Maximum Peak-Gain Margin (Mp-GM) Tuning Method for Two Degree of Freedom PID Controller

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Additional information is available at the end of the chapter

Abstract

The specification of controller setting for a standard controller typically requires a trade-off between set point tracking and disturbance rejection. For this reason, two simple strategies can be used to adjust the set point and disturbance responses independently. These strategies are referred as controllers that have two degree of freedom. Unfortunately, the tuning parameters of the model uncertainty at two degree of freedom structure controller are difficult to obtain. Maximum peak-gain margin (Mp-GM) tuning method has been introduced to obtain the setting parameters of two degree of freedom structure controller based on model uncertainty. This tuning method is able to obtain reasonable controller parameters even under process uncertainties on standard two degree of freedom IMC. This research was conducted to develop maximum peak-gain margin tuning method for another two degree of freedom structure controller such as two degree of freedom IMC by Kaya [9] and two degree of freedom PID. The simulation results show that the maximum peak-gain margin tuning method can give a good target set point tracking, disturbance rejection, and robustness in two degree of freedom structure controller system.

Keywords: two degree of freedom structure controller, IMC, PID, maximum peak, gain margin

1. Introduction

The process control is one of the important component parts in industries which is useful to keep and maintain the operating conditions of processes working on the desired performance. The development of this issue had begun since 1940. It is characterized by using PID controller in industries. Nowadays, PID control system is widely used as the basic control technology,

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because the PID controller uses a simple control algorithm [1]. Although the development of PID controller is rapid, it still does not produce maximum results especially for the process with large time delay. This is due to the disturbance that is not detected immediately (only can be detected until a certain time with delay) and also control actions based on the delay that are not in accordance with the purpose of information and need some time to determine its effects on the process.

To overcome this weakness, a new structure controller has been developed. This structure controller is called as internal model control (IMC) controller (**Figure 1**) [2, 3]. The philosophy of this structure stated that if the process model is an exact representation of the process that will be controlled, then it is possible to obtain the ideal control in 1DOF-IMC without any feedback. But in fact, the process model may not be invertible and some disturbances may enter the system so that the feedback path control is still necessary. Unfortunately, IMC design is intended only for the set point problem and the disturbance rejection responses still cannot be expected in many cases. So, this controller provides a good response for the set point tracking and a very slow response for the disturbance rejection case [4].

The specification of controller settings for a standard controller typically requires a trade-off between set point tracking and disturbance rejection. For many single-loop controllers, it is extremely difficult to obtain the specification in one degree of freedom structure controller settings. Fortunately, there are two simple strategies that can be used to adjust the set point and disturbance responses independently. These strategies are referred as controllers with two degree of freedom structure controller [5]. The design of these control systems is a multiobjective problem, so that a two degree of freedom (abbreviated as 2DOF) controller system has more advantages than a one degree of freedom (abbreviated as 1DOF) controller system. This fact was already stated by Horowitz, but it did not attract the general attention from engineers for a long time, until 1984, two decades after Horowitz's work, when a research to exploit the advantages of the 2DOF structure for PID control systems was eventually started [6].
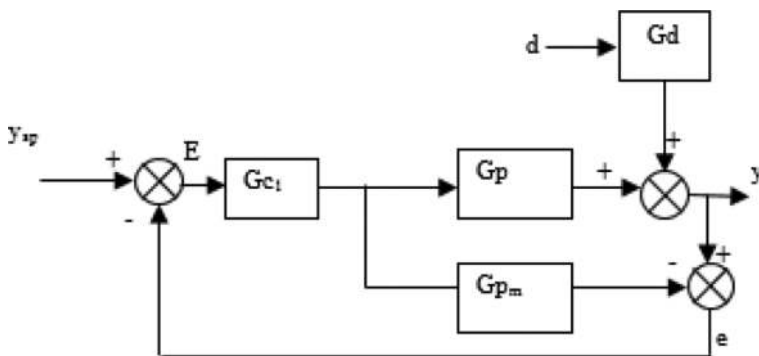

Figure 1. The structure of one degree of freedom IMC controller.

Many researches have proposed new various configurations of 2DOF structure control for PID, IMC, fuzzy logic controller, etc. Unfortunately, this is not followed by the study of 2DOF controller tuning method. The research conducted for 2DOF tuning method is still very rare, especially for the process with uncertainty. Maximum peak-gain margin (Mp-GM) tuning method has been proposed to obtain setting parameter of 2DOF structure controller based on model uncertainty. This tuning method is able to obtain the good controller parameter even under process uncertainties on standard 2DOF IMC structure controller [7]. The stability and robust Mp-GM tuning method has potential to be implemented into the other 2DOF structure controllers, both 2DOF PID controller and 2DOF IMC controller. This chapter studies the analytical procedure of implementation of Mp-GM tuning method to the other 2DOF structure controller under process uncertainties.

2. Two degree of freedom PID structure controller

For many single-loop controls, disturbance rejection is more precedent to be attained than set point tracking. Hence, the tuning methods hold a dominant role to reach this goal. Unfortunately, 1DOF structure controller can only arrange one parameter so that a trade-off between set point tracking and disturbance rejection cannot be reached. If the parameters give good enough response for set point tracking, it will give a slow response for the disturbance rejection and vice versa. This leads to the difficulty for stabilizing the control response simultaneously between set point tracking and disturbance rejection [5]. To overcome this weakness, a new simple control strategy has been developed to arrange the set point tracking and disturbance rejection controller independently without affecting each other. This method is called as (2DOF) strategy controller. The research of 2DOF strategy control for PID controller began since 1984. In 2DOF PID structure control, controller which is used to control set point tracking and disturbance rejections can be in PI, PD, or PID form controller. In 2003, there are some new variations developed for 2DOF PID structure controller such as 2DOF PID filter set point as shown in **Figure 2** [6].
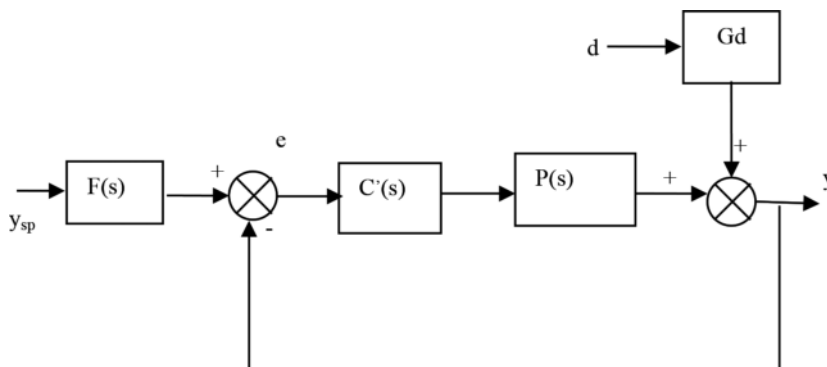

Figure 2. 2DOF-PID filter set point.

The structure of 2DOF-PID filter set point was developed by adding filter function in PID controller conventional (F(s)) that was used for controlling set point tracking, whereas PID parallel controller with approximate derivative was used for controlling disturbance rejection. Algorithm of F(s) and C'(s) controller for controlling set point tracking and disturbance rejection can be seen in Eqs. (1) and (2), respectively.

$$F(s) = \frac{1 + (1-a)\tau_I(s) + (1-\beta)\tau_I(s)\tau_D D(s)}{1 + \tau_I(s) + \tau_I(s)\tau_D D(s)} \tag{1}$$

$$C'(s) = k_p \left[1 + \frac{1}{\tau_I s} + \tau_D D(s) \right] \tag{2}$$

$$D(s) = \frac{s}{1 + \tau s} \tag{3}$$

Another variation of 2DOF-PID that is showed in **Figure 3** was developed by added feedback loop from output y directly to input u which will be compared with conventional PID controller (C_y(s)), which is called as feedback compensator that is used for controlling disturbance rejection). C_r(s) will be used as set point tracking controller. Algorithm for C_r(s) and C_y(s) controller was given by Eqs. (4) and (5) [6]:

$$C_r(s) = k_p \left[(1-\alpha) + \frac{1}{\tau_I s} + (1-\beta)\tau_D D(s) \right] \tag{4}$$

$$C_y(s) = k_p \left[\alpha + \beta \tau_D D(s) \right] \tag{5}$$

In 2011, another structure called as 2DOF-PID Vilanova was developed and was given in **Figure 4**. **Figure 4** shows that $C_{sp}(s)$ is used as set point tracking, $C_{yd}(s)$ as disturbance rejection control, and P(s) as transfer function process. $C_{yd}(s)$ was placed in the feedback loop to give a significant influence in maintaining stability without depending on the weighting factor set point tracking. For set point tracking controller, a filter is inserted in the path of the
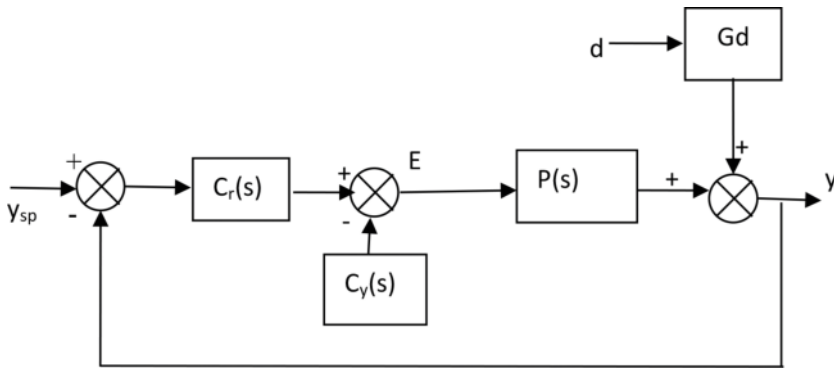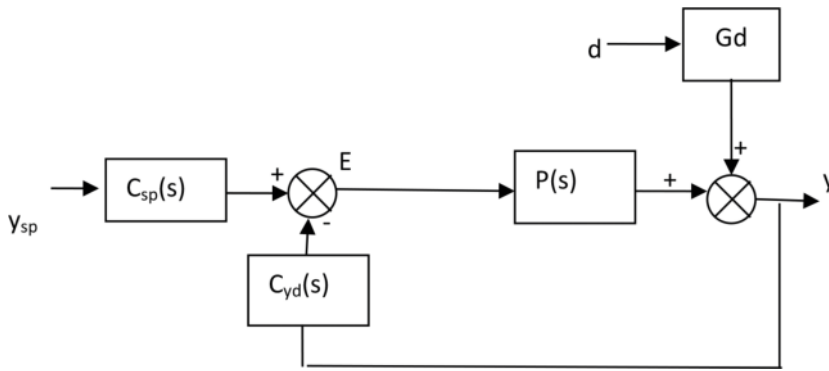

Figure 3. 2DOF-PID feedback.

Figure 4. 2DOF-PID Vilanova.

conventional PID controller. Transfer function of C_{sp} and C_{yd} was given by Eqs. (6) and (7), respectively [8].

$$C_r(s) = k_c\left[\beta + \frac{1}{\tau_I s}\right] \qquad (6)$$

$$C_y(s) = k_c\left[1 + \frac{1}{\tau_I s} + \tau_D\right] \qquad (7)$$

where k_p is proportional to gain controller, τ_I the is integral time constant, τ_D is the derivative time constant as "basic parameters," and α and β variables as parameters for 2DOF controller. The range value of parameters α and β is between 0 and 1. All parameters in 2DOF-PID filter set point and feedback will be treated as adjustable parameters. The τ parameter in approximate derivative Eq. (3) is set as τ_D/δ, where δ is called the derivative gain. The fixed value of δ can be determined by traditional step. The research stated that the change of δ does not influence the optimal value of all parameters in this structure drastically [6], while in 2DOF-PID Vilanova, the controller parameters will be determined by analytical robust tuning (ART) method. This tuning method used approach of the robustness-performance to determine controller parameters [8].

Beside 2DOF-PID, research on 2DOF controller also performed on controller with model principle like 2DOF-IMC. 2DOF IMC (**Figure 5**) structure controller was developed which aimed to cover a very slow response for disturbance rejection at 1DOF-IMC. This controller consists of controller for set point tracking (G_{c1}) in the open loop and disturbance rejection (G_{c2}) in the feedback path as shown in **Figure 6**. This structure configuration shows if there are no errors in the model and there are no disturbance enter to the process, it will need open loop path control only to get the ideal control response where the output will be same with set point. In fact, none of the models exactly same with the process and disturbance will always enter to the process in the field so that will be required a feedback loop to overcome these problems [2].

$$y = \frac{G_{c1}G_p y_{sp} + \left(1 - G_{c2}G_{pm}\right)G_d}{1 + G_{c2}\left(G_p - G_{pm}\right)} \tag{8}$$

If $G_p = G_{pm}$,

$$e = \left(1 - G_p G_{c2}\right)d - \left(1 - G_p G_{c2}\right)Y_{sp} \tag{9}$$

From Eq. (9), it can be assumed that G_{c2} was designed for disturbance rejection (d). If G_{c2} was designed exactly with G_{c1}, then the disturbance rejections cannot be eliminated optimally. Therefore, it is necessary to do tuning to get an optimal control result [2]. Unfortunately, the research for 2DOF controller tuning method is still extremely rare. Most studies were conducted only on the development of the new structure configuration of 2DOF structure controller. As in 2004, a new structure configuration was proposed for 2DOF IMC called as 2DOF IMC Kaya. This structure was designed for controlling integrating process with small time delays. Besides that, this structure is also used for the tuning of proportional derivative
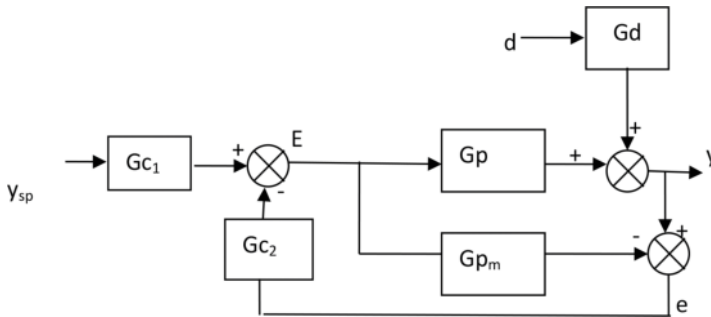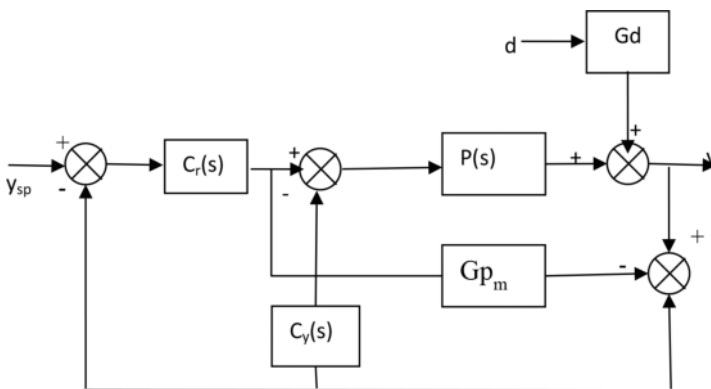

Figure 5. 2DOF-IMC standard.

Figure 6. 2DOF-IMC Kaya.

(PD) controller using gain and phase margin stability principle. As show in **Figure 6**, G_{c1} and G_{c2} in 2DOF-IMC Kaya are going to be located in the close loop of the structure. G_{c1} will be used for set point tracking and G_{c2} for disturbance rejection [9]. Besides 2DOF IMC controller, there is another controller that has model principle like Smith Predictor (SP), and nowadays, it is developed in 2DOF controller form. 2DOF SP structure controller has been applied on the integrating process with large time delay. The results show that 2DOF SP controller is able to gain fast and stable response for disturbance rejection [10].

3. Tuning method for two degree of freedom structure controller

The purpose of controller tuning is to determine the controller parameter to obtain appropriate control parameters in order to achieve stable closed-loop performance robustly. The controller performance is expected to be stable and robust when the variable control at desired set point and the disturbance can be eliminated as soon as possible [11]. The proposed 2DOF controller tuning method has been started since the structure developed in 1984. Tuning of 2DOF controller is developed in the form of proportional derivative (PD) or proportional integral derivative (PID) controller. Unfortunately, this tuning did not provide an analytical explanation for the controller parameters. Besides that, there is no guarantee that a stable response and robust process can be produced [4]. Furthermore, another tuning has been developed for 2DOF PID structure controller with principle multiplication from dominant pole on sensitivity and complementary sensitivity function [12]. This tuning has only been developed for the integrating process with small time delay. Additionally, this tuning involves weighting factor in variables for both proportional and derivative part in PID controller which is used for both set point tracking and disturbance rejections. In 2008, another research has been done to develop a tuning for 2DOF PI/PID structure controller with analytical approaching. This tuning was called as analytical robust tuning (ART), which is also using a weighting factor in variable of proportional controller for the case with perfect models. Analytical approaching in this tuning depends on the process being controlled. To control FOPDT process, the proportional integral (PI) controller will be used for set point tracking and disturbance rejection. Nevertheless, when SOPDT process is to be controlled, the proportional integral derivative (PID) controller will be used [4]. Tuning for 2DOF-PID filter set point has been done by Zhang et al. at 2006, but the tuning was used for integrating process and the dead time of process is approximated with two-order Pade approximation so that the equations become more complicated [13].

For the 2DOF IMC structure controller tuning, most of them are still being developed for the case with perfect model, where the transfer functions process and model are exactly equal. One of the researchers who developed a tuning for the case of uncertainties is Brosilow and Joseph. They used the principle of the resonant peak of the complementary sensitivity function to develop a tuning for 2DOF IMC structure. The tuning was called as maximum peak (Mp) tuning [14]. Unfortunately, this tuning can only be used for 1DOF IMC structure. Furthermore, it can be done by using the maximum peak (Mp) principle that was developed by Brosilow and Joseph, Stryczek et al. to propose IMCTUNE. This tuning can be implemented not only in

the 1DOF and 2DOF structure controller but also on the other structures, such as 1DOF PID and model state feedback (MSF)-IMC. Unfortunately, IMCTUNE needs partial sensitivity functions from the transfer function of disturbance which is difficult to be modeled [3]. To overcome this weakness, in 2013, maximum peak-gain margin (Mp-GM) tuning has been proposed to obtain setting parameter of 2DOF structure controller based on model uncertainty. By using maximum value of complementary sensitivity function of 1DOF IMC structure to determine parameter control for set point tracking and gain margin (GM) values to determine parameter control for disturbance rejection, this tuning method is able to obtain a good controller parameter when it is even under process uncertainties on standard 2DOF IMC. The steps for Mp-GM tuning will be explained more clearly in the next section [7].

4. Maximum peak-gain margin tuning method

One of the newest tuning method that was developed to handle the case control with para-metric uncertainty is maximum peak-gain margin (Mp-GM) tuning method. This tuning method consists of three steps with all figure to determine the parameter value of Mp-GM tuning given in **Figure 7**. The initial step in Mp-GM tuning is determining the worst case of uncertainty model. Worst case is a condition when transfer function process is not same with model. The worst case can be found from the limit of the uncertainty model in terms of upper and lower on process model parameters. This condition usually occurs at the uncertainty model with the larger (upper limit) steady-state gain process, the larger the (upper limit) time delay, the smaller the (lower limit) process time constant. The worst case can be identified as the biggest maximum value of magnitude of frequency response of complementary sensitivity
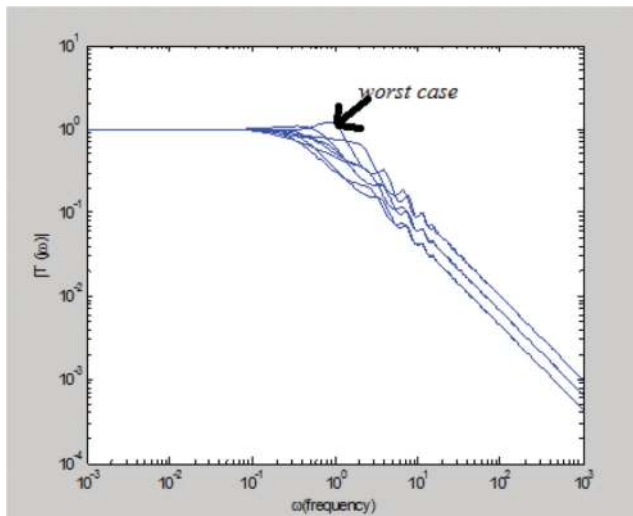

Figure 7. Magnitude of |T(jw)| vs. frequency response (w) to get the worst case.

function which can be seen in the **Figure 7**. When determining the worst case, the filter time constant (τ) value will be set equal to the time delay of no error in the model [7].

The second step is specifying the parameter of set point controller (Gc_1) using complementary sensitivity function of 1DOF-IMC structure, based on the maximum peak stability criterion. By using algorithm of Eq. (10) below:

$$G_{c1} = \frac{1}{k} \frac{\tau_p s + 1}{\lambda_1 s + 1} \tag{10}$$

where k is the gain process, τ is the time constant process, and λ_1 is the filter time constant parameter, the parameter λ_1 is the parameter of set point controller. The filter time constant parameter can be obtained by looping the value of λ_1 (the filter time constant Gc_1) in calculating complementary sensitivity function so that acquired max $|T(j\omega)|$ will be 1.05 in the range of frequency ω equal to 10^{-3}–10^3. For the first looping, λ_1 will be set equal to the time delay (θ) of no error in the model divided by 20. Calculation results are displayed in the graphical frequency form which is shown in **Figure 8** [7].

The third step is obtaining parameter of disturbance rejection controller (G_{c2}) using open loop transfer function of 2DOF structure controller based on the gain margin criterion. The disturbance rejection parameter is obtained by looping the value of α in calculating transfer function open loop so that the acquired GM will be 2.4. For the first looping, α is set equal to the filter
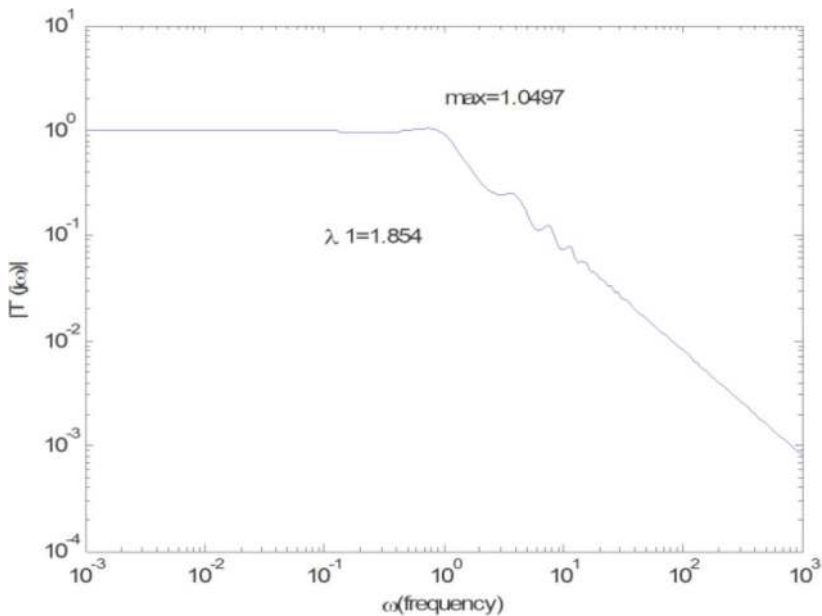

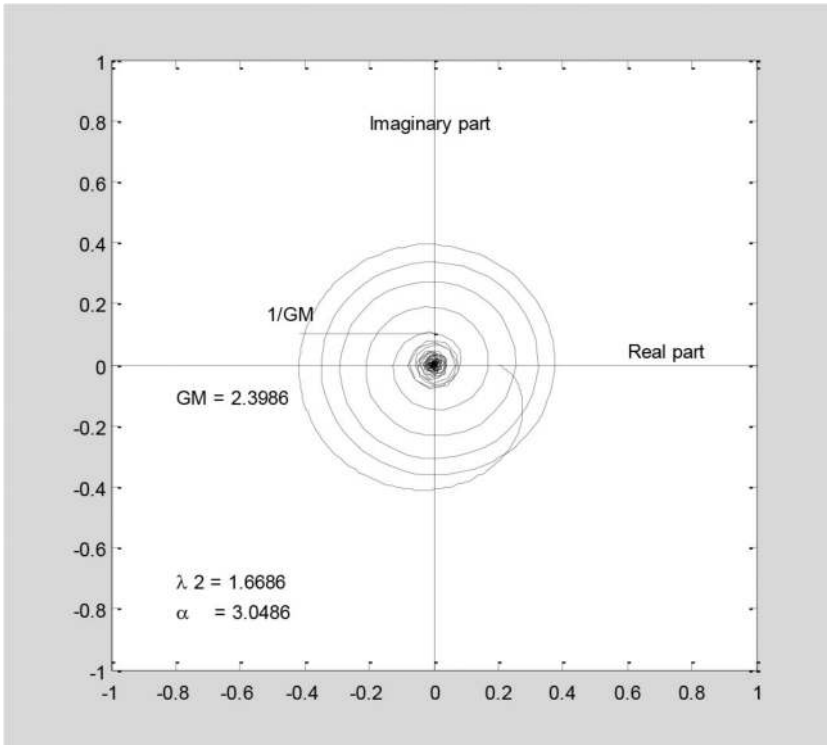**Figure 8.** Magnitude of $|T(jw)|$ vs. frequency response (w) to determine λ_1.

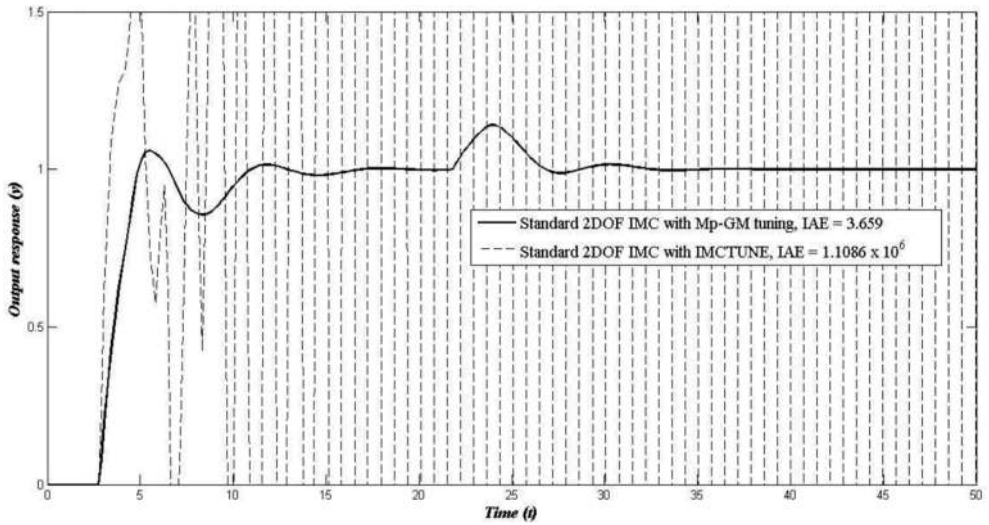**Figure 9.** Nyquist plot to determine λ_2 and α.

Figure 10. Comparison of responses between Mp-GM and IMCTUNE on 2 DOF-IMC standard.

time constant parameter disturbance rejection controller (λ_2) by setting ratio of λ_2 to λ_1 as much as 0.9. This calculation is using Eq. (11) below:

$$G_{c2} = \frac{1}{k}\frac{\tau_p s + 1}{\lambda_1 s + 1}\frac{\alpha s + 1}{\lambda_2 s + 1} \tag{11}$$

where λ_2 and α are the filter time constant parameter and lead parameter at disturbance rejection controller, respectively. The result will be plotted into the Nyquist plot as can be seen in the **Figure 9** [7].

To see the results of Mp-GM tuning, the used IMCTUNE will be required as the comparison. To get parameter controller by IMCTUNE, Mp-tuning software was used [2]. Based on **Figure 10**, it can be seen that this tuning method is able to obtain a good controller parameter when it is even under process uncertainties on standard 2DOF IMC [7].

5. Maximum peak-gain margin tuning method for 2DOF IMC Kaya and 2DOF PID feedback

Four examples of FOPDT cases can be considered to illustrate the use of the Mp-GM tuning method on 2DOF structure control. The examples cover FOPDT cases model with $\frac{\theta}{\tau} < 1$ and $\frac{\theta}{\tau} > 1$ where process time constant or dead time is fixed. The assumption for uncertainty model is the deviation $\pm 20\%$. As described earlier, the worst case will be determined as the maximum value of the calculation of complementary sensitivity function of 1DOF-IMC controller that was given in Eq. (13), with Eq. (12) as process and model transfer function.

$$G_p = G_{pm} = \frac{ke^{-\theta s}}{\tau s + 1} \tag{12}$$

$$T(j\omega) = \frac{G_{c1}G_p}{1 + G_{c1}(G_p - G_{pm})}y_{sp} \tag{13}$$

The first FOPDT case model where the variables are gain and dead time with $\frac{\theta}{\tau} < 1$ is described as below.

$$Gp = \frac{ke^{-\theta s}}{\tau s + 1}, 0.8 \leq k \leq 1.2,\ 16 \leq \tau \leq 24\ \text{ and }\ 9.6 \leq \theta \leq 12 \tag{14}$$

$$Gp_m = \frac{e^{-10s}}{20s + 1} \tag{15}$$

$$Gd = \frac{0.5}{2s + 1} \tag{16}$$

By using Mp-GM tuning, it is obtained that the worst case of the plant is the condition with k = 1.2, τ = 16, and θ = 12. The second FOPDT case where the variables are gain and dead time with $\frac{\theta}{\tau} > 1$ is described as below.

$$Gp = \frac{ke^{-\theta s}}{\tau s + 1}, 0.8 \le k \le 1.2, 1.6 \le \tau \le 2.4 \text{ and } 9.6 \le \theta \le 12 \tag{17}$$

$$Gp_m = \frac{e^{-10s}}{2s + 1} \tag{18}$$

where the parameter of the worst case of the plant is the condition with k = 1.2, τ = 2.4, and θ = 12, respectively. The third FOPDT case model where the variables are gain and process time constant with $\frac{\theta}{\tau} < 1$ is described as below.

$$Gp = \frac{ke^{-\theta s}}{\tau s + 1}, 1.6 \le k \le 2.4, 2.4 \le \tau \le 3.6 \text{ and } 1.2 \le \theta \le 1.8 \tag{19}$$

$$Gp_m = \frac{e^{-1.5s}}{3s + 1} \tag{20}$$

The worst case plant is obtained under the condition with k = 2.4, τ = 2.4, and θ = 1.8. The fourth FOPDT case model where the variables are gain and process time constant with $\frac{\theta}{\tau} > 1$ is described as below.

$$Gp = \frac{ke^{-\theta s}}{\tau s + 1}, 1.6 \le k \le 2.4, 2.4 \le \tau \le 3.6 \text{ and } 6.4 \le \theta \le 9.6 \tag{21}$$

$$Gp_m = \frac{e^{-8s}}{3s + 1} \tag{22}$$

The worst case plant is obtained under the condition with k = 2.4, τ = 3.6, and θ = 9.6.

Parameter value of set point tracking (λ_1) of 2DOF-IMC Kaya was also determined by calculation in Eq. (13), so that acquired maximum value of complementary sensitivity function will be 1.05. The implementation of the Mp-GM tuning in 2DOF-PID feedback has been done with the same method as the one of 2DOF-IMC Kaya. Therefore, by using Eq. (23) for approximation of set point tracking, controller form in structure 2DOF-PID feedback was obtained. The value of filter time constant G_{c1} (λ_1) of 2DOF-IMC Kaya was also used to get parameter controller in 2DOF-PID feedback.

$$C_r(s) = \frac{G_{c1}(s)}{1 - G_{pm}G_{c1}(s)} \tag{23}$$

In order to improve the controller's performance, the dead time can be approximated using a first-order Taylor series expansion such as Eq. (24);

$$e^{-\theta s} = 1 - \theta s \tag{24}$$

By substituting Eqs. (10) and (12) into the Eq. (23), Eq. (25) can be obtained. Eq. (25) can be approximated into the proportional integral (PI) controller form as Eq. (26).

$$C_r(s) = \frac{1}{k} \frac{\tau s + 1}{\lambda_1 s + \theta s} \tag{25}$$

$$C_r(s) = k_c \left(1 + \frac{1}{\tau_I s} \right) \tag{26}$$

where:

$$k_c = \frac{1}{k} \frac{\tau_p}{\lambda_1 + \theta}$$

$$\tau_I = \tau_p$$

To determine the parameter value of λ_2 and α as parameter disturbance rejection controller in 2DOF-IMC Kaya, the same steps are used. By using Eq. (27), one can obtain parameter disturbance rejection by looping the value of α in calculating transfer function open loop, so that the acquired GM will be 2.4 by setting ratio of λ_2 to λ_1 as much as 0.9.

$$G_{ol} = G_{c2}G_p + G_{c1}(G_p - G_{pm}) \tag{27}$$

whereas disturbance rejection controller at 2DOF-PID feedback will have same transfer function form with controller that is used in 2DOF-IMC Kaya. The transfer function is given in Eq. (11). As a correction factor, parameter gain of disturbance rejection controller will be multiplied by 0.5 in 2DOF-IMC Kaya and by 0.3 in 2DOF-PID feedback. Parameter values of 2DOF-IMC Kaya and 2DOF-PID feedback are given in **Table 1**.

The implementation of Mp-GM tuning method into 2DOF-PID filter set point and Vilanova structure control has been done with the analogies that of 2DOF-IMC standard, so that one uses Eq. (29) for approximation of set point tracking controller form in structure 2DOF-PID filter set point and Eq. (30) for approximation of set point tracking controller form in structure 2DOF-PID Vilanova. The parameter λ_1 (the filter time constant) was obtained by using Eq. (13) so that acquired maximum value of complementary sensitivity function will be 1.05.

Variation of FOPDT cases that used	Parameter values of PI for set point tracking controller		Parameter values of 2DOF-IMC controller and disturbance rejection in 2DOF-PID feedback controller		
	k_c	τ_I	λ_1	λ_2	α
First case	0.7051	20	18.366	16.5294	22.2694
Second case	0.2539	2	14.792	13.3128	16.4128
Third case	0.0807	3	2.776	2.4984	3.3784
Fourth case	0.0759	3	11.772	10.5948	13.0948

Table 1. Parameter values of 2DOF-IMC Kaya and 2DOF-PID feedback.

To get the parameter value of λ_2 and α, one can use Eq. (28) as the open-loop transfer function of 2DOF-IMC standard. By looping the value of α in the calculation of open-loop transfer function, one can get the acquired GM of 2.4 by setting ratio of λ_2 to λ_1 as much as 0.9.

$$G_{ol} = G_{c2}\left(G_p - G_{pm}\right) \tag{28}$$

$$F(s) = \frac{G_{c1}(s)}{G_{c2}(s)} \tag{29}$$

$$C_{sp}(s) = \frac{G_{c1}(s)}{1 - G_{pm}G_{c2}(s)} \tag{30}$$

Substituting Eqs. (10) and (11) into Eq. (29) will give PD controller as set point tracking controller in 2DOF-PID filter set point;

$$F(s) = \frac{\lambda_2 s + 1}{\alpha s + 1} \tag{31}$$

Substituting Eqs. (10) and (12) into Eq. (30), one can obtain Eq. (32). This equation will be used to approximate the function into PID series with derivative filter controller form in Eq. (33) as set point tracking controller in 2DOF-PID Vilanova.

$$C_{sp}(s) = \frac{1}{k} \frac{\lambda_2 \tau s^2 + (\lambda_2 + \tau)s + 1}{(\lambda_1 \lambda_2 + \alpha\theta)s^2 + (\lambda_1 + \lambda_2 - \alpha + \theta)s} \tag{32}$$

$$C_{sp}(s) = k_C \left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s + 1}{A\tau_D s + 1}\right) \tag{33}$$

where:

$$k_c = \frac{0.5 \times \tau}{k(\lambda_1 + \lambda_2 + \theta - \alpha)}$$

$$\tau_I = \tau$$

$$\tau_D = \lambda_2$$

$$A = \frac{\lambda_1\lambda_2 + \alpha\theta}{\lambda_2(\lambda_1 + \lambda_2 + \theta - \alpha)}$$

For the disturbance rejection on 2DOF-PID filter set point and Vilanova controller, one can obtain the same controller form like Eqs. (34) and (35).

$$C'^{(s)} = C_{yd}(s) = \frac{G_{c2}(s)}{1 - G_{pm}G_{c2}(s)} \tag{34}$$

$$C'(s) = C_{yd}(s) = \frac{1}{k} \frac{\alpha\tau s^2 + (\alpha + \tau)s + 1}{(\lambda_1 \lambda_2 + \alpha\theta)s^2 + (\lambda_1 + \lambda_2 - \alpha + \theta)s} \tag{35}$$

Eq. (35) for disturbance rejection controller will be approximated into PID series with derivative filter form as Eq. (36)

$$C'(s) = k_C \left(\frac{\tau_I s + 1}{\tau_I s} \right) \left(\frac{\tau_D s + 1}{A \tau_D s + 1} \right) \tag{36}$$

where:

$$k_c = \frac{0.5 \times \tau}{k(\lambda_1 + \lambda_2 + \theta - \alpha)}$$

$$\tau_I = \tau$$

$$\tau_D = \alpha$$

$$A = \frac{\lambda_1 \lambda_2 + \alpha \theta}{\alpha(\lambda_1 + \lambda_2 + \theta - \alpha)}$$

The gain parameter controller of PID series with derivative filter that is used on 2DOF-PID filter set point and 2DOF-PID Vilanova will be multiplied with weighting factor equal to 0.5 as factor correction. As a comparison to see performance of Mp-GM tuning, analytical robust tuning (ART) proposed by Vilanova was used [4]. Parameter values of 2DOF-PID filter set point are given in **Table 2** and 2DOF-PID Vilanova is given in **Table 3**.

The response of 2DOF IMC Kaya and 2DOF-PID which had been tuned with Mp-GM in the FOPDT case model with variations of ratio of dead time (θ) and process time constant (τ) for θ fixed is presented in **Figures 11** and **12**, while τ fixed is presented in **Figure 12**. The worst case in FOPDT case with ratio dead time and time constant process lower than 1 was found from the larger (upper limit) steady-state gain process, the larger the (upper limit) time delay, the smaller the (lower limit) process time constant. On the other hand, in FOPDT case with ratio dead time and process time constant more than 1, the worst case was found on the upper limit on all parameters of process model. **Figures 11** and **12** with the control action of 2DOF-IMC Kaya that was tuned by Mp-GM showed that processes with ratio dead time and process time constant more than 1 at fixed dead time gave smaller IAE and faster settling time toward desired set point. On the other hand, processes with ratio less than 1 produce sluggish control

Variation of FOPDT cases that used	Parameter values of 2DOF-IMC standard			Parameter values of PID series with derivative for disturbance rejection controller			
	λ_1	λ_2	A	k_c	τ_I	τ_D	A
First case	12.35	11.115	20.295	1.5186	20	20.295	1.2789
Second case	11.365	16.4128	23.7185	0.2539	2	23.7185	1.8922
Third case	1.854	1.6686	3.0486	0.7599	3	3.0486	1.2739
Fourth case	9.543	8.5887	20.1687	0.2516	3	20.1687	2.0231

Table 2. Parameter values of 2DOF-PID filter set point.

Cases	Mp-GM tuning								Parameter values of PI controller using ART method	
	Parameter values of PID series with derivative for set point tracking controller				Parameter values of PID series with derivative for disturbance rejection controller					
	k_c	τ_I	τ_D	A	k_c	τ_I	τ_D	A	k_c	τ_I
1st	1.5186	20	11.115	2.3241	1.5186	20	20.295	1.2728	0.582	19.867
2nd	0.2539	2	10.229	4.3877	0.2539	2	23.719	1.8922	0.116	1.7867
3rd	0.7598	3	1.6686	2.3275	0.7598	3	3.0486	1.2739	0.291	2.98
4th	0.2515	3	8.5887	4.7508	0.2515	3	20.169	2.0231	0.091	2.7055

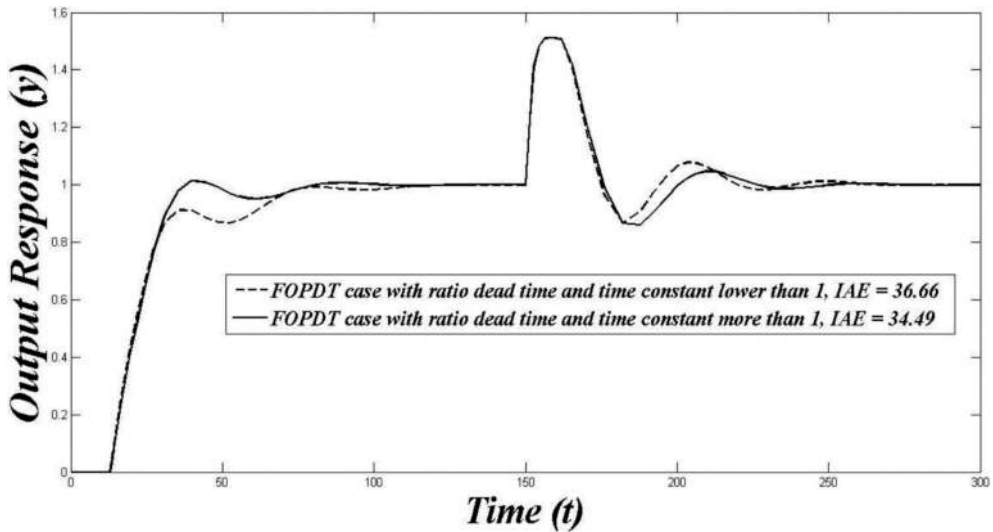**Table 3.** Parameter values of 2DOF-PID Vilanova.

Figure 11. Responses of 2DOF-IMC Kaya with dead time fixed.

action. The reason was that at processes with ratio less than 1, it produced bigger process time constant, so that it gave sluggish control action. While for the case in which process time constant is fixed, then processes with ratio dead time and process time constant greater than 1 produce smaller IAE and faster settling time to reach desired set point. Processes with ratio less than 1 have a smaller dead time so that it can produce faster control action with smaller overshoot. The use of the same transfer function of disturbance rejection cause control action that was produced in 2DOF-PID feedback was almost the same as response that was resulted in 2DOF-IMC Kaya controller.

Using the 2DOF-IMC standard that was tuned by Mp-GM method to be applied for 2DOF-PID filter set point and 2DOF-PID Vilanova causes both of the them to produce somewhat the same response. **Figures 13** and **14** showed that the processes with ratio dead time and process time
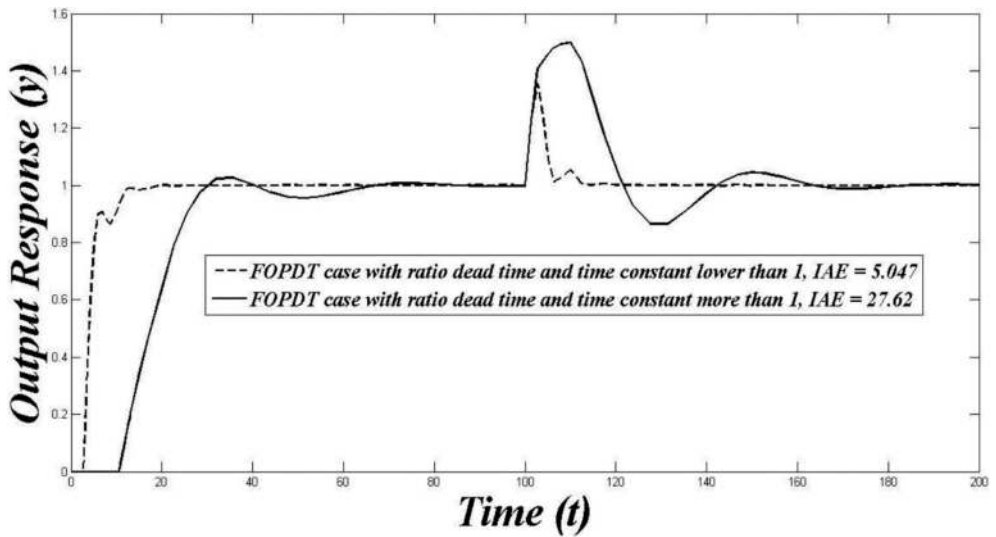
Figure 12. Responses of 2DOF-IMC Kaya with fixed time constant process.

constant more than 1 produce a faster response with smaller IAE and overshoot in either dead time or process time constant is fixed. The output response of 2DOF-PID Vilanova structure which was tuned by Mp-GM tuning and ART method showed that Mp-GM produced control action with smaller overshoot and smoother than ART method even though Mp-GM method gave bigger IAE value with dead time fixed. On the other hand, Mp-GM gives sluggish control action with bigger IAE than ART method in case FOPDT with ratio dead time and process time
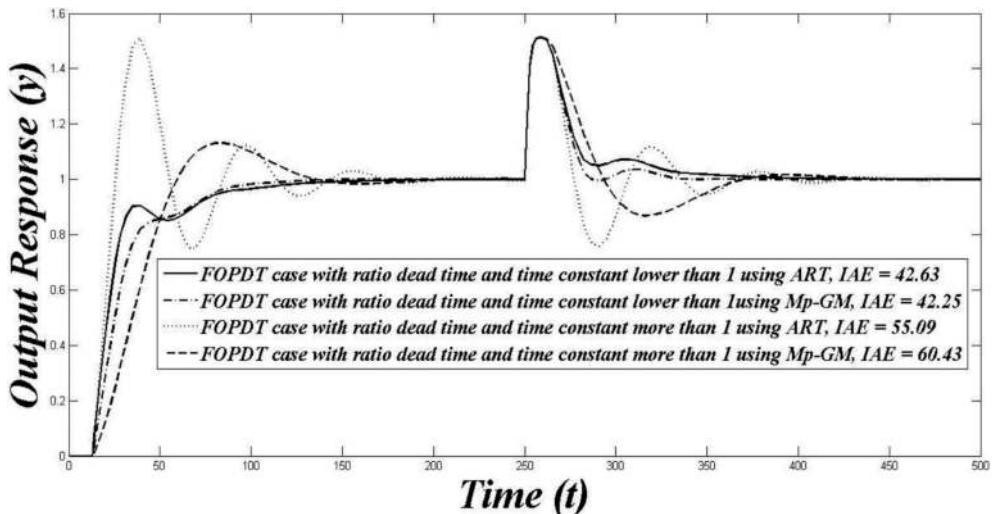

Figure 13. Responses of 2DOF-PID Vilanova with fixed dead time.
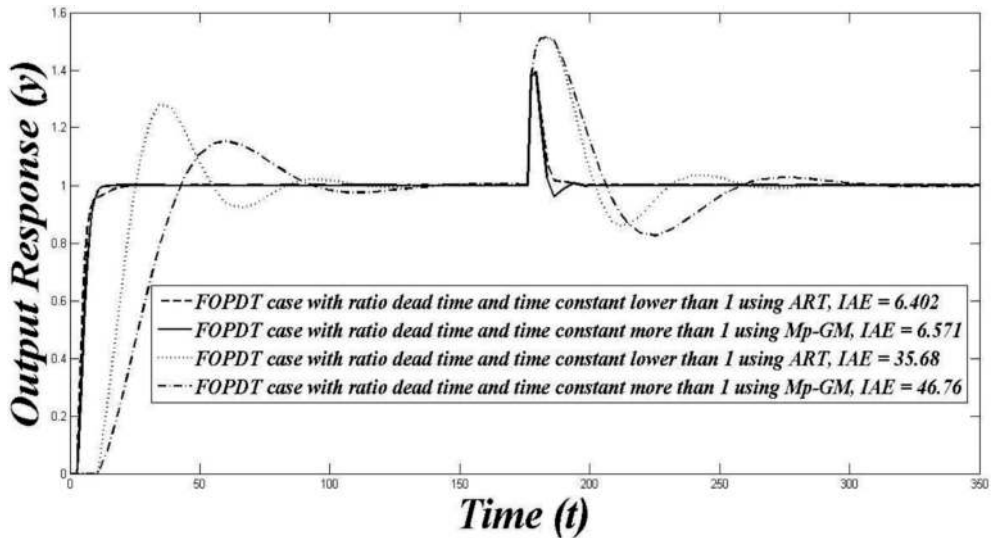
Figure 14. Responses of 2 DOF-PID Vilanova with fixed time constant process.

constant larger than 1 at process time constant fixed. As for the case with ratio less than 1, Mp-GM and ART methods gave somewhat same results. All cases showed that Mp-GM can give same and better response with an easier way than ART method. But in overall, all of the FOPDT cases that are used showed good results for set point tracking and disturbance rejection both on 2DOF-IMC Kaya or all of 2DO-PID controller that used in this research. This can be seen from controller response, which can be returned to its desired condition when there is a change of the set point and the load. The weighting factor which was added as a correction factor at the equation for calculation of parameter gain controller can have faster response, so that it needs less time to reach a desired set point. These results show that Mp-GM tuning method can be implemented in other 2DOF structure controllers.

6. Mp-GM implementation for simulation of temperature control on CSTR reactor using Simulink and HYSYS

In the previous section, the Mp-GM tuning has been proven capable of being implemented on 2DOF controllers to control various processes using Simulink simulation to see the control response. Furthermore, Mp-GM method will also be used for tuning the control of a real process modeling using HYSYS program. The process to be used as a model is the process of hydrolysis of propylene oxide to produce propylene glycol. The hydrolysis reaction is assumed to be of one-order with the expected 50% reaction conversion. Propylene oxide as limiting reactant and water as an excess reactant. This reaction is a type of exothermic reaction, so that a CSTR reactor with coolant is used as a heat absorbing medium generated from the reaction. Design data for the CSTR are provided in **Table 4**.

Based on the derivation of the equation, one can obtain the function transfer equation in the form of second-order Laplace transform for the influence of the feed temperature to the reaction temperature as in Eq. (37)

$$G_p = \frac{1,4582s + 12,249}{s^2 + 3,1937s + 13,1413} \tag{37}$$

To facilitate the implementation of the Mp-GM tuning method, the second-order function transfer equation is approximated by the Skogestad's "Half rule"

$$G_p = \frac{e^{-0,12s}}{0,082s^2 + 0,261s + 1,0728} \tag{38}$$

Furthermore, to obtain the first-order function transfer form, Eq. (38) is then approximated by Panda method so that the Eq. (39)

$$G_p = \frac{0,932e^{-0,202s}}{0,243s + 1} \tag{39}$$

The inconsistency parameter is assumed to be ±20% of the transfer of the process model function in Eq. (39) as $0.7456 \le k_c \le 1.1184$; $0.1616 \le \tau \le 0.2424$ dan $0.1944 \le \theta \le 0.2916$.

	Input	Output		
		Manual	Simulation	Error (%)
The concentration of propylene oxide (lbmol/ft^3)	0.132	0.066	0.06587	0.002
The concentration of propylene glycol (lbmol/ft^3)	—	0.066	0.06613	0.002
Temperature (°F)	60	102.64	102.64	—
Pressure (psia)	16.17	16.17	16.17	—
Energy *Coolant* (Btu/hr)	−7.837 x 10^5			—

Table 4. The simulation data of input and output of CSTR reactor on propylene oxide hydrolysis process to produce propylene glycol.
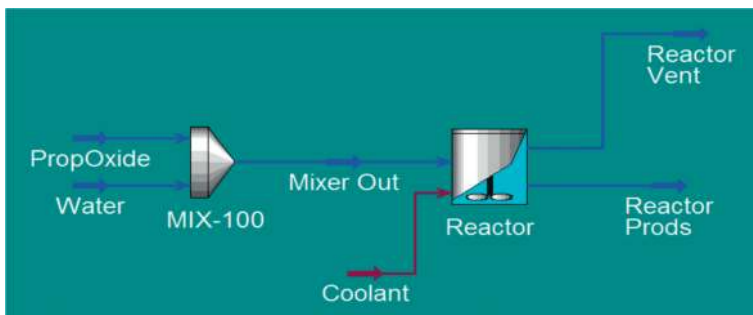

Figure 15. Steady-state simulation of propylene oxide hydrolysis process to produce propylene glycol using CSTR reactor with HYSYS.
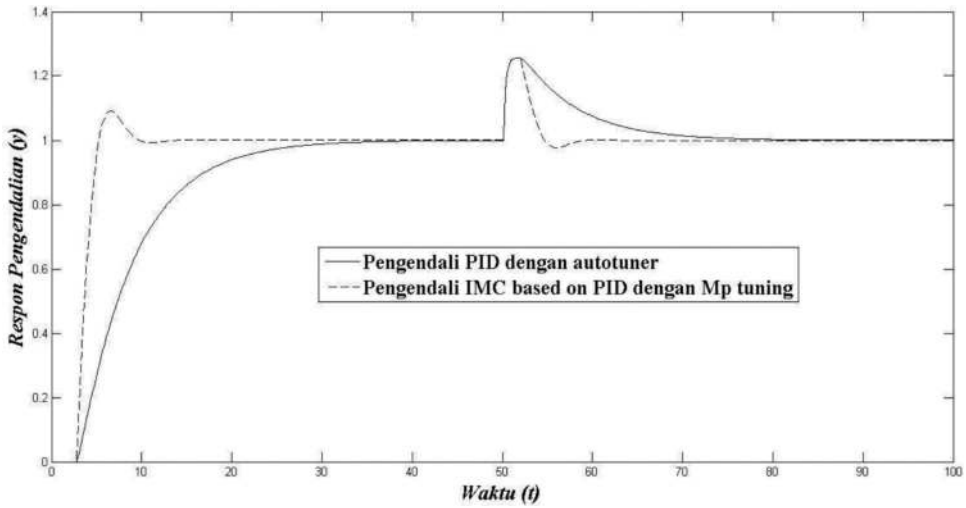
Figure 16. Comparison of temperature control responses in the process of hydrolysis of propylene oxide with CSTR reactor with disturbance +20% change in feed temperature simulation Simulink.

Based on transfer function of process, the value of process model parameters, respectively, for k_p, τ_p, and θ is 3.22, 0.97, and 0.15 was obtained. The parameter values for the worst case process are each of 3.864, 0.776, and 0.18.

Based on the simulation using Simulink and HYSYS software, the control result profile for disturbance change +20% from the propylene oxide feed temperature is given in **Figures 16** and **17**. **Figures 16** and **17** show that the resulting control profile gives almost the same result. From simulation using Simulink and HYSYS, it is shown that the use of Mp-GM tuning gives faster control response to achieve stability with smaller IAE compared with autotuner method.
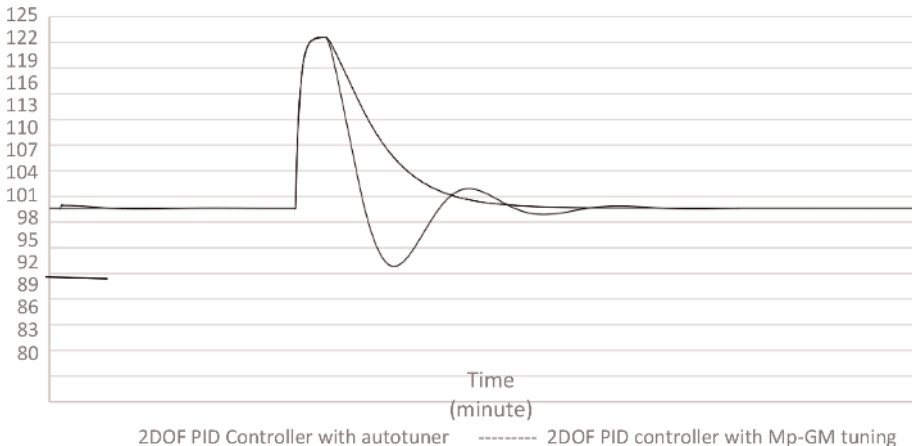

Figure 17. Comparison of temperature control responses in the process of hydrolysis of propylene oxide with CSTR reactor with disturbance +20% change in feed temperature simulation HYSYS.

Based on the calculation with Simpson rule method 1/3 obtained IAE value for 2DoF PID controller with autotuner and 2DoF PID controller with a Mp-GM tuning of 1221.721 and 528.3267. Similar results were obtained from the control response profile with disturbance −20% of the propylene oxide feed temperature as given in **Figures 16** and **17**. The control response with 2DOF PID controller with Mp-GM tuning gives better results when viewed from the control response profile or the resulting IAE value. Where based on Simpson rule method 1/3 obtained IAE value for 2DoF PID controller with autotuner and Mp-GM tuning is equal to 924.2412.

7. Conclusion

A maximum peak-gain margin (Mp-GM) tuning method has been used for 2DOF-IMC Kaya and 2DOF-PID. The simulation results show that the maximum peak gain margin tuning method can give a good target set point tracking, disturbance rejection, and robustness in system 2DOF structure controller with a little addition step. All of the process of FOPDT with different ratio of dead time and process time constant showed good responses. Mp-GM tuning is able to give better response than analytical robust tuning (ART) at the 2DOF-PID Vilanova structure control. The implementations of Mp-GM tuning on another model controller like 2DOF-IMC Kaya follow the similar steps by adding a correction factor of 0.5 multiplied by transfer function disturbance rejection. The implementations of Mp-GM tuning on another 2DOF-PID consist of three ways:

1. Determining the worst case as maximum value of complementary sensitivity function of 1DOF-IMC controller.

2. Determining parameter λ_1 by looping λ_1 in calculating Eq. (13) so that acquired maximum value of complementary sensitivity function will be 1.05 (for first looping, λ_1 will be set equal to θ), while for parameter λ_2 and α will be obtained by looping the value of α in calculating Eq. (27) for 2DOF-PID feedback and Eq. (28) for 2DOF-PID filter set point and Vilanova so that the acquired GM will be 2.4 by setting ratio of λ_2 to λ_1 as much as 0.9.

3. Substituting the value of k, τ, θ, λ_1, λ_2, and α into the previous equations that have been derived to obtain parameter value of PID controller (k_c, τ_I, τ_D, A) that will be used in 2DOF-PID controller.

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