

# PID Controller Tuning Based on the Classification of Stable, Integrating and Unstable Processes in a Parameter Plane

Tomislav B. Šekara and Miroslav R. Mataušek  
*University of Belgrade/Faculty of Electrical Engineering,  
Serbia*

## 1. Introduction

Classification of processes and tuning of the PID controllers is initiated by Ziegler and Nichols (1942). This methodology, proposed seventy years ago, is still actual and inspirational. Process dynamics characterization is defined in both the time and frequency domains by the two parameters. In the time domain, these parameters are the velocity gain  $K_v$  and dead-time  $L$  of an Integrator Plus Dead-Time (IPDT) model  $G_{ZN}(s)=K_v \exp(-Ls)/s$ , defined by the reaction curve obtained from the open-loop step response of a process. In the frequency domain these parameters are the ultimate gain  $k_u$  and ultimate frequency  $\omega_u$ , obtained from oscillations of the process in the loop with the proportional controller  $k=k_u$ . The relationship between parameters in the time and frequency domains is determined by Ziegler and Nichols as

$$L = \frac{\pi}{2\omega_u}, K_v = \varepsilon \frac{\omega_u}{k_u}, \varepsilon = \varepsilon_{ZN} = \frac{4}{\pi}. \quad (1)$$

However, for the process  $G_p(s)=G_{ZN}(s)$  in the loop with the proportional controller  $k$ , one obtains from the Nyquist stability criterion the same relationship (1) with  $\varepsilon=1$ . As a consequence, from (1) and the Ziegler-Nichols frequency response PID controller tuning, where the proportional gain is  $k=0.6k_u$ , one obtains the step response tuning  $k=0.3\varepsilon\pi/(K_vL)$ . Thus, for  $\varepsilon=\varepsilon_{ZN}$  one obtains  $k=1.2/(K_vL)$ , as in (Ziegler & Nichols, 1942), while for  $\varepsilon=1$  one obtains  $k=0.9425/(K_vL)$ , as stated in (Aström & Hägglund, 1995a). According to (1), the same values of the integral time  $T_i=\pi/\omega_u$  and derivative time  $T_d=0.25\pi/\omega_u$  are obtained in both frequency and time domains, in (Ziegler & Nichols, 1942) and from the Nyquist analysis. This will be discussed in more detail in Section 2.

Tuning formulae proposed by Ziegler and Nichols, were improved in (Hang et al., 1991; Aström & Hägglund, 1995a; 1995b; 2004). Besides the ultimate gain  $k_u$  and ultimate frequency  $\omega_u$  of process  $G_p(s)$ , the static gain  $K_p=G(0)$ , for stable processes, and velocity gain  $K_v = \lim_{s \rightarrow 0} sG_p(s)$ , for integrating processes, are used to obtain better process dynamics characterization and broader classification (Aström et al., 1992). Stable processes are approximated by the First-Order Plus Dead-Time (FOPDT) model  $G_{FO}(s)=K_p \exp(-Ls)/(Ts+1)$  and classified into four categories, by the normalized gain  $\kappa_1=K_p k_u$  and normalized dead-

time  $\theta_1=L/T$ . Integrating processes are approximated by the Integrating First-Order Plus Dead-Time (IFOPDT) model  $G_{IF}(s)=K_v \exp(-Ls)/(s(T_v s+1))$  and classified into two categories, by the normalized gain  $\kappa_2=K_v k_u/\omega_u$  and normalized dead-time  $\theta_2=L/T_v$ . The idea of classification proposed in (Aström et al., 1992) was to predict the achievable closed-loop performance and to make possible performance evaluation of feedback loops under closed-loop operating conditions.

In the present chapter a more ambitious idea is presented: define in advance the PID controller parameters in a classification plane for the purpose of obtaining a PID controller guaranteeing the desired performance/robustness tradeoff for the process classified into the desired region of the classification plane. It is based on the recent investigations related to: I) the process modeling of a large class of stable processes, processes having oscillatory dynamics, integrating and unstable processes, with the ultimate gain  $k_u$  (Šekara & Mataušek, 2010a; Mataušek & Šekara, 2011), and optimizations of the PID controller under constraints on the sensitivity to measurement noise, robustness, and closed-loop system damping ratio (Šekara & Mataušek, 2009,2010a; Mataušek & Šekara, 2011), II) the closed-loop estimation of model parameters (Mataušek & Šekara, 2011; Šekara & Mataušek, 2011b, 2011c), and III) the process classification and design of a new Gain Scheduling Control (GSC) in the parameter plane (Šekara & Mataušek, 2011a).

The motive for this research was the fact that the thermodynamic, hydrodynamic, chemical, nuclear, mechanical and electrical processes, in a large number of plants with a large number of operating regimes, constitutes practically an infinite batch of transfer functions  $G_p(s)$ , applicable for the process dynamics characterization and PID controller tuning. Since all these processes are nonlinear, some GSC must be applied in order to obtain a high closed-loop performance/robustness tradeoff in a large domain of operating regimes. A direct solution, mostly applied in industry, is to perform experiments on the plant in order to define GSC as the look-up tables relating the controller parameters to the chosen operating regimes. The other solution, more elegant and extremely time-consuming, is to define nonlinear models used for predicting accurately dynamic characteristics of the process in a large domain of operating regimes and to design a continuous GSC (Mataušek et al., 1996). However, both solutions are dedicated to some plant and to some region of operating regimes in the plant. The same applies for the solution defined by a nonlinear controller, for example the one based on the neural networks (Mataušek et al., 1999).

A real PID controller is defined by Fig. 1, with  $C(s)$  and  $C_{ff}(s)$  given by

$$C(s) = \frac{k_d s^2 + k_s s + k_i}{s(T_f s + 1)} F_C(s), \quad C_{ff}(s) = \frac{k_{ff} s + k_i}{s} F_C(s), \quad k_{ff} = b k, \quad F_C(s) \equiv 1, \quad 0 \leq b. \quad (2)$$

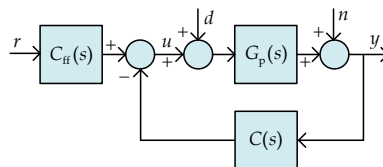


Fig. 1. Process  $G_p(s)$  with a two-degree-of-freedom controller.

An effective implementation of the control system (2) is defined by relations

$$U(s) = \left( k(bR(s) - Y_f(s)) + \frac{k_i}{s}(R(s) - Y_f(s)) - k_d s Y_f(s) \right) F_C(s), \quad Y_f(s) = \frac{Y(s)}{T_f s + 1}, \quad (3)$$

for  $F_C(s) \equiv 1$  as in (Panagopoulos et al., 2002; Mataušek & Šekara, 2011). When the proportional, integral, and derivative gains ( $k$ ,  $k_i$ ,  $k_d$ ) and derivative (noise) filter time constant  $T_f$  are determined, parameter  $b$  can be tuned as proposed in (Panagopoulos et al., 2002). The PID controller (2),  $F_C(s) \equiv 1$ , can be implemented in the traditional form, when noise filtering affects the derivative term only if some conditions are fulfilled (Šekara & Mataušek, 2009). The derivative filter time constant  $T_f$  must be an integral part of the PID optimization and tuning procedures (Isaksson & Graebe, 2002; Šekara & Mataušek, 2009).

For  $F_C(s)$  given by a second-order filter, one obtains a new implementation of the Modified Smith Predictor (Mataušek & Micić, 1996, 1999). The MSP-PID controller (3) guarantees better performance/robustness tradeoff than the one obtained by the recently proposed Dead-Time Compensators (DTC's), optimized under the same constraints on the sensitivity to measurement noise and robustness (Mataušek & Ribić, 2012).

Robustness is defined here by the maximum sensitivity  $M_s$  and maximum complementary sensitivity  $M_p$ . The sensitivity to measurement noise  $M_{n_r}$ ,  $M_{s_r}$ , and  $M_p$  are given by

$$M_s = \max_{\omega} \left| \frac{1}{1 + L(i\omega)} \right|, \quad M_p = \max_{\omega} \left| \frac{L(i\omega)}{1 + L(i\omega)} \right|, \quad M_n = \max_{\omega} |C_{nu}(i\omega)|, \quad (4)$$

where  $L(s)$  is the loop transfer function and  $C_{nu}(s)$  is the transfer function from the measurement noise to the control signal. In the present chapter, the sensitivity to the high frequency measurement noise is used  $M_n = M_{n_{\infty}}$ , where  $M_{n_{\infty}} = |C_{nu}(s)|_{s \rightarrow \infty}$ .

## 2. Modeling and classification of stable, integrating, and unstable plants

A generalization of the Ziegler-Nichols process dynamics characterization, proposed by Šekara and Mataušek (2010a), is defined by the model

$$G_m(s) = \frac{A\omega_u \exp(-\tau s)}{s^2 + \omega_u^2 - A\omega_u \exp(-\tau s)} \frac{1}{k_u}, \quad \tau = \frac{\varphi}{\omega_u}, \quad A = \frac{\omega_u k_u G_p(0)}{1 + k_u G_p(0)}, \quad (5)$$

where  $\varphi$  is the angle of the tangent to the Nyquist curve  $G_p(i\omega)$  at  $\omega_u$  and  $G_p(0)$  is the gain at the frequency equal to zero. Thus, for integrating processes  $G_p(0) = \pm\infty$  and  $A = \omega_u$ . Adequate approximation of  $G_p(s)$  by the model  $G_m(s)$  is obtained for  $\omega_u = \omega_{\pi}$ , where  $\arg\{G_p(i\omega_{\pi})\} = -\pi$ . It is demonstrated in (Šekara & Mataušek, 2010a; Mataušek & Šekara, 2011, Šekara & Mataušek, 2011a) that this extension of the Ziegler-Nichols process dynamics characterization, for a large class of stable processes, processes with oscillatory dynamics, integrating and unstable processes, guarantees the desired performance/robustness tradeoff if optimization of the PID controller, for the given maximum sensitivity  $M_s$  and given sensitivity to measurement noise  $M_{n_r}$ , is performed by applying the frequency response of the model (5) instead of the exact frequency response  $G_p(i\omega)$ .

Ziegler and Nichols used oscillations, defined by the impulse response of the system

$$G_p^*(s) = \frac{k_u G_p(s)}{1 + k_u G_p(s)}, \quad (6)$$

to determine  $k_u$  and  $\omega_u$ , and to define tuning formulae for adjusting parameters of the P, PI and PID controllers, based on the relationship between the quarter amplitude damping ratio and the proportional gain  $k$ . Oscillations defined by the impulse response of the system (6) are used in (Šekara & Mataušek, 2010a) to define model (5), obtained from  $G_m(s) \approx G_p(s)$  and the relation

$$\frac{k_u G_m(s)}{1 + k_u G_m(s)} = \frac{A \omega_u \exp(-\tau s)}{s^2 + \omega_u^2}. \quad (7)$$

Then, by analyzing these oscillations, it is obtained in (Šekara & Mataušek, 2010a) that amplitude  $A = \omega_u \kappa / (1 + \kappa)$ ,  $\kappa = k_u G_p(0)$ , and dead-time  $\tau$  is defined by  $\omega_u$  and a parameter  $\varphi$ , given by

$$\varphi = \arg \left( \frac{\partial G_p(\mathbf{i}\omega)}{\partial \omega} \right) \Bigg|_{\omega=\omega_u}. \quad (8)$$

Other interpretation of amplitude  $A = A_0$ , obtained in (Mataušek & Šekara, 2011), is defined by

$$A_0 = \frac{2}{k_u} \left| \frac{\partial G_p(\mathbf{i}\omega)}{\partial \omega} \right|_{\omega=\omega_u}^{-1}. \quad (9)$$

Amplitudes  $A$  and  $A_0$  are not equal, but they are closely related for stable and unstable processes, as demonstrated in (Mataušek & Šekara, 2011) and Appendix. Parameter  $A_0$  is not used for integrating processes, since for these processes  $A = \omega_u$ .

The quadruplet  $\{k_u, \omega_u, \varphi, A\}$  is used for classification of stable processes, processes with oscillatory dynamics, integrating and unstable processes in the  $\rho$ - $\varphi$  parameter plane, defined by the normalized model (5), given by

$$G_n(s_n, \rho, \varphi) = \frac{\rho \exp(-\varphi s_n)}{s_n^2 + 1 - \rho \exp(-\varphi s_n)}, \quad s_n = \frac{s}{\omega_u}, \quad (10)$$

where  $\rho = A/\omega_u$ . From the Nyquist criterion it is obtained that the region of stable processes is defined by  $0 < \varphi < \pi / \sqrt{\rho + 1}$ ,  $0 < \rho < 1$  (Šekara & Mataušek, 2011a). Integrating processes, since  $A = \omega_u$ , are classified as  $\rho = 1$ ,  $0 < \varphi < \pi / \sqrt{2}$  processes, while unstable processes are outside these regions. It is demonstrated that a large test batch of stable and integrating processes used in (Aström & Hägglund, 2004) covers a small region in the  $\rho$ - $\varphi$  plane.

To demonstrate that besides  $k_u$  and  $\omega_u$ , parameters  $\varphi$  and  $G_p(0)$  must be used for the classification of processes, Nyquist curves are presented in Fig. 2 for stable, integrating and

unstable processes having the same values  $k_u=1$  and  $\omega_u=1$ . For processes having also the same values of  $\varphi$ , the distinction of the Nyquist curves in the broader region around the critical point requires the information about gain  $G_p(0)$ , as demonstrated in Fig. 2-a. On the other hand, the results presented in Fig. 2-b to Fig. 2-d demonstrate that for the same values of  $k_u$ ,  $\omega_u$ , and  $G_p(0)$  the distinction of the Nyquist curves in the region around the critical point is obtained by applying parameter  $\varphi$ . This fact confirms importance of parameter  $\varphi$  in process modeling for controller tuning, taking into account that optimization of the PID controller under constraints on the robustness is performed in the region around  $\omega_u$ .

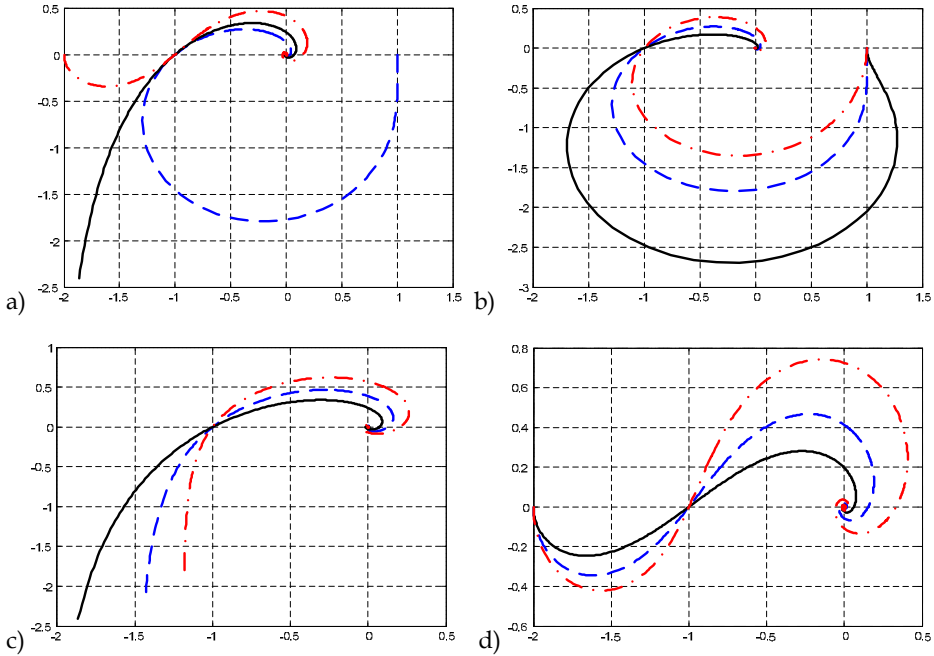


Fig. 2. Nyquist curves of processes with the same values  $k_u=1$ ,  $\omega_u=1$ : a)  $\varphi=\pi/4$ , stable  $G_p(0)=1$  (dashed), integrating  $G_p(0)=\infty$  (solid), unstable  $G_p(0)=-2$  (dashed-dotted); b) stable processes with  $G_p(0)=1$ , for  $\varphi=\pi/4$  (dashed),  $\varphi=\pi/6$  (solid),  $\varphi=\pi/3$  (dashed-dotted); c) integrating processes with  $\varphi=1$  (dashed),  $\varphi=\pi/4$  (solid),  $\varphi=1.2$  (dashed-dotted); d) unstable processes with  $G_p(0)=-2$ , for  $\varphi=\pi/4$  (dashed),  $\varphi=\pi/6$  (solid),  $\varphi=\pi/3$  (dashed-dotted).

For the lag dominated process

$$G_{p1}(s) = 1 / \cosh \sqrt{2s} , \tag{11}$$

and the corresponding models, the step and impulse responses, with the Nyquist curves around  $\omega_u$  are presented in Fig. 3. The models are Ziegler-Nichols IPDT model  $G_{ZN}(s)=K_v \exp(-Ls)/s$  and model (5), with  $A=\omega_u k_u G_p(0)/(1+k_u G_p(0))$  and  $A=A_0$ . The set-point and load disturbance step responses of this process, in the loop with the optimal PID controller (Mataušek & Šekara, 2011) and PID controller tuned as proposed by Ziegler and Nichols (1942), are compared in Fig. 4-a. In this case  $k_u=11.5919$ ,  $\omega_u=9.8696$  and  $K_v=0.9251$ ,

$L=0.1534$ . The PID controller tuned as proposed by Ziegler and Nichols is implemented in the form

$$U(s) = k(bR(s) - Y(s)) + \frac{k_i}{s}(R(s) - Y(s)) - \frac{k_d s}{T_f s + 1} Y(s), \quad b = 0, k_i = \frac{k}{T_i}, k_d = kT_d, T_f = \frac{T_d}{N_d}, \quad (12)$$

where  $k=0.6k_u$ ,  $T_i=\pi/\omega_u$ ,  $T_d=\pi/(4\omega_u)$ , for the frequency domain ZN tuning (ZN PID1). For the time domain ZN tuning (ZN PID2) the parameters are  $k=1.2/(K_v L)$ ,  $T_i=2L$ ,  $T_d=L/2$ , or, as suggested by the earlier mentioned Nyquist analysis, proportional gain  $k$  is adjusted to  $k=0.943/(K_v L)$ , denoted as the modified time domain ZN tuning (ZN ModifPID2). In  $M_n = (N_d + 1)|k|$  parameter  $N_d$  is adjusted to obtain the same value of  $M_n=76.37$  used in the constrained optimization of the PID in (3),  $F_C(s) \equiv 1$ , where  $M_n = |k_d| / T_f$ .

Parameters of the PID controllers and performance/robustness tradeoff are compared in Table 1. It is impressive that Ziegler and Nichols succeeded in defining seventy years ago an excellent experimental tuning for the process  $G_{p1}(s)$ , which is an infinite-order system that can be represented in simulation by the following high-order system  $G_{p1}(s) \approx \exp(-Ls) / \prod_{k=1}^{20} (T_k s + 1)$ ,  $L=0.01013$  (Mataušek & Ribić, 2009). Also, it should be noted here, that Ziegler and Nichols succeeded seventy years ago in obtaining an excellent tuning with the IPDT model defined by  $K_v=0.9251$ ,  $L=0.1534$ , which is an extremely crude approximation of the real impulse response of the process  $G_{p1}(s)$ , as in Fig. 3-b.

Tuning method	$k$	$k_i$	$k_d$	$T_f$	$N_d$	IAE	$M_n$	$M_s$	$M_p$
optPID	6.5483	18.4321	0.6345	0.0094	-	0.0609	76.37	2.00	1.45
ZN PID1	6.9551	21.8502	0.5535	0.0080	9.980	0.0538	76.37	2.20	1.72
ZN PID2	8.4560	27.5621	0.6486	0.0096	8.031	0.0429	76.37	2.82	2.23
ZN ModifPID2	6.6450	21.6592	0.5097	0.0073	10.49	0.0587	76.37	2.16	1.78

Table 1. Process  $G_{p1}(s)$ : comparison of the optimization (optPID) and the Ziegler-Nichols tuning in the frequency domain (ZN PID1) and time domain (ZN PID2, ZN ModifPID2).

The Nyquist curves of  $G_{p1}(s)$ ,  $G_{m1}(s)$ , and  $G_{m2}(s)$  are almost the same around  $\omega_u$ . This is important since the PID controller optimization, based on the experimentally determined frequency response of the process, under constraints on  $M_s$  or on  $M_s$  and  $M_p$ , is performed around the ultimate frequency  $\omega_u$ . Amplitudes  $A$  and  $A_0$  are closely related for the stable and unstable processes, as demonstrated in (Mataušek & Šekara, 2011) and Appendix. For integrating processes  $A=\omega_u$ . This means, that the Ziegler-Nichols parameters  $k_u$  and  $\omega_u$ , and the Šekara-Mataušek parameters  $\varphi$  and  $A=A_0$ , for the stable and unstable processes, and  $A=\omega_u$ , for integrating processes, constitute the minimal set of parameters, measurable in the frequency domain, necessary for obtaining PID controller tuning for the desired performance/robustness tradeoff. This will be demonstrated in the subsequent sections.

### 3. Optimization of PI/PID controllers under constraints on the sensitivity to measurement noise, robustness, and closed-loop system damping ratio

PID controllers are still mostly used control systems in the majority of industrial applications (Desborough & Miller, 2002) and "it is reasonable to predict that PID control

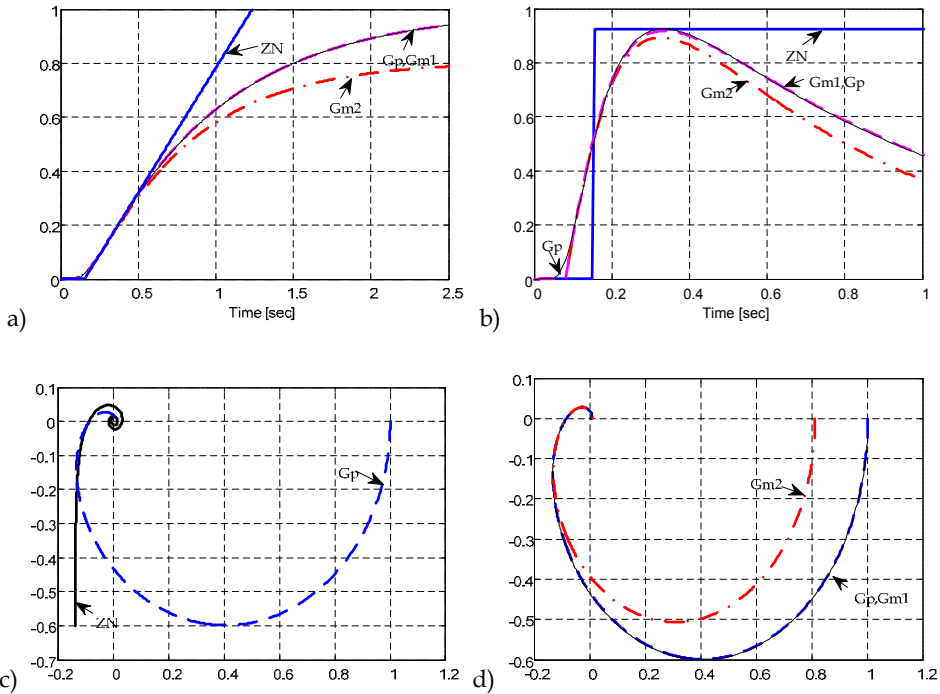


Fig. 3. Process  $G_{p1}(s)$ , denoted as ( $G_p$ ), and models  $G_{mj}(s)$ ,  $j=1,2$ ,  $k_u=11.5919$ ,  $\omega_u=9.8696$ ,  $\tau=0.0796$  for  $A=9.0858$  ( $G_{m1}$ ) and  $A=A_0=8.9190$  ( $G_{m2}$ ), and  $G_{ZN}(s)=K_v \exp(-Ls)/s$ ,  $K_v=0.9251$ ,  $L=0.1534$  (ZN): a) step responses, b) impulse responses, c) Nyquist curves of  $G_{p1}(s)$  and  $G_{ZN}(s)$ , d) Nyquist curves of  $G_{p1}(s)$ ,  $G_{m1}(s)$  and  $G_{m2}(s)$  are almost the same around  $\omega_u$ .

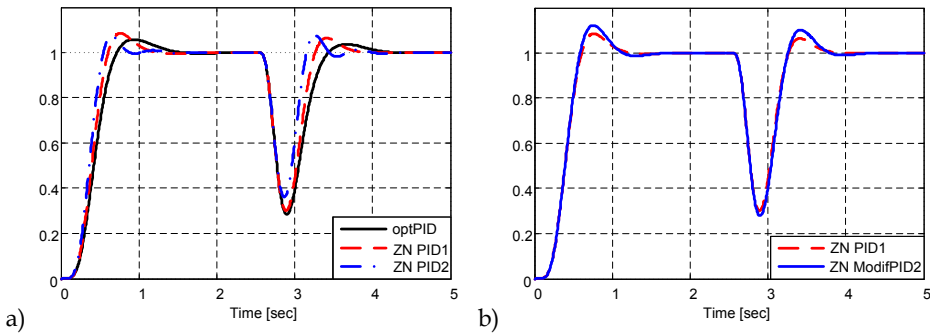


Fig. 4. Comparison of the optimization and the Ziegler-Nichols (ZN) tuning. Process  $G_{p1}(s)$  in the loop with the optPID or ZN PID, tuned by using the rules: frequency domain (ZN PID1), time domain (ZN PID2), and time domain with the modified proportional gain  $k=0.943/(K_v L)$  (ZN ModifPID2). In all controllers  $b=0$  and  $D(s)=-5 \exp(-2.5s)/s$ .

will continue to be used in the future" (Aström & Hägglund, 2001). They operate mostly as regulators (Aström & Hägglund, 2001) and rejection of the load step disturbance is of

primary importance to evaluate PID controller performance under constraints on the robustness (Shinsky, 1990), measured by the Integrated Absolute Error (IAE). Inadequate tuning and sensitivity to measurement noise are the reasons why derivative action is often excluded in the industrial process control. This is the main reason why PI controllers predominate (Yamamoto & Hashimoto, 1991). However, for lag-dominated processes, processes with oscillatory dynamics and integrating/unstable processes PID controller guarantees considerably better performance than PI controller, if adequate tuning of the PID controller is performed (Mataušek & Šekara, 2011). Moreover, PID controller is a “prerequisite for successful advanced controller implementation” (Seki & Shigemasa, 2010).

Besides PI/PID controllers, in single or multiple loops (Jevtović & Mataušek, 2010), only Dead-Time Compensators (DTC) are used in the process industry with an acceptable percentage (Yamamoto & Hashimoto, 1991). They are based on the Smith predictor (Smith, 1957; Mataušek & Kvaščev, 2003) or its modifications. However, the area of application of PID controllers overlaps deeply with the application of DTC's, as confirmed by the Modified Smith Predictor, which is a PID controller in series with a second-order filter, applicable to a large class of stable, integrating and unstable processes (Mataušek & Ribić, 2012).

Optimization of the performance may be carried out under constraints on the maximum sensitivity to measurement noise  $M_n$ , the maximum sensitivity  $M_s$  and maximum complementary sensitivity  $M_p$ , as done in (Mataušek & Ribić, 2012). In this case it is recommended to use some algorithm for global optimization, such as Particle Swarm Optimization algorithm (Rapaić, 2008), requiring good estimates of the range of unknown parameters. Other alternatives, presented here, are recently developed in (Šekara & Mataušek, 2009, 2010a; Mataušek & Šekara, 2011). For the PID controller (3), for  $F_C(s) \equiv 1$  defined by four parameters  $k$ ,  $k_i$ ,  $k_d$  and  $T_f$ , optimization under constraints on  $M_n$  and  $M_s$  is reduced in (Šekara & Mataušek, 2009) to the solution of a system of three algebraic equations with adequate initial values of the unknown parameters. The adopted values of  $M_n$  and  $M_s$  are satisfied exactly for different values of  $\zeta_z$ . Thus, by repeating calculations for a few values of the damping ratio of the controller zeros  $\zeta_z$  in the range  $0.5 \leq \zeta_z$ , the value of  $\zeta_z$  corresponding to the minimum of IAE is obtained. Optimization methods from (Šekara & Mataušek, 2009) are denoted as max(k) and max(ki) methods.

The improvement of the max(k) method is proposed in (Šekara & Mataušek, 2010a). It consists of avoiding repetition of calculations for different values of  $\zeta_z$  in order to obtain the minimal value of the IAE for a desired value of  $M_s$ . In this method, denoted here as method optPID, the constrained optimization is based on the frequency response of model (5).

For the PI optimization, an improvement of the performance/robustness tradeoff is obtained by applying the combined performance criterion  $J_c = \beta k_i + (1-\beta)\omega$  (Šekara & Mataušek, 2008). Thus, one obtains

$$\max_{k_i, \omega} J_c, \quad (13)$$

$$F(\omega, k, k_i) = 0, \quad \partial F(\omega, k, k_i) / \partial \omega = 0, \quad (14)$$

where  $0 \leq \omega < \infty$  and  $\beta$  is a free parameter in the range  $0 < \beta \leq 1$ . The calculations are repeated for a few values of  $\beta$ , in order to find  $\beta$  corresponding to the minimum of IAE. The optimization



in this method, denoted here as opt2, is performed for the desired value of  $M_s$ . For  $\beta=1$  one obtains the same values of parameters  $k$  and  $k_i$  as obtained by the method proposed in (Aström et al., 1998), denoted here as opt1.

The most general is the new tuning and optimization procedure proposed in (Mataušek & Šekara, 2011). Besides the tuning formulae, the optimization procedure is derived. For the PID and PI controllers it requires only obtaining the solution of two nonlinear algebraic equations with adequate initial values of the unknown parameters. PID optimization is performed for the desired closed-loop system of damping ratio  $\zeta$  and under constraints on  $M_n$  and  $M_s$ . Thus, for  $\zeta=1$  the critically damped closed-loop system response is obtained. PI optimization is performed under constraint on  $M_s$  for the desired value of  $\zeta$ . The procedure proposed in (Mataušek & Šekara, 2011) will be discussed here in more details, since it is entirely based on the concept of using oscillators (6)-(7) for dynamics characterization of the stable processes, processes having oscillatory dynamics, integrating and unstable processes. The method is derived by defining a complex controller  $C(s)=k_u(1+C^*(s))$ , where the controller  $C^*(s)$ , given by

$$C^*(s) = \frac{s^2 + \omega_u^2}{A\omega_u\Lambda(s)} \frac{E(s) / \Lambda(s)}{1 - E(s)\exp(-\tau s) / \Lambda(s)^2}, \quad E(s) = \eta_2 s^2 + \eta_1 s + 1, \quad \Lambda(s) = \lambda^2 s^2 + 2\zeta\lambda s + 1, \quad (15)$$

is obtained by supposing that in Fig. 1 process  $G_p(s)$  is defined by oscillator  $G_p^*(s)$  in (6), approximated by (7). Complex controller  $C(s)=k_u(1+C^*(s))$  is defined by the parameters  $k_u$ ,  $\omega_u$ ,  $\tau$ ,  $A$  and by the two tuning parameters  $\lambda$  and  $\zeta$ , with the clear physical interpretation. Parameter  $\lambda$  is proportional to the desired closed-loop system time constant. Parameter  $\zeta$  is the desired closed-loop system damping ratio. Then, by applying Maclaurin series expansion, the possible internal instability of the complex controller  $C(s)$  is avoided and parameters of PID controller  $C(s)$  in Fig. 1 are obtained, defined by:

$$T_f = \frac{\eta_2 - \beta_2(\eta_1 - \beta_2) - \beta_3 + 1 / \omega_u^2}{\beta_2 - \eta_1 - (1 - M_n / |k_u|) / \beta_1}, \quad (16)$$

$$k = k_u (\beta_1(T_f + \eta_1 - \beta_2) + 1), \quad k_i = k_u \beta_1, \quad (17)$$

$$k_d = k_u \beta_1 (\eta_2 + (T_f - \beta_2)(\eta_1 - \beta_2) - \beta_3 + 1 / \omega_u^2) + k_u T_f. \quad (18)$$

Parameters  $\eta_1$ ,  $\eta_2$ ,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$ , from (Mataušek & Šekara, 2011), depends on  $\lambda$ ,  $\zeta$  and  $k_u$ ,  $\omega_u$ ,  $\tau$ ,  $A$ . They are given in Appendix. Generalization of this approach is presented in (Šekara & Trifunović, 2010; Šekara et al., 2011).

For the desired closed-loop damping ratio  $\zeta=1$ ,  $\lambda=1/\omega_u$ , and for

$$T_f = 1 / (N\omega_u), \quad (19)$$

one obtains (Mataušek & Šekara, 2011) the PID tuning that guarantees set-point and load disturbance step responses with negligible overshoot for a large class of stable processes, processes with oscillatory dynamics, integrating and unstable processes. Tuning formulae

defined by (17)-(19) are denoted here as method  $tun\lambda_u$ . Absolute value of the Integrated Error (IE), approximating almost exactly the obtained IAE, is given by  $|IE| = 1/(|k_u|\beta_1)$ . Here the value  $T_f = 1/(10\omega_u)$  is used, as in (Mataušek & Šekara, 2011).

To demonstrate the relationship between PID controller, tuned by using the method  $tun\lambda_u$ , and complex controller  $C(s) = k_u(1+C^*(s))$ , obtained for  $\lambda = 1/\omega_u$  and  $\zeta = 1$ , the frequency responses of these controllers, tuned for the process

$$G_{p2}(s) = 1 / (s + 1)^4, \tag{20}$$

are presented in Fig. 5-a. For this process, parameters  $k_u$ ,  $\omega_u$ ,  $\tau$ ,  $A$ ,  $\rho$  and  $\phi$  are given in Appendix. The load disturbance unite step responses, obtained for  $G_{p2}(s)$  in the loop with the PID controller and complex controller  $C(s)$ , are presented in Fig. 5-b. Further details about the relationship between these controllers are presented in (Mataušek & Šekara, 2011; Trifunović & Šekara, 2011; Šekara et al., 2011).

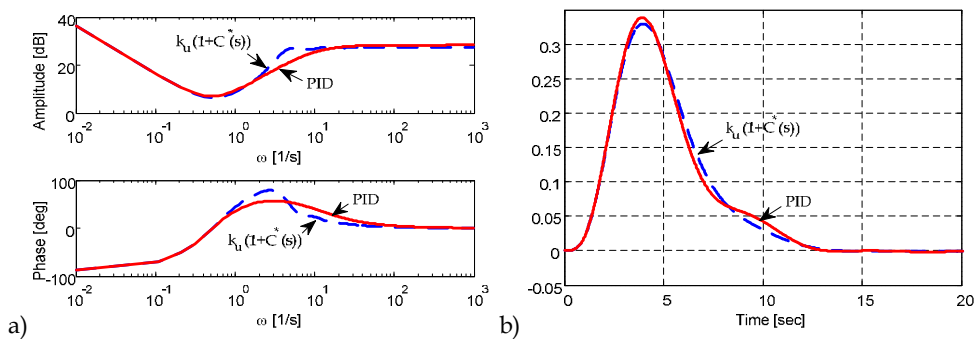


Fig. 5. Comparison of the complex controller  $C(s) = k_u(1+C^*(s))$  with PID controller, both tuned for  $G_{p2}(s)$ : a) Bode plots of the controllers and b) the load unite step disturbance responses of  $G_{p2}(s)$  in the loop with these controllers.

By applying tuning formulae (17)-(19), the desired closed-loop damping ratio  $\zeta = 1$  is obtained with the acceptable values of maximum sensitivity  $M_s$  and maximum sensitivity to measurement noise  $M_n$ . However, when a smaller value of  $M_n$  is required for a desired value of  $M_s$  and the desired closed-loop damping ratio  $\zeta$ , the other possibility is to determine the closed-loop time constant  $\lambda$  and the corresponding  $\omega_0$ , by using (16)-(18) and by solving two algebraic equations:

$$|1 + C(i\omega)G_m(i\omega)|^2 - 1 / M_s^2 = 0, \tag{21}$$

$$\partial(|1 + C(i\omega)G_m(i\omega)|^2) / \partial\omega = 0. \tag{22}$$

In this case, the PID controller in (3),  $F_C(s) \equiv 1$ , is obtained for the desired critical damping ratio  $\zeta = 1$  of the closed-loop system and the desired values of  $M_n$  and  $M_s$ . This is the unique possibility of the procedure (16)-(18) and (21)-(22) proposed in (Mataušek & Šekara, 2011). Moreover, by repeating the calculations for a few values of  $\zeta$ , the value of  $\zeta$  is obtained

guaranteeing, for desired  $M_n$  and  $M_s$ , almost the same value of the IAE as obtained by the constrained PID optimization based on the exact frequency response  $G_p(i\omega)$ . This PID optimization method is denoted here as the method opt2A, when the quadruplet  $\{k_u, \omega_u, \varphi, A\}$  is used, or opt2A<sub>0</sub>, when the quadruplet  $\{k_u, \omega_u, \varphi, A_0\}$  is used. It should be noted here, that for  $k_d=0$  and  $T_f=0$ , by relations (17) and (21)-(22) a new effective constrained PI controller optimization is obtained, denoted here as opt3. It is successfully compared (Mataušek & Šekara, 2011) with the procedure proposed in (Aström et al., 1998), opt1.

Now, tuning defined by (17)-(19) with  $N=10$ ,  $\lambda=1/\omega_u$  and  $\zeta=1$ , method  $\text{tun}\lambda_u$  will be compared with the optimization defined by (16)-(18), (21)-(22), method opt2A. Both procedures guarantee desired critical damping  $\zeta=1$ , however only the second one guarantees the desired values of  $M_n$  and  $M_s$ . Thus, for  $\zeta=1$  and for the maximum sensitivity  $M_s$  obtained by applying method  $\text{tun}\lambda_u$ , the smaller value of sensitivity to measurement noise  $M_n$  will be used by applying PID optimization method opt2A. The results of this analysis are presented in Table 2 and Fig. 6. As in Table 1, controller is tuned by using the model  $G_m(s)$  in (5) and then applied to processes  $G_{p3}(s)$  to obtain IAE,  $M_s$  and  $M_p$ , where

$$G_{p3}(s) = \frac{1.507(3.42s + 1)(1 - 0.816s)}{(577s + 1)(18.1s + 1)(0.273s + 1)(104.6s^2 + 15s + 1)}. \quad (23)$$

Lower value of IAE is obtained, for almost the same robustness, by using higher value of the sensitivity to measurement noise. However, for the lower value of  $M_n$  the controller and, as a result, the actuator activity is considerably reduced. Thus, the comparison of the IAE, obtained by the PID controllers with the same robustness, is meaningless if the sensitivity to measurement noise  $M_n$  is not specified, as demonstrated in Fig. 6. This fact is frequently ignored.

method	$\lambda$	$k$	$k_i$	$k_d$	$T_f$	IAE	$M_n$	$M_s$	$M_p$
tun $\lambda_u$	17.3310	22.3809	0.2778	377.2723	1.7331	3.62	217.7	2.14	1.58
opt2A	20.4849	18.2791	0.1944	345.5996	5.0893	5.17	67.91	2.12	1.51

Table 2. Process  $G_{p3}(s)$  in the loop with the PID controllers. Tuning method (17)-(19),  $\text{tun}\lambda_u$  and optimization (16)-(18), (21)-(22), opt2A for  $\zeta=1$ .

Concluding this section, the constrained PI/PID controller optimization methods proposed in (Mataušek & Šekara, 2011) is compared with the constrained PID controller optimization method proposed in (Šekara & Mataušek, 2010a), optPID1, and the constrained PI controller optimization method proposed in (Šekara & Mataušek, 2008), opt2. The test batch of stable processes, processes having oscillatory dynamics, integrating and unstable processes used in this analysis is defined by transfer functions  $G_{p1}(s)$ ,  $G_{p2}(s)$ ,  $G_{p3}(s)$  and

$$G_{p4}(s) = \frac{e^{-5s}}{(s + 1)^3}, G_{p5}(s) = \frac{e^{-s}}{9s^2 + 0.24s + 1}, \quad (24)$$

$$G_{p6}(s) = \frac{e^{-5s}}{s(s + 1)(0.5s + 1)(0.25s + 1)(0.125s + 1)}, G_{p7}(s) = \frac{2e^{-5s}}{(10s - 1)(2s + 1)}, \quad (25)$$

with parameters  $k_u, \omega_u, \tau, A, A_0, \rho, \varphi$  presented in Appendix. Comparison of the methods for PID controller tuning is presented in Table 3. Comparison of the methods for PI controller tuning is presented in Table 4 and Fig. 7.

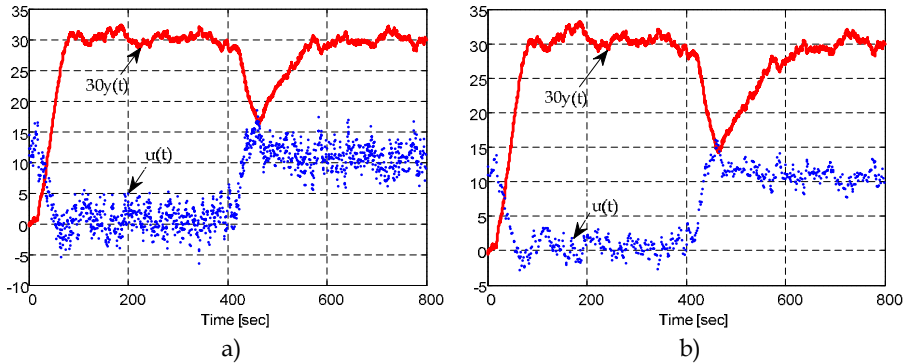


Fig. 6. Set-point,  $R(s)=1/s$ , and load disturbance,  $D(s)=-10\exp(-400s)/s$ , step responses.  $G_{p3}(s)$  and PID controllers tuned by: a)  $tun\lambda_u, b=0.5$ ; b)  $opt2A, b=0.6$ . Measurement noise is obtained by passing uniform random noise  $\pm 1$  through a low-pass filter  $F(s)=0.5/(10s+1)$ .

Process/ method	$k$	$k_i$	$k_d$	$T_i$	IAE	$M_n$	$M_s$	$M_p$	$\zeta_z$	$\zeta$
$G_{p3}/\max(k)$	17.0778	0.2372	320.06	4.7131	4.83	67.91	2.00	1.56	0.98	-
$G_{p3}/optPID$	17.1037	0.2303	315.14	4.6407	4.84	67.91	2.00	1.54	-	-
$G_{p3}/opt2A$	17.1994	0.1788	316.59	4.6621	5.62	67.91	2.00	1.41	-	1
$G_{p3}/opt2A_0$	16.9411	0.2670	312.65	4.6040	4.87	67.91	2.00	1.69	-	0.75
$G_{p3}/opt2A$	16.8802	0.2083	268.32	3.9513	4.92	67.91	2.00	1.59	-	0.80
$G_{p5}/\max(k_i)$	-0.3090	0.0654	0.8640	1.7597	21.17	0.49	2.00	1.03	0.65	-
$G_{p5}/optPID$	-0.3032	0.0651	0.8280	1.6864	21.87	0.49	2.00	1.07	-	-
$G_{p5}/opt2A$	-0.4139	0.0336	0.9398	1.9140	30.04	0.49	2.00	1.04	-	1
$G_{p5}/opt2A_0$	-0.3369	0.0583	0.8948	1.8223	20.29	0.49	2.00	1.02	-	0.65
$G_{p5}/opt2A$	-0.3542	0.0528	0.8860	1.8044	20.30	0.49	2.00	1.02	-	0.70
$G_{p6}/\max(k)$	0.1177	0.0063	0.3961	0.8353	207.22	0.47	2.00	1.76	1.18	-
$G_{p6}/optPID$	0.1181	0.0054	0.3736	0.7878	208.65	0.47	2.00	1.63	-	-
$G_{p6}/opt2A$	0.1133	0.0043	0.2373	0.5003	234.73	0.47	2.01	1.60	-	1
$G_{p6}/opt2A$	0.1160	0.0043	0.2709	0.5712	233.50	0.47	2.01	1.55	-	1.05
$G_{p7}/\max(k)$	0.8608	0.0158	3.3101	0.1418	73.50	23.35	3.61	3.39	1.88	-
$G_{p7}/optPID$	0.8609	0.0150	3.2946	0.1411	75.00	23.35	3.61	3.33	-	-
$G_{p7}/opt2A$	0.8543	0.0106	2.9385	0.1258	93.96	23.35	3.54	3.18	-	1.3
$G_{p7}/opt2A_0$	0.8060	0.0093	2.3759	0.1017	107.41	23.35	3.61	3.77	-	1.1

Table 3. PID controllers, obtained by applying model  $G_m(s)$  and tuning methods:  $\max(k)$ ,  $\max(k_i)$ ; (31)-(35)  $optPID$ ; (16)-(18), (21)-(22)  $opt2A$  and  $opt2A_0$ .

In Table 3 optimization (16)-(18), (21)-(22) is performed for stable  $G_{p3}(s)$ ,  $G_{p5}(s)$  and unstable  $G_{p7}(s)$  processes by using  $G_m(s)$  with two quadruplets:  $\{k_u, \omega_u, \varphi, A\}$ , denoted as  $opt2A$ , and

$\{k_u, \omega_u, \varphi, A_0\}$ , denoted  $\text{opt2}A_0$ . As mentioned previously, for integrating processes  $A=\omega_u$ . Almost the same performance/robustness tradeoff is obtained for  $A$  and  $A_0$ , as supposed in Section 2. This result is important since it confirms that an adequate approximation of the frequency response of the stable and unstable processes around  $\omega_u$  can be used in the optimization (16)-(18) and (21)-(22), instead of the model  $G_m(i\omega)$  in (5). Obviously, the same applies for integrating processes. The advantage of the constrained PID controller optimization (16)-(18) and (21)-(22) is that only two nonlinear algebraic equations have to be solved, with very good initial conditions for the unknown parameters  $\lambda$  and  $\omega_b$ . Moreover, the optimization is performed for the desired values of  $M_s$ ,  $M_n$  and for the desired closed-loop system damping ratio  $\zeta$ .

Finally, the results of the PI controller optimization are demonstrated in Table 4 and in Fig. 7. By repeating calculations for a few values of  $\zeta$ , for the same values of  $M_s$  and  $M_p$ , the same (minimal) value of the IAE is obtained by applying method  $\text{opt3}$ , defined by (17) and (21)-(22), and the method  $\text{opt2}$ , defined by (13)-(14). As mentioned previously, method  $\text{opt2}$  is an improvement of the method proposed in (Aström et al., 1998), denoted here as method  $\text{opt1}$ .

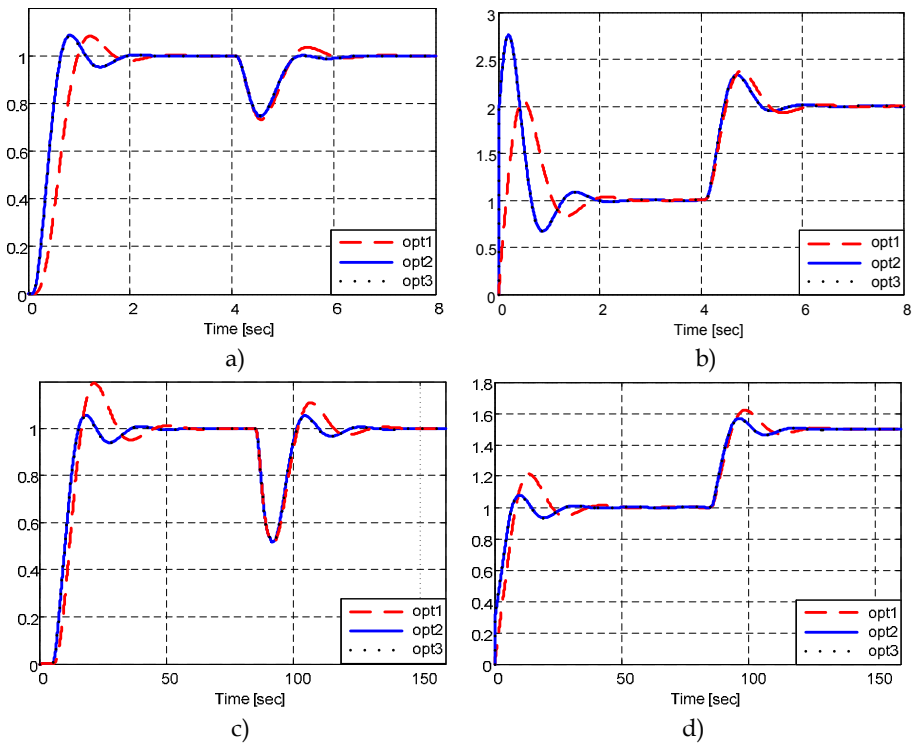


Fig. 7. Set-point and load disturbance step responses:  $y(t)$  (left) and  $u(t)$  (right). PI controllers from Table 4:  $\text{opt1 } b=0$ ,  $\text{opt2 } b=0.6$ ,  $\text{opt3 } b=0.6$ . In a) and b)  $G_{p1}(s)$ ,  $D(s)=-\exp(-4s)/s$ ; in c) and d)  $G_{p4}(s)$ ,  $D(s)=-0.5\exp(-80s)/s$ .

Process/method	$k$	$k_i$	IAE	$M_s$	$M_p$	$\beta$	$\zeta$
$G_{p1}/\text{opt1}$	2.6707	6.4739	0.19	1.98	1.58	1	-
$G_{p1}/\text{opt2}$	3.1874	6.1391	0.16	1.99	1.48	0.52	-
$G_{p1}/\text{opt3}$	3.2119	6.1083	0.16	1.99	1.48	-	0.95
$G_{p3}/\text{opt1}$	7.4060	0.0692	15.61	1.92	1.65	1	-
$G_{p3}/\text{opt2}$	8.1456	0.0680	14.72	1.94	1.60	0.48	-
$G_{p3}/\text{opt3}$	8.1355	0.0679	14.73	1.94	1.60	-	0.90
$G_{p4}/\text{opt1}$	0.3248	0.1259	12.04	2.16	1.35	1	-
$G_{p4}/\text{opt2}$	0.4608	0.1137	10.23	2.11	1.18	0.69	-
$G_{p4}/\text{opt3}$	0.4651	0.1128	10.19	2.10	1.18	-	0.90

Table 4. PI controllers, obtained for  $M_s=2$  by applying model (5) and methods: (Aström et al., 1998) opt1, (13)-(14) opt2, and (17), (21)-(22) opt3.

#### 4. Closed-loop estimation of model parameters

Approximation of process dynamics, around the operating regime, can be defined by some transfer function  $G_p(s)$  obtained from the open-loop or closed-loop process identification. One two step approach (Hjalmarsson, 2005) is based on the application of the high-order ARX model identification in the first step. In the second step, to reduce the variance of the obtained estimate of frequency response of the process, caused by the measurement noise, this ARX model is reduced to a low-order model  $G_p(s)$ . By applying this procedure an adequate approximation  $G_p(i\omega)$  of the unknown Nyquist curve can be obtained in the region around the ultimate frequency  $\omega_u$ . As demonstrated for the Ziegler-Nichols tuning, in Fig. 3-c and Fig. 4-b, such approximation of the unknown Nyquist curve is of essential importance for designing an adequate PID controller. The same applies for the successful PID optimization under constraints on the desired values of  $M_n$  and  $M_s$ , as demonstrated in Table 5 for the value of  $A$  defined as in (5) and for  $A=A_0$ .

The Closed-Loop (CL) system identification can be performed by using indirect or direct identification methods. In indirect CL system identification methods it is assumed that the controller in operation is linear and a priori known. Direct CL system identification methods are based only on the plant input and output data (Agüero et al., 2011). Finally, the identification can be based on the simple tests, as initiated by Ziegler and Nichols (1942), to obtain an IPDT model (1). Later on, this approach is extended to obtain FOPDT model and the Second-Order Plus Dead-Time (SOPDT) model, for integrating processes characterized by the IFOPDT model. The SOPDT model can be obtained from  $k_u$ ,  $\omega_u$ ,  $\varphi$ ,  $A$ . In this case it is defined by

$$G_{SO}(s) = \frac{e^{-Ls}}{as^2 + bs + c}, \quad (26)$$

where parameters  $a$ ,  $b$ ,  $c$  and  $L$  are functions of  $k_u$ ,  $\omega_u$ ,  $\varphi$  and  $A$ , obtained from the tangent rule (Šekara & Mataušek, 2010a). This model (26) is an adequate SOPDT approximation of the Nyquist curve  $G_p(i\omega)$  in the region around the ultimate frequency  $\omega_u$ , for a large class of stable processes, processes with oscillatory dynamics, integrating and unstable processes.

The recently proposed new Phase-Locked Loop (PLL) estimator (Mataušek & Šekara, 2011), its improvement (Šekara & Mataušek, 2011c), and new relay SheMa estimator (Šekara & Mataušek 2011b) make possible determination of parameters  $k_u$ ,  $\omega_u$ ,  $\varphi$  and  $A_0$  of the model  $G_m(s)$  in the closed-loop experiments, without breaking the control loop in operation. This property of the proposed PLL and SheMa estimators is important for practice, since breaking of control loops in operation is mainly ignored by plant operators, especially in the case of controlling processes with oscillatory dynamics, integrating or unstable processes. The PLL estimator can be applied in the case when the controller in operation is an unknown linear controller, while the SheMa estimator can be applied when the controller in operation is unknown and nonlinear. In that sense, the SheMa estimator belongs to the direct CL system identification methods, based only on the plant input and output data, as in (Agüero et al., 2011). Both procedures, SheMa and PLL, are based on the parameterization presented in (Šekara & Mataušek, 2010a; Mataušek & Šekara, 2011). Estimates of parameters  $k_u^-$ ,  $k_u$ ,  $k_u^+$  and  $\omega_u^-$ ,  $\omega_u$ ,  $\omega_u^+$ , obtained for  $\arg G_p(i\omega) = -\pi + \varphi$ ,  $\varphi = 0$  and  $\varphi = \varphi^\pm = \pm\pi/36$ , are used for determining  $\varphi$  and  $A_0$ , as defined in (Mataušek & Šekara, 2011).

In this section, an improvement of the new PLL estimator from (Mataušek & Šekara, 2011) is presented in Fig. 8. The improvement, proposed by Šekara and Mataušek (2011c), consists of adding two integrators at the input to the PLL estimator from (Mataušek & Šekara, 2011). Inputs to these integrators are defined by outputs of the band-pass filters  $AF_1$ , used to eliminate the load disturbance. Outputs of these integrators are passed through a cascade of the band-pass filters  $AF_m$ ,  $m=2,3,4$ . All filters  $AF_m$ ,  $m=1,2,3,4$ , are tuned to the ultimate frequency. Such implementation of the PLL estimator eliminates the effects of the high measurement noise and load disturbance. Blocks  $AF_m$ ,  $j=1,2,3,4$ , are implemented as presented in (Mataušek & Šekara, 2011), while implementation of blocks for determining  $\arg\{G_p(i\omega)\}$  and  $|G_p(i\omega)|$  are presented in (Šekara & Mataušek, 2011c).

PLL estimator from Fig. 8 is applied to processes  $G_{ps}(s)=\exp(-s)/(2s+1)$  and  $G_{p0}(s)=4\exp(-2s)/(4s-1)$  in the loop with the known PID controller. Estimation of parameters  $k_u^-$ ,  $k_u$ ,  $k_u^+$  and  $\omega_u^-$ ,  $\omega_u$ ,  $\omega_u^+$  is presented in Fig. 9. Highly accurate estimates of  $k_u^-$ ,  $k_u$ ,  $k_u^+$  and  $\omega_u^-$ ,  $\omega_u$ ,  $\omega_u^+$  are obtained in the presence of the high measurement noise and load disturbance. Since these parameters are used to determine  $\varphi$  and  $A_0$ , this experiment demonstrates that highly accurate estimate of the quadruplet  $\{k_u, \omega_u, \varphi, A_0\}$  can be obtained, in the presence of the high measurement noise and load disturbance, by the PLL estimator from (Šekara & Mataušek 2011c). In Fig. 10, estimation of the unknown Nyquist curve of the unstable process in the loop with the PID controller is demonstrated.

The PLL estimator from (Mataušek & Šekara, 2011; Šekara & Mataušek 2011c) is a further development of the idea firstly proposed in (Crowe & Johnson, 2000) and used in (Clarke & Park, 2003). The SheMa estimator is a further development of the estimator proposed by Aström and Hägglund (1984) as an improvement of the Ziegler-Nichols experiment.

The Ziegler and Nichols (1942) experiment, used to determine  $k_u$  and  $\omega_u$  of a process is performed by setting the integral and derivative gains to zero in the PID controller  $C(s)$  in

operation. However, in this approach the amplitude of oscillations is not under control. This drawback is eliminated by Aström and Hägglund (1984). The factors influencing the critical point estimation accuracy in this conventional relay setup are: the use of describing function method is faced with the fact that higher harmonics are not efficiently filtered out by the process, presence of the load disturbance  $d$ , and presence of the measurement noise  $n$ . The first drawback of the conventional relay experiment is eliminated by the modified relay setup (Lee et al., 1995).

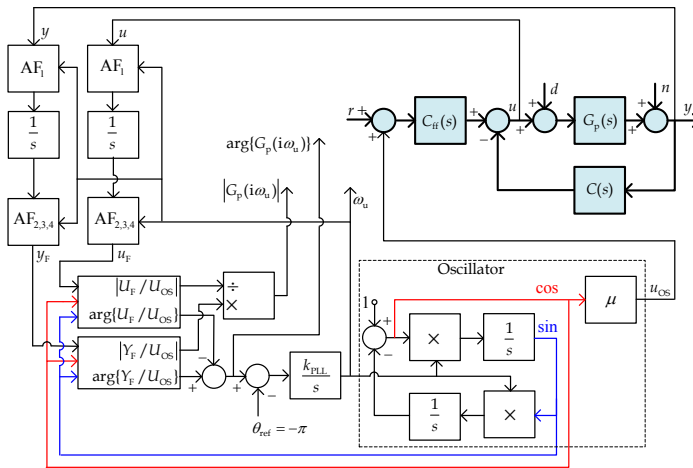


Fig. 8. Improved PLL estimator.  $AF_{2,3,4}$  is the cascade of band-pass filters  $AF_m, m=1,2,3,4$ .

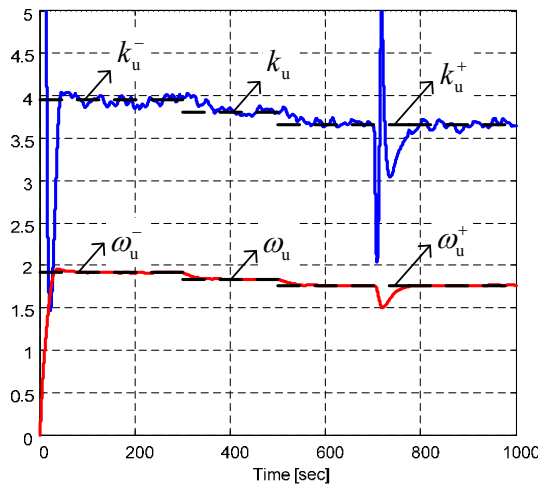


Fig. 9. PLL estimates of  $k_u^-, k_u, k_u^+$  and  $\omega_u^-, \omega_u, \omega_u^+$ , in the presence of the high measurement noise and step load disturbance at  $t=700$  s. Process  $G_{ps}(s)=\exp(-s)/(2s+1)$ , for:  $\phi^- = -\pi/36$  for  $0 \leq t \leq 300$  s,  $\phi = 0$  for  $300 < t \leq 500$  s and  $\phi^+ = \pi/36$  for  $500 < t \leq 1000$  s.



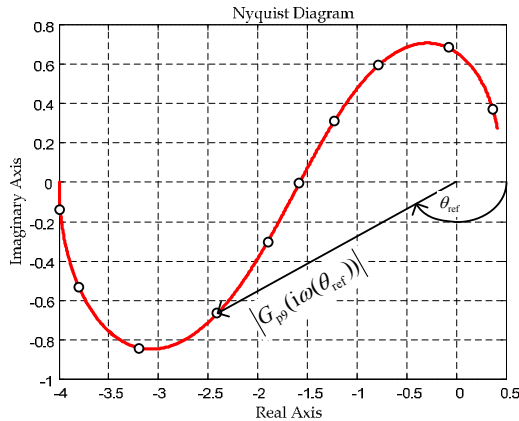


Fig. 10. Estimates (circles) of the Nyquist curve (solid) obtained by the PLL estimator for the desired values  $\theta_{ref} = \arg\{G_p(i\omega)\}$ . Process  $G_p(s) = 4\exp(-2s)/(4s-1)$ , the noise-free case.

Due to its simplicity, the relay-based setup proposed by Åström and Hägglund (1984) is still a basic part of different methods developed in the area of process dynamics characterization. For example, it is used to generate signals to be applied for determining FOPDT and SOPDT models, using a biased relay (Hang et al., 2002). However, from the viewpoint of the process control system in operation, the estimation based on this setup, and its modifications, is performed in an open-loop configuration: the loop with the controller  $C(s)$  in operation is opened and the process output is connected in feedback with a relay.

In the paper (Šekara & Mataušek, 2011b) a new relay-based setup is developed, with the controller  $C(s)$  in operation. It consists of a cascade of variable band-pass filters  $AF_{m_r}$  from (Clarke & Park, 2003), a new variable band-pass filter  $F_{mod}$  proposed by Šekara and Mataušek (2011b) and a notch filter  $F_{NF} = 1 - F_{mod}$ . Center frequencies of variable band-pass filters  $AF_m$  and  $F_{mod}$  are at  $\omega_{li}$ .

Highly accurate estimates of  $\omega_{li}$  and  $k_{li}$  are obtained in the presence of the measurement noise and load disturbance. Also, highly accurate estimates of the Nyquist curve  $G_p(i\omega)$  at the desired values of  $\arg\{G_p(i\omega)\}$  are obtained by including into the SheMa the modified relay instead of the ordinary relay. The amplitude  $\mu$  of both relays is equal to  $\mu = \pi k_{u,0} y_{ref} \varepsilon_0 / 4$ , where  $k_{u,0}$  is the ultimate gain obtained in the previous activation of the SheMa,  $y_{ref}$  is the amplitude of the set-point  $r$  and  $\varepsilon_0$  is a small percent of  $y_{ref}$ , for example  $\varepsilon_0 = 0.1\%$  in the examples presented in (Šekara & Mataušek, 2011b). The proposed closed-loop procedure can be activated or deactivated with small impact on the controlled process output. Further details of the SheMa estimator, including the stability and robustness analyses, and implementation details, are presented in (Šekara & Mataušek 2011b).

### 5. Gain scheduling control of stable, integrating, and unstable processes, based on the controller optimization in the classification parameter plane

For a chosen region in the  $\rho$ - $\varphi$  classification plane, presented in Fig. 11, the normalized parameters  $k_n(\rho, \varphi)$ ,  $k_{in}(\rho, \varphi)$ ,  $k_{dn}(\rho, \varphi)$  and  $T_{in} = |k_{dn}(\rho, \varphi)| / m_n$  of a virtual  $PID_n$  controller are calculated in advance by using the process-independent model  $G_n(i\omega_n, \rho, \varphi)$  in (10).

Then, parameters  $k$ ,  $k_i$ ,  $k_d$  and  $T_f$  of the PID controller (3),  $F_C(s) \equiv 1$ , are obtained, for the process classified in the chosen region of the  $\rho$ - $\varphi$  plane, by using the estimated  $k_u$ ,  $\omega_u$ ,  $\varphi$ ,  $A$  and the following relations

$$k = k_u k_n, \quad k_i = k_u \omega_u k_{in}, \quad k_d = k_u k_{dn} / \omega_u, \quad T_f = T_{fn} / \omega_u. \quad (27)$$

Depending on the method applied to obtain parameters  $k_n$ ,  $k_{in}$ ,  $k_{dn}$  and  $T_{fn} = |k_{dn}| / m_n$  of a PID<sub>n</sub> controller, parameters  $k$ ,  $k_i$ ,  $k_d$  and  $T_f$  of the PID controller (3),  $F_C(s) \equiv 1$ , guarantee the desired  $M_s$  and the sensitivity to measurement noise equal to  $M_n = |k_u| m_n$ , or guarantee the

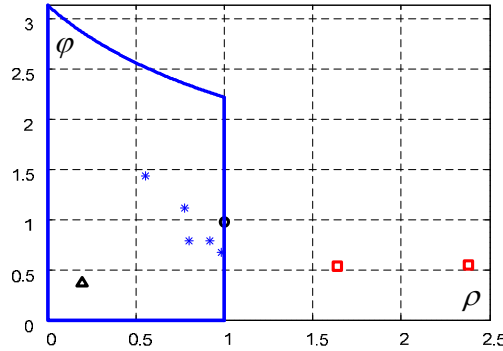


Fig. 11. Classification  $\rho$ - $\varphi$  parameter plane, with processes  $G_{pj}(s)$ ,  $j=1,2,\dots,9$ . Stable processes are classified in the region  $0 < \rho < 1$ ,  $0 < \varphi < \pi / \sqrt{\rho + 1}$ , integrating processes are classified as  $\rho = 1$ ,  $0 < \varphi < \pi / \sqrt{2}$  processes. Unstable processes are classified outside this region.

desired  $M_s$ ,  $\zeta$  and  $M_n = |k_u| m_n$ . Since parameters  $k_n$ ,  $k_{in}$ ,  $k_{dn}$  and  $T_{fn} = |k_{dn}| / m_n$  are determined in advance, they can be memorized as look-up tables in the  $\rho$ - $\varphi$  plane. Besides, this can be done for different values of  $M_s$ ,  $m_n$  and  $\zeta$ . These look-up tables define a new Gain Scheduling Control (GSC) concept. Important feature of this GSC is that these look-up tables, obtained for some values of  $M_s$ ,  $m_n$  and  $\zeta$  from the model  $G_n(i\omega_n, \rho, \varphi)$ , are process-independent. Enormous resources are avoided, required for performing experiments on the plant in order to define the standard GSC as the look-up tables of PID controller parameters for this plant and the desired region of operating regimes. Thus, the important and exclusive feature of the new GSC is that a desired performance/robustness tradeoff can be obtained for a large region of dynamic characteristics of processes in different plants and different operating regimes, covered by the look-up tables of parameters  $k_n$ ,  $k_{in}$ ,  $k_{dn}$  in the  $\rho$ - $\varphi$  classification plane.

Now, this GSC PID controller tuning, performed by using (27), will be demonstrated by the two different procedures applied for obtaining parameters  $k_n$ ,  $k_{in}$ ,  $k_{dn}$  and  $T_{fn} = |k_{dn}| / m_n$  of the PID<sub>n</sub> controller for integrating and stable processes. Stable processes having a weakly damped impulse response are denoted as processes having oscillatory dynamics, while processes with damped impulse response are denoted as stable processes.

For integrating processes, parameters  $k_n$ ,  $k_{in}$ ,  $k_{dn}$  and  $T_{fn} = |k_{dn}| / m_n$  of the PID<sub>n</sub> controller depend only on angle  $\varphi$ , since  $\rho=1$ . In this case, for desired values of  $M_s$  and  $m_n$ , PID controller parameters (27) are obtained from tuning formulae for  $k_n(\varphi)$ ,  $k_{in}(\varphi)$  and  $k_{dn}(\varphi)$

(Šekara & Mataušek, 2011a). Thus, for integrating process  $G_{p6}(s)$  parameters of the PID<sub>n</sub> controller are obtained by applying angle  $\varphi=0.9716$  in the tuning formulae defined in (Šekara & Mataušek, 2011a) for  $M_s=2$  and  $m_n=2$ , given in Appendix as tun1. The results are presented in Table 5,  $G_{p6}$ -tun1.

For processes having the oscillatory dynamics look-up tables and tuning formulae are derived in (Šekara & Mataušek, 2011a) for  $M_s=2$  and  $m_n=40$ , in the region  $0.1 \leq \rho \leq 0.2$ ,  $0.1745 \leq \varphi \leq 1.0472$  of the  $\rho$ - $\varphi$  classification plane of Fig. 11. These tuning formulae, in Appendix denoted as tun2, are applied to determine parameters  $k$ ,  $k_i$ ,  $k_d$  and  $T_f$  for the process having the oscillatory dynamics  $G_{p5}(s)$ , classified as process  $\rho=0.1971$ ,  $\varphi=0.3679$  (Table 5,  $G_{p5}$ -tun2). To illustrate the direct application of the look-up tables from (Šekara & Mataušek, 2011a, Table A4) and interpolation procedure defined in Appendix, Fig. 17, since this process is classified as  $\rho=0.1971$ ,  $\varphi=21.0791^\circ$  ( $0.3679$ ), the following points are determined from (Šekara & Mataušek, 2011a, Table A4) and Appendix, Fig. 17:  $\rho_{1,1}=0.15$ ,  $\varphi_{1,1}=20^\circ$ ,  $\rho_{1,2}=0.2$ ,  $\varphi_{1,2}=20^\circ$  and  $\rho_{2,2}=0.2$ ,  $\varphi_{2,2}=30^\circ$ . Parameters ( $k_{in}$ ,  $k_{in}$ ,  $k_{dn}$ ) are defined by: (-2.4122, 0.5988, 3.9353) for  $\rho_{1,1}$ ,  $\varphi_{1,1}$ , (-1.7022, 0.4125, 2.8783) for  $\rho_{1,2}$ ,  $\varphi_{1,2}$  and (-1.6626, 0.4164, 2.3017) for  $\rho_{2,2}$ ,  $\varphi_{2,2}$ . Then, by using three point interpolation from Appendix, upper triangle ( $a_{ru}=0.0578$ ,  $\beta_{ru}=0.1971$ ), one obtains parameters in Table 5,  $G_{p5}$ -GSC:  $k=-0.4220$ ,  $k_i=0.0384$ ,  $k_d=1.9116$ ,  $T_f=1.947$ .

For stable processes, in a large region of the  $\rho$ - $\varphi$  plane, look-up tables of parameters  $k_n$ ,  $k_{in}$  and  $k_{dn}$  are defined for  $M_s=2$  and  $m_n=2$  (Šekara & Mataušek, 2011a, Tables A1-A3). These look-up tables are applied in the present paper to determine parameters  $k$ ,  $k_i$ ,  $k_d$  and  $T_f$  for the stable process  $G_{p3}(s)$ . This process is classified as process  $\rho=0.9808$ ,  $\varphi=0.6783$  ( $38.8637^\circ$ ). Thus, for  $G_{p3}(s)$  parameters ( $k_n$ ,  $k_{in}$ ,  $k_{dn}$ ) can be obtained from the three points in the  $\rho$ - $\varphi$  classification plane (Appendix, Fig. 17):  $\rho_{1,1}=0.95$ ,  $\varphi_{1,1}=30^\circ$ ;  $\rho_{2,1}=0.95$ ,  $\varphi_{2,1}=40^\circ$  and  $\rho_{2,2}=1$ ,  $\varphi_{2,2}=40^\circ$  ( $0.6981$ ). Two points are used for stable processes (0.5086, 0.1349, 0.6569) for  $\rho_{1,1}$ ,  $\varphi_{1,1}$  and (0.5013, 0.1261, 0.5332) for  $\rho_{2,1}$ ,  $\varphi_{2,1}$  from the look-up tables (Šekara & Mataušek, 2011a, Tables A1-A3), while data (0.5036, 0.1109, 0.5332) for  $\rho_{2,2}$ ,  $\varphi_{2,2}$  are obtained from tuning formulae derived for integrating processes in (Šekara & Mataušek, 2011a), given in Appendix as tun1. Then, by using three point interpolation from Appendix, Fig. 17 lower triangle ( $a_{ll}=0.6166$ ,  $\beta_{ll}=0.1136$ ), one obtains parameters presented in Table 5,  $G_{p3}$ -GSC:  $k=17.0973$ ,  $k_i=0.2307$ ,  $k_d=315.2928$  and  $T_f=4.6430$ .

Process-method	$k$	$k_i$	$k_d$	$T_f$	IAE	$M_n$	$M_s$	$M_p$
$G_{p3}$ -GSC	17.0973	0.2307	315.29	4.6430	4.84	67.91	2.00	1.54
$G_{p5}$ -tun2	-0.4220	0.0380	1.8758	0.1910	26.32	9.82	1.99	1.08
$G_{p5}$ -GSC	-0.4269	0.0384	1.9116	0.1947	26.04	9.82	2.01	1.09
$G_{p6}$ -tun1	0.1182	0.0054	0.3746	0.7970	209.10	0.47	2.00	1.62

Table 5. PID controllers: stable process  $G_{p3}(s)$ , method GSC-Appendix; stable process having oscillatory dynamics  $G_{p5}(s)$ , method tun2 and method GSC-Appendix; integrating process  $G_{p6}(s)$ , method tun1.

### 5.1 Experimental results

Experimental results, presented in Fig. 12, are obtained by using the laboratory thermal plant. It consists of a thin plate made of aluminum,  $L_a=0.1\text{m}$  long and  $h=0.03\text{m}$  wide (Mataušek & Ribić, 2012). Temperature  $T(x,t)$  is distributed along the plate, from  $x=0$  to

$x=L_a$ , and measured by precision sensors LM35 (TO92), at  $x=0$  and  $x=L_a$ . The plate is heated by a terminal adjustable regulator LM317 (TO 220) at position  $x=0$ . The manipulated variable is the dissipated power of the heater at  $x=0$ . The input to the heater is the control variable  $u(t)$  (%), defined by the output of the PID controller. The controlled variable is  $y(t)=T(L_a,t)$ , measured by the sensor at position  $x=L_a$ . Temperature sensor at  $x=0$  is used in the safety device, to prevent overheating when  $70^\circ\text{C} \leq T(0,t)$ . The anti-windup implementation of the PID controller (3),  $F_C(s)\equiv 1$ , is given by

$$u_C = T_{aw} \left( \frac{bks + k_i}{T_{aw}s + 1} r - \frac{k_d s^2 + ks + k_i}{(T_{aw}s + 1)(T_f s + 1)} y \right) + \frac{1}{T_{aw}s + 1} u \cdot \tag{28}$$

The saturation element is defined by the input  $u_C(t)$  and output  $u(t)$ :

$$u = \begin{cases} l_{low}, & u_C \leq l_{low} \\ u_C, & l_{low} < u_C < l_{high} \\ l_{high}, & u_C \geq l_{high} \end{cases} \tag{29}$$

Obviously, in the linear region  $l_{low} < u_C(t) < l_{high}$  of the saturation element, for  $u_C(t) \equiv u(t)$  one obtains (3),  $F_C(s) \equiv 1$ , from (28).

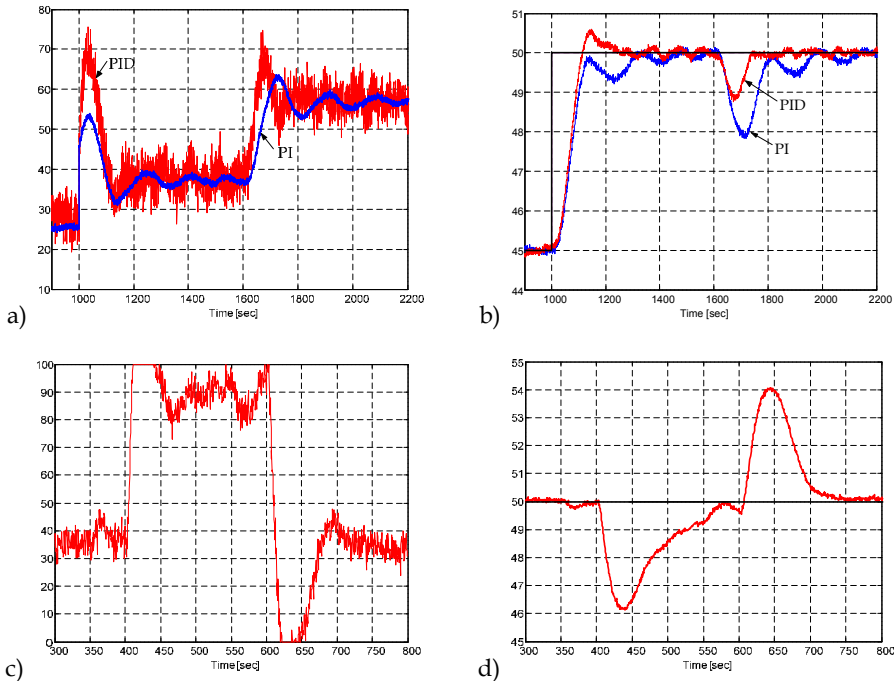


Fig. 12. Experimental results. Set-point and load step (-20% change of the controller output at  $t=1600$  s) responses of the real plant, with the PI and PID controller: a) control variable  $u(t)$  and b) controlled variable  $y(t)$ . The real plant, with the anti-windup PID controller under the disturbance induced by activating/deactivating the fan: c)  $u(t)$  and d)  $y(t)$ .

Transfer function  $G_{p3}(s)$ , used for determining parameters of the PID controller applied in the real-time experiment, is obtained previously in (Mataušek & Ribić, 2012). By applying a Pseudo-Random-Binary-Sequence for  $u(t)$ , the open-loop response  $y(t)$  of the laboratory thermal plant is obtained. From these  $u(t)$  and  $y(t)$  a 100-th order ARX model is determined and reduced then to the 5-th order transfer function  $G_{p3}(s)$  in (Mataušek & Ribić, 2012). This model of the process is used here to determine the quadruplet  $\{k_u, \omega_u, \varphi, A\}$  presented in the Appendix. Thus, the laboratory thermal plant is classified as the process  $\rho=0.9808, \varphi=0.6783$ . Then, PID controller applied to the real thermal plant is determined by using look-up tables of parameters  $k_n(\rho, \varphi), k_{in}(\rho, \varphi), k_{dn}(\rho, \varphi)$ , for stable processes, and parameters  $k_n(\varphi), k_{in}(\varphi), k_{dn}(\varphi)$ , for integrating processes, previously determined in (Šekara & Mataušek, 2011a). This procedure, used to obtain PID in Table 5, row  $G_{p3}$ -GSC, and results obtained by this PID controller, presented in Fig. 12, demonstrate that in advance determined look-up tables of parameters  $k_n, k_{in}$  and  $k_{dn}$  defines a process-independent GSC applicable for obtaining the desired performance/robustness tradeoff for a real plant classified in the  $\rho$ - $\varphi$  parameter plane. For  $T_i=k/k_i$  and  $T_d=k_d/k$ , parameter  $T_{aw}=15s$  is obtained from  $T_{aw}=pT_i+(1-p)T_d$ , for  $p=0.2$ , and  $l_{low}=0, l_{high}=100\%, b=0.25$ .

Closed-loop experiment in Fig. 12-a and Fig. 12-b is used to demonstrate advantages of the designed PID controller, compared with the PI controller, from Table 4, row  $G_{p3}/opt3$  defined by:  $k=8.1355, k_i=0.0679$ , and  $b=0.5$ . This experiment starts from temperature  $T(L_a, t) \approx 45^\circ C$ , as presented in Fig. 12-b. Then at  $t=1000$  s the set point is changed to  $r=45^\circ C+r_0, r_0=5^\circ C$ . At  $t=1600$  s a load disturbance is inserted as a step change of the controller output equal to -20%. Improvement of the performance obtained by the PID controller is evident. As expected, this is obtained with the greater variation of the control signal  $u_{PID}(t)$  than that obtained by  $u_{PI}(t)$ . This is the reason why PID controller from Table 2, row  $tun_{\lambda_u}$ , having a greater value of  $M_n=217.7$ , is not applied to the real thermal plant.

The closed-loop experiment presented in Fig. 12-c and Fig. 12-d starts from the steady state temperature  $T(L_a, t) \approx 50^\circ C$  by activating a fan at  $t=400$  s. Then, at  $t=600$  s the fan is switched-off. Action of the fan induced a strong disturbance, as seen from the control signal  $u(t)$  in Fig. 12-c. It should be observed that anti-windup action is activated two times, around 410 s and 625 s. Anti-windup action is effective and rejection of the disturbance is fast, as seen from Fig. 18-d.

## 6. Conclusion

The extension of the Ziegler-Nichols process dynamics characterization, developed in (Šekara & Mataušek, 2010a; Mataušek & Šekara, 2011), is defined by the model (5). Based on this model, a procedure is derived for classifying a large class of stable, integrating and unstable processes into a two-parameter  $\rho$ - $\varphi$  classification plane (Šekara & Mataušek, 2011a). As a result of this classification, a new CSC concept is developed. In the  $\rho$ - $\varphi$  classification plane, parameters  $g_n(\rho, \varphi)=\{k_n(\rho, \varphi), k_{in}(\rho, \varphi), k_{dn}(\rho, \varphi)\}$  and  $T_{in}(\rho, \varphi)=|k_{dn}(\rho, \varphi)|/m_n$ , of a virtual  $PID_n$  controller can be calculated in advance, to satisfy robustness defined by  $M_s$  and sensitivity to measurement noise defined by  $m_n$ . Also it is possible to satisfy  $M_s, m_n$  and the closed-loop system damping ratio  $\zeta$ . Calculation of parameters  $g_n(\rho, \varphi)$  and  $T_{in}(\rho, \varphi)$  is process-independent. The calculation is performed by using model  $G_n(s_n, \rho, \varphi)$ , defined by the values of  $\rho$  and  $\varphi$  for stable processes in the range  $0 < \rho < 1, 0 < \varphi < \pi / \sqrt{\rho + 1}$ , for integrating processes in the range  $\rho = 1, 0 < \varphi < \pi / \sqrt{2}$ , for unstable processes by the values of the  $\rho$  and  $\varphi$  outside these regions.

Parameters  $g_n(\rho, \varphi)$ , calculated for a given region in the  $\rho$ - $\varphi$  classification plane, are memorized as process-independent look-up tables. Then, for the process  $G_p(s)$  classified into this region of the  $\rho$ - $\varphi$  classification plane, parameters of a real PID controller  $k, k_i, k_d, T_i$  are obtained directly from  $g_n(\rho, \varphi)$ ,  $T_{in}(\rho, \varphi)$  and the estimated quadruplet  $\{k_u, \omega_u, \varphi, A\}$  or  $\{k_u, \omega_u, \varphi, A_0\}$  for stable/unstable processes, and the triplet  $\{k_u, \omega_u, \varphi\}$  for integrating processes. It is demonstrated by simulations that for the real  $M_n$  equal to  $M_n = |k_u| m_n$ , the desired  $M_s$  and  $\zeta$  are obtained when a real PID controller, obtained by the proposed GSC, is applied to the process  $G_p(s)$ . The desired performance/robustness tradeoff can be accurately predicted. Namely, performance index IAE and robustness index  $M_s$ , obtained on the model  $G_m(s)$  in (5) are almost the same as those obtained for the process  $G_p(s)$ , as confirmed here and by a large test batch considered in (Šekara & Mataušek, 2010a; Mataušek & Šekara, 2011; Šekara & Mataušek, 2011a).

A set of new constrained PID optimization techniques is derived for determining the four parameters  $k, k_i, k_d, T_i$  of the PID controller. The one of them has a unique property. The unknown parameters are obtained as the solution of only two nonlinear algebraic equations, with the good initial values of the unknown two parameters, determined to satisfy the desired values  $M_s$  and  $M_n$ , given desired value of the closed-loop system damping ratio  $\zeta$ . Thus, the critically damped closed-loop system response is obtained for  $\zeta=1$ . Two extensions of the PLL-based and relay-based procedures are derived in (Mataušek & Šekara, 2011; Šekara & Mataušek, 2010b; 2011c; 2011b) for determining the quadruplet  $\{k_u, \omega_u, \varphi, A_0\}$ . These procedures can be applied for the closed-loop PID controller tuning/retuning, in the presence of the measurement noise and load disturbance, without breaking the loop of the controller in operation.

Process-independent look-up tables of parameters  $g_n(\rho, \varphi)$ , defining the process-independent GSC, can be applied by using any process dynamics characterization defined by the estimated frequency response of the process around the ultimate frequency. This is demonstrated in the present chapter by applying a model obtained previously by a high-order ARX identification of a laboratory thermal plant, and reduced then to the fifth order  $G_{p3}(s)$ , used here to determine the quadruplet  $\{k_u, \omega_u, \varphi, A\}$ . This quadruplet is applied to determine parameters of the real PID, by using the look-up tables of parameters  $g_n(\rho, \varphi)$  calculated previously in (Šekara & Mataušek, 2011a). As confirmed by the experimental results, the method of the proposed process-independent GSC is effective. Finally, it is believed that material presented in this chapter will initiate further development of the proposed process-independent GSC and its implementation in advanced controllers.

## 7. Appendix

Parameters  $\eta_1, \eta_2, \beta_1, \beta_2$  and  $\beta_3$

$$\eta_1 = \frac{\alpha_1 \sin(\omega_u \tau) + \alpha_2 \cos(\omega_u \tau)}{\omega_u^2}, \quad \eta_2 = \frac{\alpha_2 \sin(\omega_u \tau) - \alpha_1 \cos(\omega_u \tau) + 1}{\omega_u^2},$$

$$\alpha_1 = \lambda^4 \omega_u^4 - 2\lambda^2 \omega_u^2 (1 + 2\zeta^2) + 1, \quad \alpha_2 = 4\zeta \lambda \omega_u (1 - \lambda^2 \omega_u^2),$$

$$\beta_1 = \frac{\omega_u}{A(4\zeta \lambda + \tau - \eta_1)}, \quad \beta_2 = \frac{2\lambda^2(1 + 2\zeta^2) - \tau^2 / 2 + \eta_1 \tau - \eta_2}{4\zeta \lambda + \tau - \eta_1}, \quad \beta_3 = \frac{4\zeta \lambda^3 + \tau^3 / 6 - \eta_1 \tau^2 / 2 + \eta_2 \tau}{4\zeta \lambda + \tau - \eta_1},$$

are presented here, to make possible to repeat the results obtained by the PID optimization from (Mataušek & Šekara, 2011).

Tuning formulae tun1, for integrating processes for  $M_s=2$  and  $m_n=2$ , given by

$$\begin{bmatrix} k_n \\ k_{in} \\ k_{dn} \end{bmatrix} = \begin{bmatrix} 0.5904 & -0.2707 & 0.3029 & -0.1554 & 0.0311 \\ 0.1534 & -0.0826 & 0.0409 & -0.0164 & 0.0033 \\ 1.2019 & -1.5227 & 1.0714 & -0.4944 & 0.0916 \end{bmatrix} \begin{bmatrix} 1 \\ \varphi \\ \varphi^2 \\ \varphi^3 \\ \varphi^4 \end{bmatrix},$$

and tun2, for processes with the oscillatory dynamics for  $M_s=40$  and  $m_n=2$ , given by

$$\begin{bmatrix} k_n \\ k_{in} \\ k_{dn} \end{bmatrix} = \begin{bmatrix} -8.9189 & 63.0913 & 0.6494 & -135.2567 & 0.2806 & -3.5564 \\ 2.2218 & -16.5791 & 0.1361 & 37.5733 & 0.0136 & -0.6388 \\ 14.8966 & -82.7969 & -9.0810 & 145.2467 & 0.9056 & 25.1221 \end{bmatrix} \begin{bmatrix} 1 \\ \rho \\ \varphi \\ \rho^2 \\ \varphi^2 \\ \rho\varphi \end{bmatrix},$$

are defined in (Šekara & Mataušek, 2011a). The angle  $\varphi$  is in radians.

Process	$k_u$	$\omega_u$	$\tau$	$A$	$A_0$	$\rho$	$\varphi$
Gp1	11.5919	9.8696	0.0796	9.0858	8.9190	0.9206	0.7854
Gp2	4	1	0.7854	0.8	0.7071	0.8	0.7854
Gp3	33.9538	0.0577	11.7521	0.0566	0.0519	0.9808	0.6783
Gp4	1.2494	0.4	3.5881	0.2222	0.2612	0.5555	1.4352
Gp5	0.2455	0.3695	0.9956	0.0728	0.0729	0.1971	0.3679
Gp6	0.2371	0.2291	4.2403	0.2291	-	1	0.9716
Gp7	0.8625	0.1333	4.1446	0.3173	0.3211	2.3793	0.5526
Gp8	3.8069	1.8366	0.6271	1.4545	1.6054	0.7920	1.1517
Gp9	0.6341	0.5828	0.9105	0.9621	1.0000	1.6509	0.5333

Table 6. Parameters of models  $G_{mj}(s)$  of processes  $G_{pj}(s), j=1,2,\dots,9$ .

Normalized parameters of the PID<sub>n</sub> controller can be obtained by interpolation based on the three points in the  $\rho$ - $\varphi$  look-up tables of the memorized parameters  $k_n(\rho_i, \varphi_j)$ ,  $k_{in}(\rho_i, \varphi_j)$  and  $k_{dn}(\rho_i, \varphi_j)$ ,  $i=1,2,\dots,I_{mv}$ ,  $j=1,2,\dots,J_{mv}$ , determined in advance. In the present paper the look-up tables from (Šekara & Mataušek, 2011a, Tables 1-4) are used. The four points mesh in the  $\rho$ - $\varphi$  look-up tables is presented in Fig. 13. The normalized parameters of the PID<sub>n</sub> controller, for the lower triangle are given by:

$$k_n = (1 - \alpha - \beta)k_{n2,1} + \alpha k_{n2,2} + \beta k_{n1,1}, \quad k_{in} = (1 - \alpha - \beta)k_{in2,1} + \alpha k_{in2,2} + \beta k_{in1,1},$$

$$k_{dn} = (1 - \alpha - \beta)k_{dn2,1} + \alpha k_{dn2,2} + \beta k_{dn1,1},$$

where  $a=\alpha_{il}$ ,  $\beta=\beta_{il}$ . The normalized parameters of the PID<sub>n</sub> controller, for the upper triangle are given by:

$$k_n = (1 - \alpha - \beta)k_{n1,2} + \alpha k_{n1,1} + \beta k_{n2,2}, \quad k_{in} = (1 - \alpha - \beta)k_{in1,2} + \alpha k_{in1,1} + \beta k_{in2,2},$$

$$k_{dn} = (1 - \alpha - \beta)k_{dn1,2} + \alpha k_{dn1,1} + \beta k_{dn2,2},$$

where  $a=\alpha_{ru}$ ,  $\beta=\beta_{ru}$ . In both cases  $T_{in} = k_{dn} / m_n$ . Then, parameters of the PID controller are obtained from (27).

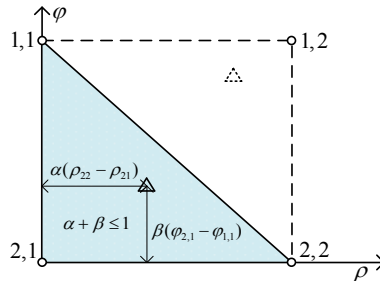


Fig. 13. The four point mesh in the  $\rho$ - $\varphi$  plane in (Šekara & Mataušek, 2011a, Tables 1-4). For lower triangle  $\alpha_{il}=(\rho_{est}-\rho_{2,1})/(\rho_{2,2}-\rho_{2,1})$  and  $\beta_{il}=(\varphi_{2,1}-\varphi_{est})/(\varphi_{2,1}-\varphi_{1,1})$ . For the upper triangle  $\alpha_{ru}=(\rho_{1,2}-\rho_{est})/(\rho_{1,2}-\rho_{1,1})$   $\beta_{ru}=(\varphi_{est}-\varphi_{1,2})/(\varphi_{2,2}-\varphi_{1,2})$ . All angles are in degrees and  $\varphi_{1,1} \leq \varphi_{est} \leq \varphi_{2,1}$ ,  $\rho_{1,1} \leq \rho_{est} \leq \rho_{2,1}$ .

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