	0	1	-1	2	1	0	0	0	0	0	2	1	0	0	0	0	0	0
1	2	0	-1	0	1	2	1	0	1	0	1	0	2	1	0	0	-1	0	1	1
1	2	0	0	0	0	2	1	0	-1	-1	0	0	2	1	0	0	-1	0	1	-1
1	2	-1	0	-1	0	2	1	-1	0	0	-1	0	2	1	-1	0	-1	0	-1	0
2	1	1	0	1	0	2	2	0	0	0	0	0	2	1	-1	1	0	0	0	1
2	1	-1	0	1	0	2	2	-1	0	0	0	-1	2	1	-1	-1	1	0	0	0
2	1	0	0	-1	1	2	2	-1	0	0	0	1	2	1	-1	1	0	0	0	-1
2	1	0	-1	-1	0	2	2	0	0	0	0	0	2	1	1	0	-1	0	-1	0
2	1	0	1	0	-1	2	2	0	-1	0	1	0	2	2	1	0	0	1	1	0
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2	1	0	0	0	0	2	2	0	0	1	-1	0	2	2	-1	0	0	1	1	0
2	2	0	-1	0	-1	2	2	0	0	1	1	0	2	2	1	1	0	1	0	0
2	2	0	1	-1	0	2	3	0	0	-1	0	-1	2	2	0	0	0	0	0	0
2	2	1	0	0	-1	2	3	0	0	-1	0	1	2	2	1	-1	0	1	0	0
2	2	-1	1	0	0	2	3	1	-1	0	0	0	2	2	0	1	1	-1	0	0
2	2	1	0	0	1	2	3	-1	0	1	0	0	2	2	-1	0	0	-1	1	0
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2	2	0	0	1	1	2	3	0	0	0	-1	-1	2	3	1	0	0	-1	0	1
					2	3	1	1	0	0	0	0	2	3	0	0	0	1	-1	1
					2	3	0	0	0	1	-1	0	2	3	0	0	1	1	0	1
													2	3	0	0	1	1	0	-1



2	3	0	0	0	-1	-1	1
2	3	0	0	0	0	0	0
2	3	0	0	0	1	-1	-1
2	3	0	0	0	-1	-1	-1
2	3	1	0	0	-1	0	-1

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