
Planting and Harvesting Decisions: A Review and Extension to the Case of Two Alternative Tree Species

Skander Ben Abdallah

Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/intechopen.72645>

Abstract

When to optimally harvest even-aged trees is a dominant concern in forest economics. In the literature, it was considered when the land is available for just one harvest (Wicksell setting) or multiple harvests (Faustmann setting). In this chapter, we will review the rotation lengths under both settings and focus on the impact of timber price variations on planting and harvesting decisions when one or two tree species are available for planting. When timber prices are rising but a species must be replaced by a more attractive species, rotation lengths before the switch are either constant and equal to the Faustmann's rotation, increasingly higher than the Faustmann's rotation, or decreasingly lower than the Faustmann's rotation. If timber prices are stochastic, forest managers prefer longer rotation lengths when the switch to the alternative tree species is about to occur as a means of postponing decision-making and waiting for more information related to timber prices. The possibility of replacing the planted species with an alternative species increases the land value, especially when timber prices are uncertain.

Keywords: forestry rotation, land value, alternative species, alternative land use

1. Introduction

A dominant research topic and practical concern in forest economics have been the age at which even-aged trees should be optimally harvested and how this age, known as the rotation length, depends on timber price levels or price change rates. In the literature, it was considered under two settings: when the land can be planted just one time, referred to as the Wicksell's rotation, and when the land can be planted an unlimited number of times, referred to as the Faustmann's rotation.

The rotation problem was first resolved using some simplifying assumptions, such as constant timber prices and the availability of just one species for planting. The rotation equation that

allows for the calculation of the optimal tree rotation length requires comparing the net marginal benefits of letting trees grow to the opportunity cost of the current harvest augmented by the land opportunity cost. The land's value stems from all future harvests or other alternative agricultural or non-agricultural uses.

Gradually, certain assumptions were relaxed. For instance, a generalized version of the Faustmann formula applies when stumpage prices and costs are known time functions in [1]. Rising timber prices were considered in [2] and changes in stumpage prices and costs in [3]. Others considered stochastic timber prices as in [4–9].

The presence of another alternative tree species was considered, for instance, in [10], in which an alternative species is available for afforestation. Two mutually exclusive tree species exist in [11, 12], but choosing one species leads to permanently losing the other. More recently, forest rotation was studied in situations in which two tree species existed with rising timber prices [13] or stochastic timber prices [14].

The rest of this chapter is organized as follows. In Section 2, we will first review the basic equations that allow for determination of the rotation lengths under Wicksell and Faustmann settings. We will then focus on the impact of timber price variations on rotation length and planting decisions.

In Section 3, we will present and discuss the model of two tree species as in [13, 14], in which two alternative tree species are available. Both species have two different stochastic timber prices that are assumed to follow a geometric Brownian motion and two deterministic age-dependent volume growth functions. We, therefore, assume the existence of two alternative species and the availability of bare land to be planted with a single species once (Wicksell setting) or an unlimited number of times (Faustmann setting). When the number of plantings is greater than one, it is possible for forest managers to replant the same species or repeatedly switch to the other species. The switch to the alternative species can also be considered a switch to an alternative agricultural or non-agricultural use.

In Section 4, we will study the problem of choosing between two alternative tree species with non-stochastic rising timber prices. This is an extension of Faustmann's problem with the presence of two alternative tree species. When the switch to the alternative species occurs, planting and harvesting the current species take place with rotations that are either increasingly higher, decreasingly lower, or equal to the Faustmann rotation.

In Section 5, we assume that timber prices are uncertain. More precisely, we discuss the problem of choosing between two alternative tree species with stochastic timber prices when the land is available for one rotation only. This is an extension of Wicksell's problem. This section helps us understand the value of waiting for more information before making decisions with irreversible consequences, a situation that will appear in Section 6 under the Faustmann's setting.

In Section 6, we will discuss the problem of choosing between two alternative tree species with stochastic timber prices when the land is available for an unlimited number of rotations. This is an extension of Faustmann's problem when two species are available. When the switch is about to take place, the rotations are increasingly longer and higher than the Faustmann's

rotation. Switching later allows forest managers to postpone decision-making and to wait for more information while trees continue growing.

Section 7 offers a conclusion and proposes future research topics.

Recall that the focus of this chapter is the economics of harvesting and planting, which are related given that harvesting opens up the option of planting, and so on. The planting decision has been overlooked in the literature despite its meaningful economic consideration, especially in a context of price uncertainty. In the remaining sections, the planting decision will be considered by assuming the land is initially bare.

2. Basic Wicksell's and Faustmann's rotation lengths

Timber prices have long-term, positive trends that are higher than the rate of increase of other goods prices (see for instance [15]). In [16], real softwood stumpage price trends are estimated at 1.8 to 3.8% (cited in [3]). At a disaggregated level, different tree species may experience differentiated timber price trends, given their distinct abilities to sequester carbon (see for instance [17]) or resist diseases and parasites. Under these conditions, forest managers may find it profitable to switch from one species to another or more generally to switch from timber to other agricultural or non-agricultural uses (see for instance [3]).

The Wicksellian rotation refers to the age at which a stand of even-aged trees will be harvested one time only. When timber price and regeneration costs are constant, the Wicksellian rotation length a_w is constant, determined implicitly as the solution to the following equation, as in [9]:

$$\frac{V_a(a_w)}{V(a_w)} = r, \tag{1}$$

where a_w is the Wicksellian rotation length, r is the discount rate, and $V(a)$ and $V_a(a)$ are, respectively, the growth volume function and its first derivative.

Eq. (1) shows that the harvesting decision is further delayed as long as the marginal benefit $V_a(a)$ from continuing to let trees grow is higher than the opportunity cost of the current harvest, which is $rV(a)$.

When timber prices evolve at a constant rate μ , the rotation length remains constant and equal to the implicit solution of the equation:

$$\frac{V_a(a_w)}{V(a_w)} = \delta, \tag{2}$$

which is the same as Eq. (1), where r is replaced by $\delta = r - \mu$ to account for a price increase resulting in higher rotation lengths.

When timber prices p are constant and it is possible to plant the land and harvest the stand an unlimited number of times according to the Faustmann's setting, the rotation length a_F ,

referred to as the static Faustmann's rotation length, is constant from one rotation to the next. This is determined implicitly by the following Faustmann's rule as, for instance, in [18]:

$$\frac{V_a(a_F)}{V(a_F) - \frac{D}{p}} = \frac{r}{1 - e^{-ra_F}} \quad (3)$$

Where, D is the constant regeneration cost.

The static Faustmann's rotation depends on the timber price level as long as the regeneration cost D is positive. In this situation, one known implication is that a one-time rise in the timber price level implies a decrease in the Faustmann's rotation length. An increase in timber revenues, therefore, makes longer rotations less attractive. Note that when the regeneration cost is equal to zero or accounted for in the timber price, the Faustmann's rotation length is independent of the price level as it is determined implicitly as the solution to:

$$\frac{V_a(a_F)}{V(a_F)} = \frac{r}{1 - e^{-ra_F}} \quad (4)$$

When timber prices evolve at a constant rate μ and regeneration costs are absent or accounted for in timber prices, the rotation length remains constant and equal to a_f , the implicit solution of the equation:

$$\frac{V_a(a_f)}{V(a_f)} = \frac{\delta}{1 - e^{-\delta a_f}} \quad (5)$$

where $\delta = r - \mu$. It can be shown that $\frac{\delta}{1 - e^{-\delta a}}$ is decreasing in δ for a given age a . Therefore, Eq. (5) admits a higher solution for the rotation length when δ decreases (or μ increases). Hence, the rotation length increases when the price rate of change μ increases. This result holds when regeneration costs are positive, as shown numerically in [2]. This result also holds when timber prices follow geometric Brownian motion and regeneration costs are absent or accounted for in the price timber as shown in [14] and discussed in Section 5.

As pointed out in [2], some confusion exists in the literature with respect to timber price impact on forest rotation length. To clear up this confusion, it is important to distinguish between a one-time static increase in the price change rate and its impact on the rotation length on the one hand, and the continuous increase of timber prices in time and their impact on successive rotation lengths on the other. For instance, when timber prices increase exponentially at a constant rate in the presence of constant regeneration costs, succeeding rotation lengths continuously decrease over time, as shown in [2]. Under these conditions, successive rotation lengths converge after some rotations to a certain length that can be higher or lower than the static Faustmann's rotation length a_F . However, when timber prices are rising or stochastic and a conversion of the site to an alternative use is possible, changes over time in the optimal rotation lengths remain uncertain, as in [3, 13, 14], for instance.

Nevertheless, when timber prices increase exponentially at a constant rate in the presence of constant regeneration costs, if the rate of change increases statically (a one-time rise in terms of

comparative analysis), then the rotation length also increases, as in [2]. This result is also verified in the case of rising or stochastic timber prices and the possibility of changing land use, as in [13, 14].

To isolate alternative land use impact on the planting decision as well as on the rotation length, regeneration costs are assumed to be nil or are accounted for in the timber prices in the following sections. When applicable, uncertainty over timber prices is considered by assuming that timber prices follow geometric Brownian processes. Along with the absence of regeneration costs, it can be shown that under these conditions, the land and stand value functions are homogeneous with respect to the timber prices as shown in [13, 14]. They can, therefore, be expressed as functions of time and the ratio of the two species' timber prices where the numeraire is one of the timber prices. This allows for the representation of the forest rotation as a function of the timber price ratio only. The rotation problem becomes autonomous in the sense that rotations do not depend on time explicitly, but rather implicitly through their dependence on timber prices. For more information, especially concerning the algebra, readers may refer to [13, 14].

3. Two-tree harvesting model

In the following sections, unless specified otherwise we assume that forest managers can plant one species among two available species, P and P' . The timber growth function for species P (respectively P') is $V(a)$ (respectively $V'(a)$). We assume that the timber price of species P (respectively P') follows a geometric Brownian motion with drift μ (resp. μ') and volatility σ (resp. σ'):

$$dp = \mu p dt + \sigma p dz \tag{6a}$$

$$dp' = \mu' p' dt + \sigma' p' dz \tag{6b}$$

where time indices have been omitted, $dz = \varepsilon \sqrt{dt}$ and $dz' = \varepsilon' \sqrt{dt}$ are the increments of Wiener processes, and ε and ε' are standardized Gaussian white noises whose correlation is ρ . For notation simplification, variables that depend on time are not indexed unless it is necessary.

The relative price $\theta = \frac{p'}{p}$ is time variable while $\delta = r - \mu > 0$ and $\delta' = r - \mu' > 0$ are constant parameters, where r is the discount rate. We assume that $\delta > 0$ and $\delta' > 0$; otherwise it would be optimal to delay the investment forever. Each tree species is characterized by a timber volume growth function with the following properties:

Assumption 1 *There exists $\underline{a} > 0$ and $\underline{a}' > 0$, such that the timber volume functions $V(a)$ and $V'(a)$ are continuous over $[0, +\infty[$, $V(a) = 0$ over $[0, \underline{a}]$, $V'(a) = 0$ over $[0, \underline{a}']$; $V(a)$ and $V'(a)$ are positive, continuous, differentiable and concave over $[\underline{a}, +\infty[$ and $[\underline{a}', +\infty[$ respectively. In addition, $\lim_{a \rightarrow +\infty} V_a(a) = 0$ and $\lim_{a \rightarrow +\infty} V'_a(a) = 0$.*

A volume growth function is usually convex and then concave. Assumption 1 allows for avoidance of the convex part that would result in complex but economically uninteresting

considerations. At the same time, this assumption reflects that trees need time to provide commercial timber volume.

4. Choosing between two alternative tree species with rising timber prices: An extension of Faustmann’s problem

In [13], timber prices rise according to the two processes in Eq. (7a) and Eq. (7b), corresponding to two alternative tree species with deterministic growth functions according to Assumption 1.

$$dp = \mu p dt \tag{7a}$$

$$dp' = \mu' p' dt \tag{7b}$$

In particular, when the planted species has a low rate of price change, a switch to the alternative species will certainly occur in the future. The same model can be applied in situations where timber harvesting must be terminated to affect the land to other agricultural or non-agricultural use. For the sake of facilitating discussion in the remaining sections, the alternative land use is assumed to be planting the alternative tree species.

Assume in the remaining sections that the price of the planted species P has a trend μ , that alternative species P' has a trend μ' , and that μ is lower than μ' . It is shown in [13] that, since the land and stand value are homogenous in timber prices, planting and harvesting decisions can be illustrated in the plan (a, θ) where a is the stand age and $\theta = \frac{p'}{p}$. The age or forest rotation of the planted species can be expressed as a function $a(\theta)$.

Figure 1 illustrates a numerical example of the planting and harvesting decisions where $\mu = 1\%$, $\mu' = 3\%$, and $r = 4\%$, as in [2]. The rotation is expressed as the harvesting age $a(\theta)$, a function of $\theta = \frac{p'}{p}$, represented by the dashed line. To focus on the role of timber prices, in [13] the same volume growth function is assumed for both species: $V(a) = V'(a) = V_\infty \left(1 - e^{-\alpha(a-\underline{a})}\right)$ where $V_\infty = 100$ is the timber volume when the age tends to infinity, $\alpha = 0.01$, and $\underline{a} = 5$ is the minimum age for positive growth. The Faustmann’s rotation for species P is $a_f = 18.05$ years.

The most crucial findings are the following. There exists a threshold θ_0 ($\theta_0 = 0.68$) such that on bare land, species P must be planted if the timber price ratio θ is lower than θ_0 , and species P' must be planted if the timber price ratio θ is higher than θ_0 . When θ is sufficiently low, that is, when p is much higher than p' , species P must be planted and harvested successively for some time. After a sufficiently long period of time, species P' will become sufficiently attractive to justify a switch from species P to species P' , as μ' will be higher than μ . As prices are assumed to be certain, a permanent switch to species P' will take place when species P is harvested at $\theta > \theta_0$. When the permanent switch to species P' occurs, its forest rotation $a_{f'}$ is constant, determined implicitly by Eq. (8) as follows:

$$\frac{V_a(a_f')}{V(a_f')} = \frac{\delta'}{1 - e^{-\delta' a_f'}} \tag{8}$$

Now reconsider the situation where species P is still attractive, as the price p is still low with respect to p' or, equivalently, θ is sufficiently below the switching threshold θ_0 . In the plan (t, θ_t) in **Figure 1**, when the stand P is growing, point (t, θ_t) moves obliquely according to:

$$\theta_t = \theta_{t=0} e^{(\mu' - \mu)t}. \tag{9}$$

As illustrated in **Figure 1**, the harvesting age $a(\theta)$ is composed of two boundaries (dashed line), an upper boundary and a lower boundary. The upper boundary is a set of downward-sloping and upward-sloping segments. The lower boundary is one upward-sloping segment starting from $(\theta_0, 0)$.

When the trees are growing, trajectory (θ_t, t) evolves obliquely according to Eq. (9) and hits the upper boundary on one of the downward-sloping segments. When the trajectory hits the upper boundary at a given downward-sloping segment, trees are harvested and the same species is replanted a given number of times. For instance, when the trajectory hits the upper boundary on the last downward-sloping segment (at θ_t higher than θ_0), trees are harvested and the alternative species are planted for the first time. The alternative species continues to be replanted and harvested forever, with rotation lengths constant and equal to a_f' given by Eq. (8). When the trajectory hits the upper boundary on the downward-sloping segment at θ higher than θ_1 but lower than θ_0 , trees are harvested and the same species P is replanted one more time before the switch to the alternative species P' occurs. The upward-sloping segments

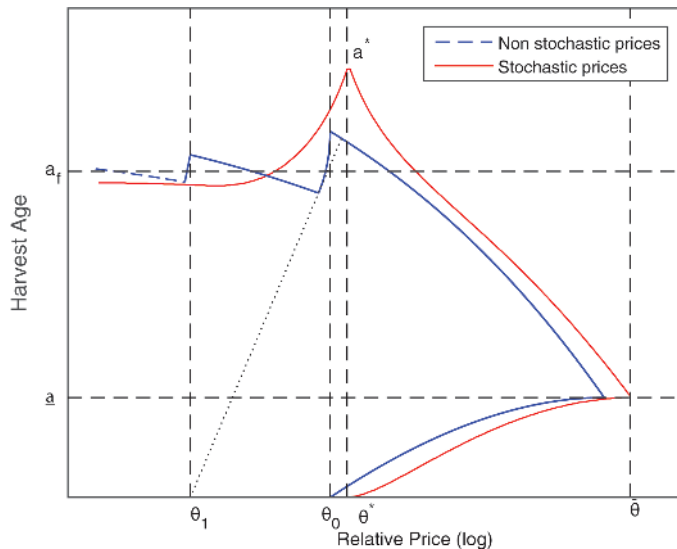


Figure 1. Harvesting boundaries when timber prices are certain and stochastic ([13, 14]).

are never optimally reached, but they signal the change in the remaining number of harvests of the same species. From left to right, the last downward-sloping segment of the upper boundary can be called a switching boundary, whereas all the remaining segments of the upper boundary constitute the replanting boundary.

Similar to the upward-sloping segments of the upper boundary, the lower boundary cannot be reached after a sequence of optimal harvests of species P . Assume that a forest manager has, for whatever reason, a stand of species P at age a and θ higher than θ_0 . Note that this situation cannot be the result of a sequence of optimal harvests. Under these conditions, the forest manager must either harvest the stand immediately if point (θ, a) is below the lower boundary or wait to let the trees grow if point (θ, a) is above the lower boundary and then harvest the stand when the upper boundary is reached. At that time, stand P must be replaced by the alternative species P' .

Most importantly, successive optimal harvests of stand P take place according to the following mutually exclusive situations in which the trajectory (θ_t, t) hits the upper boundary:

1. Successive rotations lengths are all equal to the static Faustmann rotation length $a_f = 18.05$ (y) of the planted species given by the Eq. 5. It is the case for $\theta = 0.39, 0.56,$ and 0.80 at which the switch occurs.
2. Successive rotations lengths continuously increase and remain higher than the static Faustmann rotation a_f . For instance, successive rotations are 18.45, 18.74, and 19.27 (y) when relative prices are equal respectively to 0.36, 0.52, and 0.76 at which the switch occurs.
3. Successive rotations lengths continuously decrease and remain lower than the static Faustmann rotation a_f . For instance, successive rotations are 17.67, 17.40, and 16.96 (y) when relative prices are equal respectively to 0.42, 0.60, and 0.84 at which the switch occurs.

A rise in the increase rate μ of the planted species results in a higher rotation length and a later switch to the alternative species. However, a static one-time increase in the price of the planted species (θ decreases) that is sufficiently small to not alter the number of remaining harvests of the same species before the switch results in an increase in all successive rotations lengths. Depending on the price level, a price increase sufficiently high to affect the number of remaining harvests before the switch may result in a decrease in successive rotations lengths.

5. Choosing between two alternative tree species with stochastic timber prices: An extension of Wicksell's problem

Reconsider Wicksell's problem in the presence of two alternative species P and P' and assume that the land is still bare. Timber prices are stochastic and evolve according to Eq. (6a) and Eq. (6b). Assume that the forest manager must plant a single species once at no cost. There then exists an interval (see **Figure 2**) in the timber price ratio $\theta = \frac{p'}{p}$ during which it is optimal to keep the land bare (see [13]). This situation remains until the timber prices differentiate enough for the forest manager to select the species with the highest stand value to plant. This waiting period applies despite the absence of any regeneration costs. The reason is that the availability

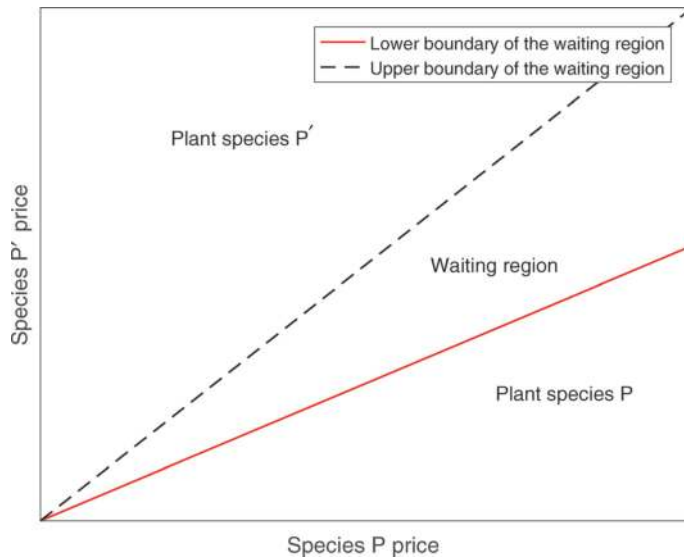


Figure 2. The one-rotation planting decision when two species are available.

of the land for a single rotation represents an opportunity cost for the forest manager if he or she commits the mistake of choosing the wrong species. The land value is proportional to the timber price once planted but follows a non-linear function of timber prices during the waiting period. This situation illustrates the value of the option to decide later once more information becomes available.

Figure 2 illustrates the decision-making process. If both timber prices (p, p') are represented by a point below the continuous line, then species P must be planted. If both timber prices (p, p') are represented by a point above the dashed line, then species P' has to be planted. Finally, as long as both timber prices (p, p') are represented by a point between the continuous and dashed lines, that is in the waiting region, the planting decision must be delayed. One way to avoid making the wrong decision is to postpone making the decision as shown in the literature related to investing under uncertainty using real options analysis (see for instance [19]). If the uncertainty increases (σ or σ' increase), then the waiting region becomes larger. If the timber price trend of one of the species increases, then the corresponding planting region becomes larger. For more information, readers may refer to [20].

6. Choosing between two alternative tree species with stochastic timber prices: An extension of Faustmann's problems

In this section, we will consider Faustmann's problem when two tree species P and P' are available with timber prices following the stochastic processes in Eq. (6a) and Eq. (6b). The

price of one species may be lower or higher than the price of the other species at any time, and the converse may occur later. Under these conditions, forest managers must choose the species to be planted, decide on rotation lengths, and decide whether to replant the same species or to plant the alternative species and whether it is optimal to keep the land bare for some time as in Section 5. It is shown in [14] that it is never optimal to keep the land bare after a harvest as in the Wicksellian setting. The reason is that instead of waiting to let timber prices differentiate clearly, forest managers are better off planting the most promising species and switching to the alternative species if it turns out that they have planted the wrong species.

Figure 1 illustrates the rotation length as a function of timber price ratio $\theta = \frac{p'}{p}$ (continuous line) when species P with the lowest trend μ is planted (recall that μ is lower than μ').

As in Section 4, when timber prices are rising and non-stochastic, if the land is bare, species P must be planted if θ is lower than a certain threshold θ^* , and species P' must be planted if θ is higher than θ^* . The harvesting boundary encompasses an upper boundary and a lower boundary. The upper boundary is composed of a replanting boundary corresponding to θ varying from zero to θ^* and a switching boundary corresponding to θ varying from θ^* to $\bar{\theta}$. When the replanting boundary is reached, species P is harvested and immediately replanted. When the switching boundary is reached, species P is harvested and immediately replaced by species P' and so on. The maximum rotation is reached at θ^* when the replanting and switching boundaries meet.

Although the converse may occur, θ increases in average with time as μ' higher than μ . Successive rotations lengths may increase or decrease similarly to the non-stochastic situation studied in Section 4, as the replanting boundary is non-monotonic (**Figure 1**). In general, the impact of a static increase in the timber price of the planted species (θ decreases) on its rotation length remains ambiguous except when the species is harvested for the last time. At that time, when timber price p increases, θ decreases and the rotation length increases. The longest rotation lengths are obtained when the planted species is about to be harvested for the last time and the switch to the alternative species is about to occur. The rotation length is higher when the switch occurs at θ closer to θ^* . The reason is that forest managers prefer delaying the decision to switch when timber prices are uncertain by choosing longer rotation lengths; it is better to delay the decision to switch while trees are still growing rather than harvest earlier and wait for timber prices to distinguish them sufficiently as in Section 5. It is also shown numerically in [14] that the rotation length increases when the price rate of change increases, as in [2] and [13].

Figure 3 represents the land value in terms of the numeraire p when only species P is available and when both species P and P' are available, with certain or stochastic timber prices. Obviously, the land has a higher value when both species are available than when only one species is available. Furthermore, when both species are available, the land has a higher value when timber prices are uncertain than when timber prices are certain. This result is well known for real options analysis. When timber prices are uncertain, the availability of two species allows forest managers to profit from an increase in timber prices but

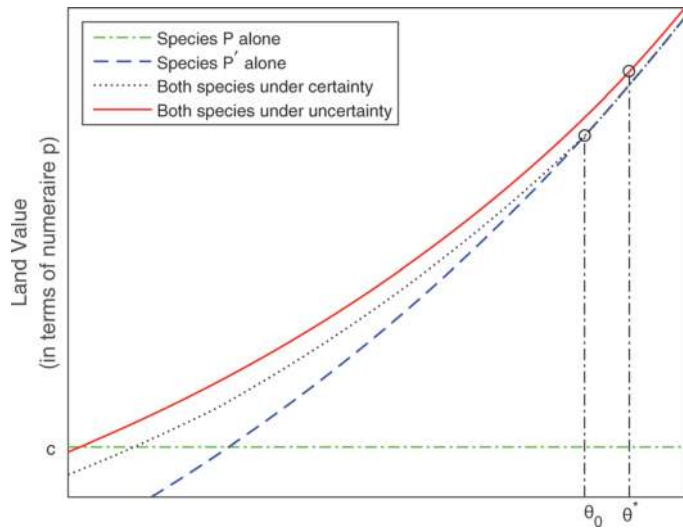


Figure 3. Land value with certain and stochastic timber prices ([14]).

protects them at least partially from price decreases, especially when the two species do not have highly correlated timber prices.

7. Conclusion

Under the Faustmann’s setting when only one species is available, regeneration costs are constant, and timber prices are rising, then successive rotations lengths decrease continuously over time. However, this result does not hold when it is possible to switch to another alternative use of the land. When timber prices are rising and it is desirable to switch to an alternative tree species, successive rotations lengths before the switch are either equal to Faustmann’s rotation length, increasingly higher, or decreasingly lower. When timber prices are uncertain, successive rotations lengths before the switch may increase or decrease. When the switch is imminent, the rotation length increases. This is an optimal way for forest managers to postpone deciding when prices are uncertain and allow trees to continue growing.

Planting and harvesting decisions are prototypes of investment decisions as decision makers face uncertain future costs and revenues as well as irreversible action, especially with respect to harvesting old and mature trees or planting with positive cost, as is the case in practice. These decisions can be managed proactively using some sources of flexibility, such as delaying irreversible decisions. Actions with irreversible consequences should be delayed until sufficient information becomes available to justify committing resources. In addition to delaying decisions, other sources of flexibility exist, such as investing sequentially, which may be applied by investors facing high levels of uncertainty.

Acknowledgements

To the memory of Pierre Lasserre.

Author details

Skander Ben Abdallah

Address all correspondence to: ben_abdallah.skander@uqam.ca

Department of Management and Technology, École des sciences de la gestion, Université du Québec à Montréal, Canada

References

- [1] Chang SJ. A generalized Faustmann model for the determination of optimal harvest age. *Canadian Journal of Forest Research*. 1998;**28**(5):652-659
- [2] Newman DH, Gilbert CB, Hyde WF. The optimal forest rotation with evolving prices. *Land Economics*. 1985;**6**(1):347-353
- [3] McConnell KE, Daberkow JN, Hardie IW. Planning timber production with evolving prices and costs. *Land Economics*. 1983;**59**(3):292-299
- [4] Brazee RJ, Mendelsohn R. Timber harvesting with fluctuating prices. *Forest Science*. 1988;**34**:359-372
- [5] Clarke HR, Reed WJ. The tree-cutting problem in a stochastic environment. The case of age-dependent growth, *Journal of Economic Dynamics and Control*. 1989;**13**:569-595
- [6] Insley M. A real options approach to the valuation of a forestry investment. *Journal of Environmental Economics and Management*. 2002;**44**:471-492
- [7] Reed WJ. The decision to conserve or harvest old-growth forest. *Ecological Economics*. 1993;**8**:45-69
- [8] Thomson T. Optimal Forest rotation when stumpage prices follow a diffusion process. *Land Economics*. 1992;**68**(3):329-342
- [9] Willassen Y. The stochastic rotation problem: A generalization of Faustmann's formula to stochastic forest growth. *Journal of Economic Dynamics and Control*. 1998;**22**:573-596
- [10] Thorsen BJ. Afforestation as a real option. Some policy implications, *Forestry Science*. 1999;**45**:171-178
- [11] Thorsen BJ, Malchow-Møller N. Afforestation as a real option: Choosing among options. In: Helles F, Strange N, Wichmann L, editors. *Recent Accomplishments in Applied Forest Economics Research*. Dordrecht: Kluwer Academic Publishers; 2003. pp. 73-80

- [12] Jacobsen BJ. The regeneration decision: A sequential two-option approach. *Canadian Journal of Forest Research*. 2007;**37**:439-448
- [13] Ben Abdallah S, Lasserre P. Forest land value and rotation with an alternative land use. *Journal of Forest Economics*. 2017;**29b**:118-217
- [14] Ben Abdallah S, Lasserre P. Asset retirement with infinitely repeated alternative replacements: Harvest age and species choice in forestry. *Journal of Economic Dynamics and Control*. 2016b;**70**:144-164
- [15] Lyon KS. Mining of the forest and the time path of the price of timber. *Journal of Environmental Economics and Management*. 1981;**8**(4):330-344
- [16] U.S. Department of Agriculture. An Analysis of the Timber Situation in the United States: 1952–2030. 1980. Forest Service Review Draft
- [17] Sohngen B, Mendelsohn R. Valuing the impact of large-scale ecological change in a market: The effect of climate change on US timber. *American Economic Review*. 1998: 686-710
- [18] Heaps T, Neher PA. The economics of forestry when the rate of harvest is constrained. *Journal of Environmental Economics and Management*. 1979;**5**:297-310
- [19] Dixit AK. *The Art of Smooth Pasting*, Harwood Academic Publishers. London: Paris; 1993
- [20] Ben Abdallah S. *Essais sur l'approche Des Options réelles appliquée à la Gestion et l'exploitation Des Ressources Naturelles*. Université du Québec à Montréal; 2009 Retrieved from <http://www.archipel.uqam.ca/2477/>

